GCE

Mathematics

Advanced Subsidiary GCE

Unit **4721:** Core Mathematics 1

Mark Scheme for January 2011

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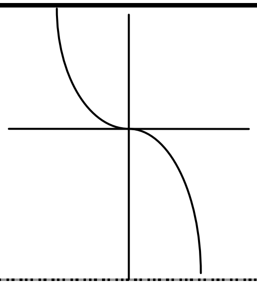
1 (i)	$\sqrt{(-2-6)^2+(7-1)^2}$	M1		Use of $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$	3 out of 4 substitutions correct Look out for no square root, $(x_2+x_1)^2$ etc. **M0**
	$= 10$	A1	2		
(ii)	$\frac{7-1}{-2-6}$	M1		uses $\frac{y_2-y_1}{x_2-x_1}$	3 out of 4 substitutions correct
	$=-\frac{3}{4}$	A1	2	o.e. **ISW**	Allow –0.75 $\frac{3}{-4}$ etc.
(iii)	Gradient of given line = $\frac{4}{3}$	M1		Attempt to rearrange equation to make *y* the subject **OR** attempt to find the gradient using points on the line	Must at least isolate *y*
	$-\frac{3}{4}\times\frac{4}{3}=-1$	B1ft		Correct conclusion for their gradients	
	So lines are perpendicular	B1	3 $\boxed{7}$	States $-\frac{3}{4}\times\frac{4}{3}=-1$ or "negative reciprocal" relating to the correct values **www**	
2	$2x^3+9x^2-2px^2-9px+10x-10p$ $=2x^3+qx^2-8x-4q$	M1*		Attempt to expand both sides **OR** to substitute 2 values of *x* into both expressions **OR** to express at least one side as a product of three factors	If expanding, minimum of 5 terms on LHS and 3terms on RHS
		DM1		Valid method to obtain either p or q	If comparing coefficients, must be of corresponding terms
	$p=2$ and $q=5$	A1	3 $\boxed{3}$	Both values correct	**SR** Spotted solutions **B1** one correct **B2** other correct
3 (i)	$8^{\frac{1}{2}}$	B1	1		Allow $8^{0.5}$ Condone p= $\frac{1}{2}$, just "$\frac{1}{2}$" seen as answer **www**
(ii)	8^{-2}	B1	1		Condone p = -2, just "-2" seen as answer **www** $\frac{1}{8^2}$ only not enough
(iii)	$2^8=\left(8^{\frac{1}{3}}\right)^8$	M1		2^8 or $2^6=8^2$ soi	Condone p= $\frac{8}{3}$, just "$\frac{8}{3}$" seen as answer **www**
		M1		$2=8^{\frac{1}{3}}$ soi	2^3= 8 not enough for second **M** mark
	$=8^{\frac{8}{3}}$	A1	3 $\boxed{5}$	o.e.	

4	$u^2 - 5u + 4 = 0$	M1*		Use the given substitution to obtain a quadratic or factorise into 2 brackets each containing $(3x-2)^2$	**No marks** if evidence of "square rooting" e.g. "$(3x-2)^2$ - 5$(3x-2)$ + 2 (or 4) = 0" **No marks** if straight to quadratic formula to get x = "1" x = "4" and no further working
	$(u-1)(u-4) = 0$	DM1		Correct method to solve a quadratic	
	$u = 1$ or $u = 4$	A1		Correct values for u	**SR 1)** If **M0** Spotted solutions **www B1** each Justifies 4 solutions exactly **B2**
	$3x-2 = \pm 1$ or $3x-2 = \pm 2$	M1		Attempt to square root and rearrange to obtain x **OR** to expand, rearrange and solve quadratic (at least one)	**SR 2)** If first 3 marks awarded, spotted solutions 2 correct **B1** Other 2 correct **B1**
	$x = 1$ or $\frac{1}{3}$ or $\frac{4}{3}$ or 0	A1		2 correct values	Justifies 4 solutions exactly **B1**
		A1	6 $\boxed{6}$	All 4 correct values ($\frac{0}{3}$ = **A0**)	<u>Alternative scheme for candidates who multiply out:</u> Attempt to expand $(3x-2)^4$ and $(3x-2)^2$ **M1** $81x^4 - 216x^3 + 171x^2 - 36x = 0$ **A1** $x = 0$ a solution or x a factor of the quartic **A1** Attempt to use factor theorem to factorise their cubic **M1*** Correct method to solve quadratic **DM1** All 4 solutions correct **A1**
5 (i)		M1		Negative cubic through (0, 0) (may have max and min)	Must be continuous. Allow slight curve towards or away from y-axis at one end, but not both.
		A1	2	Must have reasonable rotational symmetry. Cannot be a finite "plot". Allow negative gradient at origin. Correct curvature at both ends.	
(ii)	$y = -(x-3)^3$	M1		$\pm(x-3)^3$ seen	
		A1	2	or $y = (3-x)^3$	Must have "$y =$ " for A mark **SR** $y = -(x-3)^2$ **B1**
(iii)	Stretch	B1			
	scale factor 5 parallel to y-axis	B1	2 $\boxed{6}$	o.e. e.g. scale factor $\frac{1}{\sqrt[3]{5}}$ parallel to the x axis.	Allow "factor" for "scale factor" For "parallel to the y axis" allow "vertically", "in the y direction". **Do not accept** "in/on/across/up/along the y axis"

