

GCE Examinations  
Advanced Subsidiary

## **Core Mathematics C3**

Paper H

### **MARKING GUIDE**

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks could be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

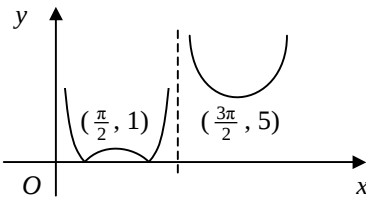


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### C3 Paper H – Marking Guide

- |       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |       |      |
|-------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|------|
| 1.    | <p>(a) <math>= f(2) = -2</math></p> <p>(b) <math>gf(x) = g(2 - x^2) = \frac{3(2 - x^2)}{2(2 - x^2) - 1} = \frac{6 - 3x^2}{3 - 2x^2}</math></p> <p><math>\therefore \frac{6 - 3x^2}{3 - 2x^2} = \frac{1}{2}, \quad 2(6 - 3x^2) = 3 - 2x^2</math></p> <p><math>x^2 = \frac{9}{4}, \quad x = \pm \frac{3}{2}</math></p>                                                                                                                                                                                                                                                                                      | M1 A1 |      |
|       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | M1 A1 |      |
|       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | M1 A1 | (6)  |
| <hr/> |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |       |      |
| 2.    | <p><math>5(\sec^2 2\theta - 1) - 13 \sec 2\theta = 1</math></p> <p><math>5 \sec^2 2\theta - 13 \sec 2\theta - 6 = 0</math></p> <p><math>(5 \sec 2\theta + 2)(\sec 2\theta - 3) = 0</math></p> <p><math>\sec 2\theta = -\frac{2}{5} \text{ or } 3</math></p> <p><math>\cos 2\theta = -\frac{5}{2} \text{ (no solutions) or } \frac{1}{3}</math></p> <p><math>2\theta = 70.529, 360 - 70.529, 360 + 70.529, 720 - 70.529</math></p> <p><math>= 70.529, 289.471, 430.529, 649.471</math></p> <p><math>\theta = 35.3^\circ, 144.7^\circ, 215.3^\circ, 324.7^\circ \text{ (1dp)}</math></p>                    | M1    |      |
|       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | M1    |      |
|       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | A1    |      |
|       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | B1 M1 |      |
|       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | A2    | (7)  |
| <hr/> |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |       |      |
| 3.    | <p>(a) <math>= \frac{(2x-3)(x+3)}{(2x-3)(x-2)} = \frac{x+3}{x-2}</math></p> <p>(b) <math>\ln(2x^2 + 3x - 9) - \ln(2x^2 - 7x + 6) = 2, \quad \ln \frac{2x^2 + 3x - 9}{2x^2 - 7x + 6} = 2</math></p> <p><math>\ln \frac{x+3}{x-2} = 2, \quad \frac{x+3}{x-2} = e^2</math></p> <p><math>x + 3 = e^2(x - 2)</math></p> <p><math>3 + 2e^2 = x(e^2 - 1)</math></p> <p><math>x = \frac{2e^2 + 3}{e^2 - 1}</math></p>                                                                                                                                                                                             | M1 A2 |      |
|       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | M1    |      |
|       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | A1    |      |
|       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | M1    |      |
|       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | A1    | (7)  |
| <hr/> |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |       |      |
| 4.    | <p>(a) </p> <p>(b) <math>(\frac{\pi}{2}, -1) \Rightarrow -1 = a + b</math></p> <p><math>(\frac{3\pi}{2}, -5) \Rightarrow -5 = a - b</math></p> <p>adding, <math>-6 = 2a \therefore a = -3, b = 2</math></p> <p>(c) <math>-3 + 2 \operatorname{cosec} x = 0, \quad \operatorname{cosec} x = \frac{3}{2}, \quad \sin x = \frac{2}{3}</math></p> <p><math>x = 0.73, \pi - 0.7297, \quad x = 0.73, 2.41 \text{ (2dp)}</math></p>                                                                                           | B3    |      |
|       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | B1    |      |
|       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | M1 A1 |      |
|       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | M1    |      |
|       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | A2    | (9)  |
| <hr/> |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |       |      |
| 5.    | <p>(a) <math>t = 3, N = 18\,000 \Rightarrow 18\,000 = 2000e^{3k}, \quad e^{3k} = 9</math></p> <p><math>k = \frac{1}{3} \ln 9 = 0.732 \text{ (3sf)}</math></p> <p>(b) <math>4000 = 2000e^{0.7324t}</math></p> <p><math>t = \frac{1}{0.7324} \ln 2 = 0.9464 \text{ hours}</math></p> <p><math>\therefore \text{doubles in 57 minutes (nearest minute)}</math></p> <p>(c) <math>N = 2000e^{0.7324t}, \quad \frac{dN}{dt} = 0.7324 \times 2000e^{0.7324t} = 1465e^{0.7324t}</math></p> <p>when <math>t = 3, \frac{dN}{dt} = 13\,200 \therefore \text{increasing at rate of 13\,200 per hour (3sf)}</math></p> | M1    |      |
|       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | M1 A1 |      |
|       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | M1    |      |
|       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | M1 A1 |      |
|       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | A1    |      |
|       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | M1 A1 |      |
|       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | A1    | (10) |

