FOR EDEXCEL

GCE Examinations
Advanced Subsidiary

Core Mathematics C4

Paper K

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks could be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

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C4 Paper K – Marking Guide

1. $= \pi \int_1^3 \frac{(3x+1)^2}{x}\, dx$ M1

$= \pi \int_1^3 \frac{9x^2+6x+1}{x}\, dx = \int_1^3 (9x + 6 + \frac{1}{x})\, dx$ A1

$= \pi[\frac{9}{2}x^2 + 6x + \ln|x|]_1^3$ M1 A1

$= \pi\{(\frac{81}{2} + 18 + \ln 3) - (\frac{9}{2} + 6 + 0)\}$ M1

$= \pi(48 + \ln 3)$ A1 **(6)**

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2. *(a)* $(1 - 3x)^{-2} = 1 + (-2)(-3x) + \frac{(-2)(-3)}{2}(-3x)^2 + \frac{(-2)(-3)(-4)}{3\times 2}(-3x)^3 + \ldots$ M1

$= 1 + 6x + 27x^2 + 108x^3 + \ldots$ A3

(b) $\left(\frac{2-x}{1-3x}\right)^2 = (2 - x)^2(1 - 3x)^{-2} = (4 - 4x + x^2)(1 + 6x + 27x^2 + 108x^3 + \ldots)$ M1

$= 4 + 24x + 108x^2 + 432x^3 - 4x - 24x^2 - 108x^3 + x^2 + 6x^3 + \ldots$ A1

$\therefore$ for small x, $\left(\frac{2-x}{1-3x}\right)^2 = 4 + 20x + 85x^2 + 330x^3$ A1 **(7)**

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3. *(a)* $\frac{7+3x+2x^2}{(1-2x)(1+x)^2} \equiv \frac{A}{1-2x} + \frac{B}{1+x} + \frac{C}{(1+x)^2}$

$7 + 3x + 2x^2 \equiv A(1 + x)^2 + B(1 - 2x)(1 + x) + C(1 - 2x)$

$x = \frac{1}{2} \Rightarrow 9 = \frac{9}{4}A \Rightarrow A = 4$ B1

$x = -1 \Rightarrow 6 = 3C \Rightarrow C = 2$ B1

coeffs $x^2 \Rightarrow 2 = A - 2B \Rightarrow B = 1$ M1

$\therefore$ f$(x) = \frac{4}{1-2x} + \frac{1}{1+x} + \frac{2}{(1+x)^2}$ A1

(b) $= \int_1^2 (\frac{4}{1-2x} + \frac{1}{1+x} + \frac{2}{(1+x)^2})\, dx$

$= [-2\ln|1 - 2x| + \ln|1 + x| - 2(1 + x)^{-1}]_1^2$ M1 A3

$= (-2\ln 3 + \ln 3 - \frac{2}{3}) - (0 + \ln 2 - 1)$ M1

$= -\ln 3 - \ln 2 + \frac{1}{3} = \frac{1}{3} - \ln 6$ $[\,p = \frac{1}{3}, q = 6\,]$ M1 A1 **(11)**

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4. *(a)* $4\lambda = 6 + 14\mu$ (1)

$-3 - 2\lambda = 3 + 2\mu$ (2) B1

(1) + 2 × (2): $-6 = 12 + 18\mu,\ \mu = -1,\ \lambda = -2$ M1 A1

$\mathbf{r} = \begin{pmatrix} 7 \\ 0 \\ -3 \end{pmatrix} - 2\begin{pmatrix} 5 \\ 4 \\ -2 \end{pmatrix} = \begin{pmatrix} -3 \\ -8 \\ 1 \end{pmatrix}$ M1 A1

(b) $a - (-5) = -3, \quad a = -8$ M1 A1

(c) $\cos\theta = \left|\frac{5\times(-5)+4\times 14+(-2)\times 2}{\sqrt{25+16+4}\times\sqrt{25+196+4}}\right|$ M1 A1

$= \frac{27}{\sqrt{45\times 15}} = \frac{9}{3\sqrt{5}\times 5} = \frac{3}{5\sqrt{5}} = \frac{3}{25}\sqrt{5}$ M1 A1 **(11)**