6 (i)	$y = 5x^{-2} - \frac{1}{4}x^{-1} + x$	M1		x^{-2} used for $\frac{1}{x^2}$ **OR** x^{-1} used for $\frac{1}{x}$ soi, **OR** x correctly differentiated	Look out for: $y = 5x^{-2} - 4x^{-1} + x$ followed by $\frac{dy}{dx} = -10x^{-3} + 4x^{-2} + 1$ and then the correct answer.
	$\frac{dy}{dx} = -10x^{-3} + \frac{1}{4}x^{-2} + 1$	A1		kx^{-3} or kx^{-2} from differentiating	This is **M1 A1 A1 A0**
		A1		Two fully correct terms	$4x^{-1}$ is **NOT** a misread
		A1	4	Completely correct	
(ii)	$\frac{d^2y}{dx^2} = 30x^{-4} - \frac{1}{2}x^{-3}$	M1		Attempt to differentiate their $\frac{dy}{dx}$ (one term correctly differentiated)	Allow a sign slip in coefficient for M mark
		A1	2 $\boxed{6}$	Completely correct	**NB** Only penalise "+ c" first time seen in the question

7 (i)	$4(x^2+3x)-3$ $=4\left[\left(x+\frac{3}{2}\right)^2-\frac{9}{4}\right]-3$ $=4\left(x+\frac{3}{2}\right)^2-12$	B1 B1 M1 A1 4	$p=4$ $q=\frac{3}{2}$ $r=-3-4q^2$ or $r=-\frac{3}{4}-q^2$ $r=-12$ (from $q=\pm1.5$)	If p, q, r found correctly, then **ISW** slips in format. $4(x+1.5)^2+12$ **B1 B1 M0 A0** $4(x+1.5)-12$ **B1 B1 M1 A1 (BOD)** $4(x+1.5x)^2-12$ **B1 B0 M1 A0** $4(x^2+1.5)^2-12$ **B1 B0 M1 A0** $4(x-1.5)^2-12$ **B1 B0 M1 A1** $4x(x+1.5)^2-12$ **B0 B1M1A1**
(ii)	$\frac{-12\pm\sqrt{12^2-4\times4\times-3}}{2\times4}$ $=\frac{-12\pm\sqrt{192}}{8}$ $=\frac{-12\pm8\sqrt{3}}{8}$ $=-\frac{3}{2}\pm\sqrt{3}$ OR: $4\left(x+\frac{3}{2}\right)^2-12=0$ $x+\frac{3}{2}=\pm\sqrt{3}$ $x=-\frac{3}{2}\pm\sqrt{3}$	M1 A1 B1 A1 M1 A1ft A1 A1 4	Correct method to solve quadratic $\frac{-12\pm\sqrt{192}}{8}$ or $\frac{-3\pm\sqrt{12}}{2}$ √192=8√3 or √12=2√3 from correct b^2-4ac $\frac{-3\pm2\sqrt{3}}{2}$ or $-\frac{12}{8}\pm\sqrt{3}$, $-\frac{6}{4}\pm\sqrt{3}$ Must have ± for method mark $x+1.5$ **ft** $x+q$ from part(i) www in LHS in part (ii) ±√3 Do not **ISW**	Not for $2(x+q)=...$ **SR** One correct root **www B1**
(iii)	$12^2-4\times4\times(-k)=0$ $144+16k=0$ $k=-9$ **OR (see next page)**	M1 A1 A1	Attempts $b^2-4ac=0$ or $\sqrt{b^2-4ac}=0$ involving k. If b^2-4ac not quoted then expression must be correct. Correct, unsimplified expression	<u>Other alternative methods</u> a) Attempt to factorise into two equal brackets, (may divide by 4 first – must be correct) **M1** Equate coefficient of x to 12 (or 3) **A1** $k=-9$ **A1** b) Uses differentiation to find x ordinate of turning point and uses this to form equation in k **M1** Correct equation in k **A1** $k=-9$ **A1**

