4722 Core Mathematics 2

1 (i)	$\int (x^3+8x-5)\,dx = \frac{1}{4}x^4+4x^2-5x+c$	M1		Attempt integration – increase in power for at least 2 terms
		A1		Obtain at least 2 correct terms
		A1	**3**	Obtain $\frac{1}{4}x^4+4x^2-5x+c$ (and no integral sign or dx)
(ii)	$\int 12x^{\frac{1}{2}}\,dx = 8x^{\frac{3}{2}}+c$	B1		State or imply $\sqrt{x}=x^{\frac{1}{2}}$
		M1		Obtain $kx^{\frac{3}{2}}$
		A1	**3**	Obtain $8x^{\frac{3}{2}}+c$ (and no integral sign or dx) (only penalise lack of + c, or integral sign or dx once)
			6	
2 (i)	$140^\circ = 140\times\frac{\pi}{180}$	M1		Attempt to convert 140° to radians
	$= \frac{7}{9}\pi$	A1	**2**	Obtain $\frac{7}{9}\pi$, or exact equiv
(ii)	arc $AB = 7\times\frac{7}{9}\pi$	M1		Attempt arc length using $r\theta$ or equiv method
	= 17.1	A1√		Obtain 17.1, $\frac{49}{9}\pi$ or unsimplified equiv
	chord $AB = 2\times 7\sin\frac{7}{18}\pi = 13.2$	M1		Attempt chord using trig. or cosine or sine rules
	hence perimeter = 30.3 cm	A1	**4**	Obtain 30.3, or answer that rounds to this
			6	
3 (i)	$u_1 = 23^1/_3$	B1		State $u_1 = 23^1/_3$
	$u_2 = 22^2/_3$, $u_3 = 22$	B1	**2**	State $u_2 = 22^2/_3$ and $u_3 = 22$
(ii)	$24 - {}^{2k}/_3 = 0$	M1		Equate u_k to 0
	$k = 36$	A1	**2**	Obtain 36
(iii)	$S_{20} = \frac{20}{2}\left(2\times 23\frac{1}{3}+19\times\frac{-2}{3}\right)$	M1		Attempt sum of AP with $n = 20$
	= 340	A1		Correct unsimplified S_{20}
		A1	**3**	Obtain 340
			7	
4	$\int_{-2}^{2}(x^4+3)\,dx = \left[\frac{1}{5}x^5+3x\right]_{-2}^{2}$	M1		Attempt integration – increase of power for at least 1 term
		A1		Obtain correct $\frac{1}{5}x^5+3x$
	$= (\frac{32}{5}+6)-(\frac{-32}{5}-6)$	M1		Use limits (any two of –2, 0, 2), correct order/subtraction
	$= 24\frac{4}{5}$	A1		Obtain $24\frac{4}{5}$
	area of rectangle = 19 x 4	B1		State or imply correct area of rectangle
	hence shaded area = $76 - 24\frac{4}{5}$	M1		Attempt correct method for shaded area
	$= 51\frac{1}{5}$	A1	**7**	Obtain $51\frac{1}{5}$ aef such as 51.2, $\frac{256}{5}$
OR				
	Area = $19 - (x^4 + 3)$	M1		Attempt subtraction, either order
	$= 16 - x^4$	A1		Obtain $16 - x^4$ (not from $x^4 + 3 = 19$)
	$\int_{-2}^{2}(16-x^4)\,dx = \left[16x-\frac{1}{5}x^5\right]_{-2}^{2}$	M1		Attempt integration
		A1		Obtain $\pm\left(16x-\frac{1}{5}x^5\right)$

