

GCE Examinations
Advanced Subsidiary

Core Mathematics C3

Paper F

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks could be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

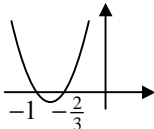
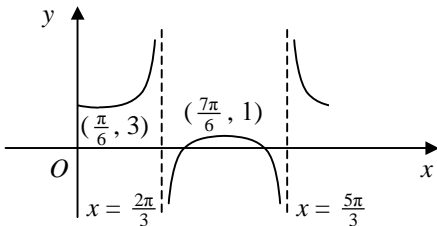


Written by Shaun Armstrong

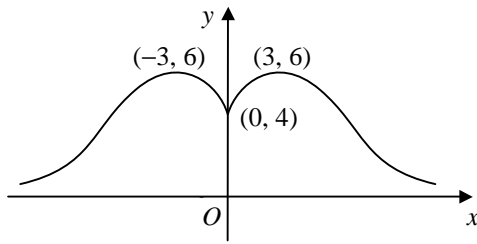
© *Solomon Press*

These sheets may be copied for use solely by the purchaser's institute.

C3 Paper F – Marking Guide

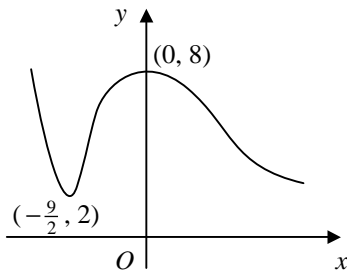
<p>1. $\frac{3}{\sin \theta} = -8 \cos \theta$ $3 = -8 \sin \theta \cos \theta = -4 \sin 2\theta$ $\sin 2\theta = -\frac{3}{4}$ $2\theta = 180 + 48.590, 360 - 48.590 = 228.590, 311.410$ $\theta = 114.3, 155.7$ (1dp)</p>	<p>M1 M1 A1 M1 A2</p>	<p>(6)</p>
<p>2. (a) $g(x) = (x+a)^2 - a^2 + 2$ $\therefore g(x) \geq 2 - a^2$</p> <p>(b) $gf(3) = g(1-3a) = (1-3a)^2 + 2a(1-3a) + 2$ $\therefore 1 - 6a + 9a^2 + 2a - 6a^2 + 2 = 7, \quad 3a^2 - 4a - 4 = 0$ $(3a+2)(a-2) = 0$ $a = -\frac{2}{3}, 2$</p>	<p>M1 A1 A1 M1 A1 M1 A1</p>	<p>(7)</p>
<p>3. (a) $3x + 1 = e^2$ $x = \frac{1}{3}(e^2 - 1)$</p> <p>(b) consider $\ln(3x^2 + 5x + 3) \geq 0$ $\Rightarrow 3x^2 + 5x + 3 \geq 1$ $3x^2 + 5x + 2 \geq 0$ $(3x+2)(x+1) \geq 0$ $x \leq -1$ or $x \geq -\frac{2}{3}$</p> <p>\therefore if (e.g.) $x = -\frac{3}{4}, \ln(3x^2 + 5x + 3) = \ln \frac{15}{16} = -0.0645\dots$ \therefore if $x = -\frac{3}{4}, \ln(3x^2 + 5x + 3) < 0 \quad \therefore$ statement is false</p>	<p>M1 M1 A1 M1 M1 A1 M1 A1</p> 	<p>(8)</p>
<p>4. (a) $\frac{dx}{dy} = 1 \times \sqrt{1-2y} + y \times \frac{1}{2}(1-2y)^{-\frac{1}{2}} \times (-2)$ $= \sqrt{1-2y} - \frac{y}{\sqrt{1-2y}} = \frac{(1-2y)-y}{\sqrt{1-2y}} = \frac{1-3y}{\sqrt{1-2y}}$ $\frac{dy}{dx} = 1 \div \frac{dx}{dy} = \frac{\sqrt{1-2y}}{1-3y}$</p> <p>(b) $y = -1, x = -\sqrt{3}, \text{grad} = \frac{1}{4}\sqrt{3}$ $\therefore y + 1 = \frac{1}{4}\sqrt{3}(x + \sqrt{3})$ $4y + 4 = \sqrt{3}x + 3$ $\sqrt{3}x - 4y - 1 = 0 \quad [p = -4, q = -1]$</p>	<p>M1 A1 M1 M1 A1 B1 M1 A1</p>	<p>(8)</p>
<p>5. (a) </p> <p>(b) $2 + \sec(x - \frac{\pi}{6}) = 0, \quad \sec(x - \frac{\pi}{6}) = -2, \quad \cos(x - \frac{\pi}{6}) = -\frac{1}{2}$ $x - \frac{\pi}{6} = \pi - \frac{\pi}{3}, \pi + \frac{\pi}{3} = \frac{2\pi}{3}, \frac{4\pi}{3}$ $x = \frac{5\pi}{6}, \frac{3\pi}{2}$</p>	<p>M2 A3 M1 B1 M1 A2</p>	<p>(10)</p>