7(iii) cont.	$4x^2+12x=k$ $4(x+\frac{3}{2})^2-9=k$	M1		Attempts completing the square in given equation or factorises to $(2x+3)^2-9=k$	Must involve k in their working to gain the method marks in this scheme
	Equal roots when $x=-\frac{3}{2}$	M1		Substitutes $x=-\frac{3}{2}$	
	$k=-9$	A1	3 $\boxed{11}$		
8 (i)	$\frac{dy}{dx}=6-2x$	M1 A1		Attempt to differentiate $\pm y$ Correct expression **cao**	One correct non-zero term
	When $x=5$, $6-2x=-4$	M1		Substitute $x=5$ into their $\frac{dy}{dx}$	
	When $x=5$, $y=12$	B1		Correct y coordinate	
	$y-12=-4(x-5)$	M1		Correct equation of straight line through (5, their y), their non-zero, numerical gradient	Allow $\frac{y-12}{x-5}$ = their gradient If using $y=mx+c$ must attempt at evaluating c
	$4x+y-32=0$	A1	6	Shows rearrangement to correct form	Allow any correct form e.g. $0=2y+8x-64$ etc.
(ii)	Q is point (8, 0)	B1ft		ft from line in (i)	.
	Midpoint of $PQ=\left(\frac{5+8}{2},\frac{12+0}{2}\right)$	M1		Uses $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$ o.e. for their P,Q	
	$=\left(\frac{13}{2},6\right)$	A1	3		Do not accept $\left(\frac{13}{2},\frac{12}{2}\right)$
(iii)	$6-2x=0$	M1		Solution of their $\frac{dy}{dx}=0$	Alternatives for Method Mark a) attempts completion of square with $\pm(x-3)^2$ b) attempts to solve quadratic (usual scheme) and to find the mid-point of the two roots c) attempts to use $x=-\frac{b}{2a}$ (allow one sign slip on substitution)
	(Line of symmetry is) $x=3$	A1	2	Allow from $\pm[16-(x-3)^2]$, $\pm[6-2x=0]$	
(iv)	$x<3$	M1		$x<$ their3 or $x>$ their3 **OR** attempt to solve their $\frac{dy}{dx}>0$	May solve $\frac{dy}{dx}=0$ then use $\frac{d^2y}{dx^2}<0$ implies maximum point for the method mark, or sketch of curve
		A1	2 $\boxed{13}$	Allow from $\pm[16-(x-3)^2]$, $\pm[6-2x=0]$ in (iii)	Allow $x\le 3$