	$= (32 - \frac{32}{5}) - (-32 - \frac{-32}{5})$	M1		Use limits – correct order / subtraction
	$= 51\frac{1}{5}$	A1		Obtain $\pm 51\frac{1}{5}$
		A1		Obtain $51\frac{1}{5}$ only, no wrong working
			7	
5 (i)	$\frac{TA}{\sin 107} = \frac{50}{\sin 3}$	M1		Attempt use of correct sine rule to find TA, or equiv
	$TA = 914$ m	A1	**2**	Obtain 914, or better
(ii)	$TC = \sqrt{914^2 + 150^2 - 2 \times 914 \times 150 \times \cos 70}$	M1		Attempt use of correct cosine rule, or equiv, to find TC
		A1√		Correct unsimplified expression for TC, following their (i)
	= 874 m	A1	**3**	Obtain 874, or better
(iii)	dist from A = 914 x cos 70 = 313 m	M1		Attempt to locate point of closest approach
	beyond C, hence 874 m is shortest dist	A1	**2**	Convincing argument that the point is beyond C,
	OR			or obtain 859, or better
	perp dist = $914 \times \sin 70 = 859$ m			**SR** B1 for 874 stated with no method shown
			7	
6 (i)	$S_\infty = \frac{20}{1-0.9}$	M1		Attempt use of $S_\infty = \frac{a}{1-r}$
	$= 200$	A1	**2**	Obtain 200
(ii)	$S_{30} = \frac{20(1-0.9^{30})}{1-0.9}$	M1		Attempt use of correct sum formula for a GP, with $n = 30$
	$= 192$	A1	**2**	Obtain 192, or better
(iii)	$20 \times 0.9^{p-1} < 0.4$	B1		Correct $20 \times 0.9^{p-1}$ seen or implied
	$0.9^{p-1} < 0.02$			
	$(p-1)\log 0.9 < \log 0.02$	M1		Link to 0.4, rearrange to $0.9^k = c$ (or >, <), introduce
	$p - 1 > \frac{\log 0.02}{\log 0.9}$			logarithms, and drop power, or equiv correct method
	$p > 38.1$	M1		Correct method for solving their (in)equation
	hence $p = 39$	A1	**4**	State 39 (not inequality), no wrong working seen
			8	
7 (i)	$6k^2a^2 = 24$	M1*		Obtain at least two of 6, k^2, a^2
	$k^2a^2 = 4$	M1dep*		Equate $6k^m a^n$ to 24
	$ak = 2$ **A.G.**	A1	**3**	Show $ak = 2$ convincingly – no errors allowed
(ii)	$4k^3a = 128$	B1		State or imply coeff of x is $4k^3a$
	$4k^3\left(\frac{2}{k}\right) = 128$	M1		Equate to 128 and attempt to eliminate a or k
	$k^2 = 16$	A1		Obtain $k = 4$
	$k = 4$, $a = \frac{1}{2}$	A1	**4**	Obtain $a = ½$
				SR B1 for $k = \pm 4$, $a = \pm\frac{1}{2}$
(iii)	$4 \times 4 \times \left(\frac{1}{2}\right)^3 = 2$	M1		Attempt $4 \times k \times a^3$, following their a and k (allow if still in terms of a, k)
		A1	**2**	Obtain 2 (allow $2x^3$)
			9	