6. (a)

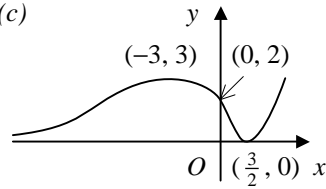


B3

(b)



(c)



M2 A2

M2 A2 (11)

7. (a)

$$\begin{aligned} f(x) &= 1 + \frac{4x}{2x-5} - \frac{15}{(2x-5)(x-1)} \\ &= \frac{2x^2 - 7x + 5 + 4x(x-1) - 15}{(2x-5)(x-1)} \\ &= \frac{6x^2 - 11x - 10}{(2x-5)(x-1)} = \frac{(3x+2)(2x-5)}{(2x-5)(x-1)} = \frac{3x+2}{x-1} \end{aligned}$$

B1

M1 A1

M1 A1

(b)

$$y = \frac{3x+2}{x-1}, \quad y(x-1) = 3x+2$$

M1

$$x(y-3) = y+2$$

M1

$$x = \frac{y+2}{y-3}$$

$$\therefore f^{-1}(x) = \frac{x+2}{x-3}$$

A1

$$f(x) = \frac{3(x-1)+5}{x-1} = 3 + \frac{5}{x-1}$$

M1

$$x < 1 \quad \therefore f(x) < 3 \quad \therefore \text{domain of } f^{-1}(x) \text{ is } x \in \mathbb{R}, x < 3$$

A1

(c)

$$f(x) = 2 \Rightarrow x = f^{-1}(2) = -4$$

M1 A1 (12)

8. (a)

$$\frac{dy}{dx} = 2x - \frac{1}{2}(4 + \ln x)^{-\frac{1}{2}} \times \frac{1}{x} = 2x - \frac{1}{2x\sqrt{4 + \ln x}}$$

M1 A1

$$x = 1, \quad y = -1, \quad \text{grad} = \frac{7}{4}$$

A1

$$\therefore y + 1 = \frac{7}{4}(x - 1)$$

M1

$$4y + 4 = 7x - 7$$

$$7x - 4y = 11$$

A1

(b)

$$\text{SP: } 2x - \frac{1}{2x\sqrt{4 + \ln x}} = 0$$

M1

$$\text{let } f(x) = 2x - \frac{1}{2x\sqrt{4 + \ln x}}, \quad f(0.3) = -0.40, \quad f(0.4) = 0.088$$

M1

sign change, $f(x)$ continuous \therefore root

A1

(c)

$$2x - \frac{1}{2x\sqrt{4 + \ln x}} = 0 \Rightarrow 2x = \frac{1}{2x\sqrt{4 + \ln x}}$$

$$x^2 = \frac{1}{4\sqrt{4 + \ln x}} = \frac{1}{4}(4 + \ln x)^{-\frac{1}{2}}$$

M1

$$x = \sqrt{\frac{1}{4}(4 + \ln x)^{-\frac{1}{2}}} = \frac{1}{2}(4 + \ln x)^{-\frac{1}{4}}$$

A1

(d)

$$x_1 = 0.38151, \quad x_2 = 0.37877, \quad x_3 = 0.37900, \quad x_4 = 0.37898 \quad (5\text{dp})$$

M1 A2 (13)

Total (75)