5.	*(a)*	$2x - 4y - 4x\frac{dy}{dx} + 4y\frac{dy}{dx} = 0$	M1 A2	
		$\frac{dy}{dx} = \frac{2x-4y}{4x-4y} = \frac{x-2y}{2x-2y}$	M1 A1	
	(b)	grad $= \frac{3}{2}$	M1	
		$\therefore\ y - 2 = \frac{3}{2}(x - 1)$	M1	
		$2y - 4 = 3x - 3$		
		$3x - 2y + 1 = 0$	A1	
	(c)	$\frac{x-2y}{2x-2y} = \frac{3}{2}$	M1	
		$2(x - 2y) = 3(2x - 2y), \quad y = 2x$	A1	
		sub. $\Rightarrow\ x^2 - 8x^2 + 8x^2 = 1$	M1	
		$x^2 = 1,\ x = 1$ (at P) or -1		
		$\therefore\ Q(-1, -2)$	A1	**(12)**
6.	*(a)*	$\frac{dN}{dt} = kN$	B1	
	(b)	$\int \frac{1}{N}\ dN = \int k\ dt$	M1	
		$\ln\lvert N\rvert = kt + c$	M1 A1	
		$t = 0,\ N = N_0 \Rightarrow \ln\lvert N_0\rvert = c$	M1	
		$\ln\lvert N\rvert = kt + \ln\lvert N_0\rvert, \quad \ln\left\lvert \frac{N}{N_0}\right\rvert = kt$	M1	
		$\frac{N}{N_0} = e^{kt}, \quad N = N_0e^{kt}$	A1	
	(c)	$2N_0 = N_0e^{6k}$	M1	
		$k = \frac{1}{6}\ln 2 = 0.116$ (3sf)	M1 A1	
	(d)	$10N_0 = N_0e^{0.1155t}$	M1	
		$t = \frac{1}{0.1155}\ln 10 = 19.932$ hours = 19 hours 56 mins	M1 A1	**(13)**
7.	*(a)*	$x + \frac{1}{x} = \sec\theta + \tan\theta + \frac{1}{\sec\theta+\tan\theta} = \frac{(\sec\theta+\tan\theta)^2+1}{\sec\theta+\tan\theta}$	M1	
		$= \frac{\sec^2\theta+2\sec\theta\tan\theta+\tan^2\theta+1}{\sec\theta+\tan\theta} = \frac{2\sec^2\theta+2\sec\theta\tan\theta}{\sec\theta+\tan\theta}$	M1 A1	
		$= \frac{2\sec\theta(\sec\theta+\tan\theta)}{\sec\theta+\tan\theta} = 2\sec\theta$	M1 A1	
	(b)	$\frac{x^2+1}{x} = \frac{2}{\cos\theta} \Rightarrow \cos\theta = \frac{2x}{x^2+1}$	M1	
		$\frac{y^2+1}{y} = \frac{2}{\sin\theta} \Rightarrow \sin\theta = \frac{2y}{y^2+1} \quad \therefore \frac{4x^2}{(x^2+1)^2} + \frac{4y^2}{(y^2+1)^2} = 1$	M1 A1	
	(c)	$\frac{dx}{d\theta} = \sec\theta\tan\theta + \sec^2\theta$	M1	
		$= \sec\theta(\tan\theta + \sec\theta) = \frac{x^2+1}{2x} \times x = \frac{1}{2}(x^2 + 1)$	M1 A1	
	(d)	$\frac{dy}{d\theta} = -\text{cosec}\,\theta\cot\theta - \text{cosec}^2\,\theta$	M1	
		$= -\text{cosec}\,\theta(\cot\theta + \text{cosec}\,\theta) = -\frac{y^2+1}{2y} \times y = -\frac{1}{2}(y^2 + 1)$	A1	
		$\therefore\ \frac{dy}{dx} = -\frac{y^2+1}{x^2+1}$	M1 A1	**(15)**

Total **(75)**

Performance Record – C4 Paper K

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	Total
Topic(s)	integration	binomial series	partial fractions	vectors	differentiation	differential equation	parametric equations	
Marks	6	7	11	11	12	13	15	75
Student								