9 (i)	Centre (4, 1)	B1		Correct centre	
	$(x-4)^2+(y-1)^2-16-1-3=0$	M1		Correct method to find r^2	r^2 =(± their 4)^2 + (±their 1)^2 + 3 soi
	$(x-4)^2+(y-1)^2=20$				
	Radius $=\sqrt{20}$	A1	3	Correct radius	$\pm\sqrt{20}$ is **A0** Ignore incorrect simplification of $\sqrt{20}$
(ii)	$k=1\pm\sqrt{20}$	M1 A1ft		y ordinate of their centre ± their radius or Both correct, unsimplified values	<u>Alternatives for method mark :</u> a) Substitutes k for y and uses $b^2-4ac=0$ to obtain quadratic in k b) Recognises $x = 4$ is equation of normal, substitutes into circle equation and solves for k.
	$k=1\pm2\sqrt{5}$	A1	3	**cao**	**SR** $k = 1+\sqrt{20}$ or $k = 1-\sqrt{20}$ or better **www B1**
(iii)	$MT^2=r^2-2^2$	M1		Correct use of Pythagoras' theorem involving MT (or SM)	**SR** ST=8 from particular S and T co-ordinates [e.g. horizontal chord calculated as (0,3) and (8,3)] **B1**
	$MT=4$	A1ft		Correct value of MT for their r	Justifies solution the same for all possible chords **B2**
	$ST=8$	A1	3	**cao**	
(iv)	$x=2y+12$	M1*		Attempt to solve equations simultaneously	Must be a clear attempt to reduce to one variable using equation of line and either form of equation of circle. Condone poor algebra for first mark. <u>If y eliminated:</u>
	$(2y+8)^2+(y-1)^2=20$	A1		Correct unsimplified expression, may be $(12+2y)^2+y^2-8(12+2y)-2y-3=0$	
	$4y^2+32y+64+y^2-2y+1=20$				
	$5y^2+30y+45=0$	A1		Obtain correct 3 term quadratic	$(x-4)^2+\left(\frac{1}{2}x-7\right)^2=20$
	$y^2+6y+9=0$				
	$(y+3)^2=0$	DM1		Correct method to solve quadratic of form $ax^2+bx+c=0$ $(b\neq0)$	Or $x^2+\left(\frac{1}{2}x-6\right)^2-8x-2\left(\frac{1}{2}x-6\right)-3=0$
	$y=-3$	A1		y value correct, no extra solutions	
	$x=6$	A1		x value correct **ISW**	Leading to $x^2-12x+36=0$
	OR				
	$y-1=-2(x-4)$	M1		Attempt to find equation of radius/normal	
		A1		Correct equation	
	Solve simultaneously with $y=\frac{1}{2}x-6$	M1			
	$x=6$	A1			
	$y=-3$	A1			
	States line is tangent as meets at one point or verifies (6, -3) lies on circle	B1	6 $\boxed{15}$	Allow showing distance between (6,-3) and (4,1) = $\sqrt{20}$	**SR** Correct coordinates spotted or from trial and improvement **www B2**

<u>Allocation of method mark for solving a quadratic</u>

e.g. $4x^2 + 12x - 3 = 0$

<u>By factorisation</u>
– when expanded, quadratic term and one other term must be correct (with correct sign):

$(2x+1)(2x-3)=0$	M1	$4x^2$ and -3 obtained from expansion
$(4x+4)(x+2)=0$	M1	$4x^2$ and $+12x$ obtained from expansion
$(4x-1)(x-3)=0$	M0	only x^2 term correct

<u>By formula</u>
- if the formula is quoted correctly first, allow one sign slip in substituting values into it:
$a = 4$, $b = 12$, $c = -3$

$$\frac{12 \pm \sqrt{(12)^2 - 4\times4\times-3}}{8}$$ gains M1 (minus sign incorrect at start of formula)

$$\frac{-12 \pm \sqrt{(12)^2 - 4\times4\times3}}{8}$$ gains M1 (3 for c instead of -3)

$$\frac{12 \pm \sqrt{(12)^2 - 4\times4\times3}}{8}$$ M0 (2 sign errors: initial sign and c incorrect)

- if the formula is not quoted, then no errors at all are allowed in substitution.

<u>By completing the square</u>

$$4x^2 + 12x - 3 = 0$$
$$4\left[\left(x+\frac{3}{2}\right)^2 - \frac{9}{4}\right] - 3 = 0$$
$$\left(x+\frac{3}{2}\right)^2 = 3$$
$$x + \frac{3}{2} = \pm\sqrt{3}$$

The method mark is awarded <u>only</u> at the last line of working
i.e. when ±√ combined constants is seen.

N.B. The value of the combined constants does not have to be correct for the M1 mark

Condone "invisible brackets" if justified by correct later working

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