8 (a)(i)	$\log_a xy = p + q$	B1	**1**	State $p+q$ cwo
(ii)	$\log_a\left(\frac{a^2x^3}{y}\right) = 2 + 3p - q$	M1		Use $\log a^b = b\log a$ correctly at least once
		M1		Use $\log \frac{a}{b} = \log a - \log b$ correctly
		A1	**3**	Obtain $2 + 3p - q$
(b)(i)	$\log_{10} \frac{x^2-10}{x}$	B1	**1**	State $\log_{10} \frac{x^2-10}{x}$ (with or without base 10)
(ii)	$\log_{10} \frac{x^2-10}{x} = \log_{10} 9$	B1		State or imply that $2\log_{10} 3 = \log_{10} 3^2$
	$\frac{x^2-10}{x} = 9$	M1		Attempt correct method to remove logs
	$x^2 - 9x - 10 = 0$	A1		Obtain correct $x^2 - 9x - 10 = 0$ aef, no fractions
	$(x - 10)(x + 1) = 0$	M1		Attempt to solve three term quadratic
	$x = 10$	A1	**5**	Obtain $x = 10$ only
			10	
9 (i)	$f(1) = 1 - 1 - 3 + 3 = 0$ **A.G.**	B1		Confirm f(1) = 0, or division with no remainder shown, or matching coeffs with $R = 0$
	$f(x) = (x - 1)(x^2 - 3)$	M1		Attempt complete division by $(x - 1)$, or equiv
		A1		Obtain $x^2 + k$
		A1		Obtain completely correct quotient (allow $x^2 + 0x - 3$)
	$x^2 = 3$	M1		Attempt to solve $x^2 = 3$
	$x = \pm\sqrt{3}$	A1	**6**	Obtain $x = \pm\sqrt{3}$ only
(ii)	$\tan x = 1, \sqrt{3}, -\sqrt{3}$	B1√		State or imply $\tan x = 1$ or $\tan x =$ at least one of their roots from (i)
	$\tan x = \sqrt{3} \Rightarrow x = {}^{\pi}/_3, {}^{4\pi}/_3$	M1		Attempt to solve $\tan x = k$ at least once
	$\tan x = -\sqrt{3} \Rightarrow x = {}^{2\pi}/_3, {}^{5\pi}/_3$	A1		Obtain at least 2 of ${}^{\pi}/_3, {}^{2\pi}/_3, {}^{4\pi}/_3, {}^{5\pi}/_3$ (allow degs/decimals)
	$\tan x = 1 \Rightarrow x = {}^{\pi}/_4, {}^{5\pi}/_4$	A1		Obtain all 4 of ${}^{\pi}/_3, {}^{2\pi}/_3, {}^{4\pi}/_3, {}^{5\pi}/_3$ (exact radians only)
		B1		Obtain ${}^{\pi}/_4$ (allow degs / decimals)
		B1	**6**	Obtain ${}^{5\pi}/_4$ (exact radians only)
				SR answer only is B1 per root, max of B4 if degs / decimals
			12	

4723 Core Mathematics 3

1 (i)	Obtain integral of form ke^{-2x}	M1		any constant k different from 8
	Obtain $-4e^{-2x}$	A1		or (unsimplified) equiv
(ii)	Obtain integral of form $k(4x+5)^7$	M1		any constant k
	Obtain $\frac{1}{28}(4x+5)^7$	A1		in simplified form
	Include … + c at least once	B1		in either part
			5	
2 (i)	Form expression involving attempts at y values and addition	M1		with coeffs 1, 4 and 2 present at least once
	Obtain $k(\ln 4 + 4\ln 6 + 2\ln 8 + 4\ln 10 + \ln 12)$	A1		any constant k
	Use value of k as $\frac{1}{3}\times 2$	A1		or unsimplified equiv
	Obtain 16.27	A1	**4**	or 16.3 or greater accuracy (16.27164…)
(ii)	State 162.7 or 163	B1√	**1**	following their answer to **(i)**, maybe rounded
			5	
3 (i)	Attempt use of identity for $\tan^2\theta$	M1		using $\pm\sec^2\theta \pm 1$; or equiv
	Replace $\frac{1}{\cos\theta}$ by $\sec\theta$	B1		
	Obtain $2(\sec^2\theta - 1) - \sec\theta$	A1	**3**	or equiv
(ii)	Attempt soln of quadratic in $\sec\theta$ or $\cos\theta$	M1		as far as factorisation or substitution in correct formula
	Relate $\sec\theta$ to $\cos\theta$ and attempt at least one value of θ	M1		may be implied
	Obtain $60°, 131.8°$	A1		allow 132 or greater accuracy
	Obtain $60°, 131.8°, 228.2°, 300°$	A1	**4**	allow 132, 228 or greater accuracy; and no others between $0°$ and $360°$
			7	
4 (i)	Obtain derivative of form $kx(4x^2+1)^4$	M1		any constant k
	Obtain $40x(4x^2+1)^4$	A1		or (unsimplified) equiv
	State $x = 0$	A1√	**3**	and no other; following their derivative of form $kx(4x^2+1)^4$
(ii)	Attempt use of quotient rule	M1		or equiv
	Obtain $\frac{2x\ln x - x^2.\frac{1}{x}}{(\ln x)^2}$	A1		or equiv
	Equate to zero and attempt solution	M1		as far as solution involving e
	Obtain $e^{\frac{1}{2}}$	A1	**4**	or exact equiv; and no other; allow from ± (correct numerator of derivative)
			7	