|     |     |                                                                                                    |         |
|-----|-----|----------------------------------------------------------------------------------------------------|---------|
| 6.  | (a) | $\frac{d}{dx}(\sec x) = \frac{d}{dx}[(\cos x)^{-1}]$                                               |         |
|     |     | $= -(\cos x)^{-2} \times (-\sin x)$                                                                | M1 A1   |
|     |     | $= \frac{\sin x}{\cos^2 x} = \frac{1}{\cos x} \times \frac{\sin x}{\cos x}$                        | M1      |
|     |     | $= \sec x \tan x$                                                                                  | A1      |
| (b) |     | $\frac{dy}{dx} = 2e^{2x} \times \sec x + e^{2x} \times \sec x \tan x = e^{2x} \sec x (2 + \tan x)$ | M1 A1   |
|     |     | $x = 0, y = 1, \text{ grad} = 2$                                                                   | M1      |
|     |     | $\therefore y = 2x + 1$                                                                            | A1      |
| (c) | SP: | $e^{2x} \sec x (2 + \tan x) = 0$                                                                   | M1      |
|     |     | $\tan x = -2$                                                                                      | M1      |
|     |     | $x = -1.11 \text{ (2dp)}$                                                                          | A1 (11) |

|     |     |                                                                                                                         |            |
|-----|-----|-------------------------------------------------------------------------------------------------------------------------|------------|
| 7.  | (a) | $f(x) = (x-1)^2 - 1 + 5 = (x-1)^2 + 4$                                                                                  | M1 A1      |
|     | (b) | $f(x) \geq 4$                                                                                                           | B1         |
|     | (c) | $y = (x-1)^2 + 4$<br>$(x-1)^2 = y-4$<br>$x-1 = \pm\sqrt{y-4}$<br>$x = 1 \pm \sqrt{y-4}$<br>$f^{-1}(x) = 1 + \sqrt{x-4}$ | M1         |
|     | (d) | translation by 4 units in negative x direction<br>translation by 1 unit in negative y direction (either first)          | M1 A1      |
| (e) |     | $\frac{dy}{dx} = \frac{1}{2}(x-4)^{-\frac{1}{2}}$                                                                       | B2         |
|     |     | $x = 8, y = 3, \text{ grad} = \frac{1}{4}$                                                                              | M1         |
|     |     | $\therefore \text{grad of normal} = -4$                                                                                 | A1         |
|     |     | $\therefore y-3 = -4(x-8) \quad [y = 35 - 4x]$                                                                          | M1 A1 (12) |

|    |     |                                                                                                                                                                                                                |         |
|----|-----|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------|
| 8. | (a) | $\frac{dy}{dx} = -e^2 x^{-2} + e^x$                                                                                                                                                                            | M1 A1   |
|    | (b) | SP: $-e^2 x^{-2} + e^x = 0$<br>let $f(x) = -e^2 x^{-2} + e^x$<br>$f(1.3) = -0.70, f(1.4) = 0.29$<br>sign change, $f(x)$ continuous $\therefore$ root                                                           | M1      |
|    | (c) | $x = 2, y = \frac{3}{2}e^2, \text{ grad} = \frac{3}{4}e^2$<br>$\therefore y - \frac{3}{2}e^2 = \frac{3}{4}e^2(x-2)$<br>$y = \frac{3}{4}e^2 x$<br>$\therefore x = 0 \Rightarrow y = 0$ so passes through origin | M1      |
|    | (d) | $x_1 = -1.125589, x_2 = -1.125803, x_3 = -1.125804 \text{ (7sf)}$<br>$\therefore x\text{-coordinate of } B = -1.1258 \text{ (5sf)}$                                                                            | M1 A2   |
|    |     |                                                                                                                                                                                                                | A1 (13) |

Total (75)

## Performance Record – C3 Paper H

[illegible]