FOR EDEXCEL

GCE Examinations
Advanced Subsidiary

Core Mathematics C4

Paper L

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks could be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

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C4 Paper L – Marking Guide

1.	*(a)*	$\frac{dn}{dt} = 0 \;\Rightarrow\; e^{0.5t} = 5$	M1	
		$t = 2 \ln 5 = 3.219$ mins $= 3$ mins 13 secs	M1 A1	
	(b)	$\int dn = \int (e^{0.5t} - 5)\ dt$		
		$n = 2e^{0.5t} - 5t + c$	M1 A1	
		$t = 0,\ n = 20 \;\Rightarrow\; 20 = 2 + c, \quad c = 18$	M1	
		$n = 2e^{0.5t} - 5t + 18$	A1	
	(c)	as t increases, n rapidly becomes very large $\therefore$ not realistic	B1	**(8)**
2.		$6x + y + x\frac{dy}{dx} - 4y\frac{dy}{dx} = 0$	M1 A2	
		$(1, 4) \;\Rightarrow\; 6 + 4 + \frac{dy}{dx} - 16\frac{dy}{dx} = 0, \quad \frac{dy}{dx} = \frac{2}{3}$	M1 A1	
		grad of normal $= -\frac{3}{2}$	M1	
		$\therefore\ y - 4 = -\frac{3}{2}(x - 1)$	M1	
		$2y - 8 = -3x + 3$		
		$3x + 2y - 11 = 0$	A1	**(8)**
3.	*(a)*	$u = 2 - x^2 \;\Rightarrow\; \frac{du}{dx} = -2x$	M1	
		$I = \int \frac{1}{u} \times (-\frac{1}{2})\ du = -\frac{1}{2}\int \frac{1}{u}\ du$	A1	
		$= -\frac{1}{2}\ln\lvert u\rvert + c = -\frac{1}{2}\ln\lvert 2 - x^2\rvert + c$	M1 A1	
	(b)	$= \int_0^{\frac{\pi}{4}} (\frac{1}{2}\sin 4x + \frac{1}{2}\sin 2x)\ dx$	M1 A1	
		$= [-\frac{1}{8}\cos 4x - \frac{1}{4}\cos 2x]_0^{\frac{\pi}{4}}$	M1 A1	
		$= (\frac{1}{8} - 0) - (-\frac{1}{8} - \frac{1}{4}) = \frac{1}{2}$	M1 A1	**(10)**
4.	*(a)*	x: 1, 2, 3; y: 0, 1.665, 3.144	B1	
		area $\approx \frac{1}{2} \times 1 \times [0 + 3.144 + 2(1.665)] = 3.24$ (3sf)	B1 M1 A1	
	(b)	volume $= \pi\int_1^3 x^2 \ln x\ dx$	M1	
		$u = \ln x,\ u' = \frac{1}{x},\ v' = x^2,\ v = \frac{1}{3}x^3$		
		$I = \frac{1}{3}x^3 \ln x - \int \frac{1}{3}x^2\ dx$	M1 A2	
		$= \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + c$	A1	
		volume $= \pi[\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3]_1^3$		
		$= \pi\{(9 \ln 3 - 3) - (0 - \frac{1}{9})\}$	M1	
		$= \pi(9 \ln 3 - \frac{26}{9})$	A1	**(11)**

5.	*(a)*	$\dfrac{5-8x}{(1+2x)(1-x)^2} \equiv \dfrac{A}{1+2x} + \dfrac{B}{1-x} + \dfrac{C}{(1-x)^2}$		
		$5 - 8x \equiv A(1-x)^2 + B(1+2x)(1-x) + C(1+2x)(1-x)$	M1	
		$x = -\frac{1}{2} \;\Rightarrow\; 9 = \frac{9}{4}A \;\Rightarrow\; A = 4$	A1	
		$x = 1 \;\Rightarrow\; -3 = 3C \;\Rightarrow\; C = -1$	A1	
		coeffs $x^2 \;\Rightarrow\; 0 = A - 2B \;\Rightarrow\; B = 2$	M1 A1	
		$f(x) = \dfrac{4}{1+2x} + \dfrac{2}{1-x} - \dfrac{1}{(1-x)^2}$		
	(b)	$f(x) = 4(1+2x)^{-1} + 2(1-x)^{-1} - (1-x)^{-2}$		
		$(1+2x)^{-1} = 1 + (-1)(2x) + \frac{(-1)(-2)}{2}(2x)^2 + \frac{(-1)(-2)(-3)}{3\times 2}(2x)^3 + \dots$	M1	
		$= 1 - 2x + 4x^2 - 8x^3 + \dots$	A1	
		$(1-x)^{-1} = 1 + x + x^2 + x^3 + \dots$	B1	
		$(1-x)^{-2} = 1 + (-2)(-x) + \frac{(-2)(-3)}{2}(-x)^2 + \frac{(-2)(-3)(-4)}{3\times 2}(-x)^3 + \dots$		
		$= 1 + 2x + 3x^2 + 4x^3 + \dots$	A1	
		$f(x) = 4(1 - 2x + 4x^2 - 8x^3) + 2(1 + x + x^2 + x^3) - (1 + 2x + 3x^2 + 4x^3)$	M1	
		$= 5 - 8x + 15x^2 - 34x^3 + \dots$	A1	
	(c)	$\lvert x \rvert < \frac{1}{2}$	A1	**(12)**
6.	*(a)*	$\dfrac{dx}{dt} = 1 + \cos t, \quad \dfrac{dy}{dt} = \cos t$	M1	
		$\dfrac{dy}{dx} = \dfrac{\cos t}{1+\cos t}$	M1 A1	
	(b)	$\dfrac{\cos t}{1+\cos t} = 0, \;\; \cos t = 0, \;\; t = \frac{\pi}{2}$	M1 A1	
		$\therefore\; (\frac{\pi}{2} + 1, 1)$	A1	
	(c)	$= \int_0^{\pi} \sin t \times (1 + \cos t)\ dt = \int_0^{\pi} (\sin t + \frac{1}{2}\sin 2t)\ dt$	M1 A1	
		$= [-\cos t - \frac{1}{4}\cos 2t]_0^{\pi}$	M1 A1	
		$= (1 - \frac{1}{4}) - (-1 - \frac{1}{4}) = 2$	M1 A1	**(12)**
7.	*(a)*	$\overrightarrow{AB} = (8\mathbf{j} - 6\mathbf{k}) - (3\mathbf{i} + 6\mathbf{j} - 8\mathbf{k}) = (-3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$	M1	
		$\therefore\; \mathbf{r} = (3\mathbf{i} + 6\mathbf{j} - 8\mathbf{k}) + \lambda(-3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$	A1	
	(b)	$3 - 3\lambda = -2 + 7\mu \quad (1)$		
		$6 + 2\lambda = 10 - 4\mu \quad (2)$		
		$-8 + 2\lambda = 6 + 6\mu \quad (3)$	B1	
		$(3) - (2)$: $-14 = -4 + 10\mu, \;\; \mu = -1, \lambda = 4$	M1 A1	
		check (1) $3 - 12 = -2 - 7$, true $\therefore$ intersect	B1	
	(c)	$\mathbf{r} = (-2\mathbf{i} + 10\mathbf{j} + 6\mathbf{k}) - (7\mathbf{i} - 4\mathbf{j} + 6\mathbf{k}) \quad \therefore (-9, 14, 0)$	M1 A1	
	(d)	$\overrightarrow{OC} = [(-2 + 7\mu)\mathbf{i} + (10 - 4\mu)\mathbf{j} + (6 + 6\mu)\mathbf{k}]$		
		$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = [(-5 + 7\mu)\mathbf{i} + (4 - 4\mu)\mathbf{j} + (14 + 6\mu)\mathbf{k}]$	M1 A1	
		$\therefore\; [(-5 + 7\mu)\mathbf{i} + (4 - 4\mu)\mathbf{j} + (14 + 6\mu)\mathbf{k}].(-3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}) = 0$	M1	
		$15 - 21\mu + 8 - 8\mu + 28 + 12\mu = 0$	A1	
		$\mu = 3 \;\therefore\; \overrightarrow{OC} = (19\mathbf{i} - 2\mathbf{j} + 24\mathbf{k})$	M1 A1	**(14)**

Total **(75)**

Performance Record – C4 Paper L

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	Total
Topic(s)	differential equation	differentiation	integration	trapezium rule, integration	partial fractions, binomial series	parametric equations	vectors	
Marks	8	8	10	11	12	12	14	75
Student								