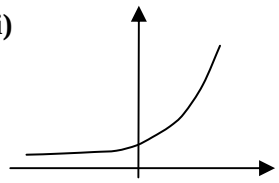
4722 Core Mathematics 2

1	**(i)**	$2(1 - \cos^2 x) = 5\cos x - 1$ $2\cos^2 x + 5\cos x - 3 = 0$ **A.G.**	M1 A1	**2**	Use $\sin^2 x = 1 - \cos^2 x$ Show given equation correctly
	(ii)	$(2\cos x - 1)(\cos x + 3) = 0$	M1		Recognise equation as quadratic in $\cos x$ and attempt recognisable method to solve
		$\cos x = ½$	M1		Attempt to find x from root(s) of quadratic
		$x = 60^o$	A1		Obtain 60^o or $^{\pi}/_3$ or 1.05 rad
		$x = 300^o$	A1√	**4**	Obtain 300^o only (or 360^o – their x) and no extra in range **SR** answer only is B1 B1
				6	
2	**(i)**	$\int (6x - 4)dx = 3x^2 - 4x + c$	M1*		Attempt integration (inc. in power for at least one term)
			A1		Obtain $3x^2 - 4x$ (or unsimplified equiv), with or without $+ c$
		$y = 3x^2 - 4x + c \Rightarrow 5 = 12 - 8 + c$ $\Rightarrow c = 1$	M1dep*		Use (2, 5) to find c
		Hence $y = 3x^2 - 4x + 1$	A1	**4**	Obtain $y = 3x^2 - 4x + 1$
	(ii)	$3p^2 - 4p + 1 = 5$	M1*		Equate their y (from integration attempt) to 5
		$3p^2 - 4p - 4 = 0$ $(p - 2)(3p + 2) = 0$	M1dep*		Attempt to solve three term quadratic
		$p = {}^{-2}/_3$	A1	**3**	Obtain $p = {}^{-2}/_3$ (allow any variable) from correct working; condone $p = 2$ still present, but A0 if extra incorrect solution
				7	
3	**(i)**	$(2 - x)^7 = 128 - 448x + 672x^2 - 560x^3$	M1		Attempt (at least) two relevant terms – product of binomial coeff, 2 and x (or expansion attempt that considers all 7 brackets)
			A1		Obtain $128 - 448x$
			A1		Obtain $672x^2$
			A1	**4**	Obtain $-560x^3$
	(ii)	$-560 \times ({}^1/_4)^3 = {}^{-35}/_4$	M1		Attempt to use coeff of x^3 from **(i)**, with clear intention to cube $^1/_4$
			A1	**2**	Obtain $^{-35}/_4$ (w^6), (allow $^{35}/_4$ from $+560x^3$ in **(i)**)
				6	

4	**(i)**	$\int_3^5 \log_{10}(2+x)\,dx \approx \frac{1}{2}\times\frac{1}{2}\times(\log 5 + 2\log 5.5 +$	M1		Attempt y-coords for at least 4 of the correct 5 x-coords only
		$2\log 6 + 2\log 6.5 + \log 7)$	M1		Use correct trapezium rule, any h, to find area between $x = 3$ and $x = 5$
			M1		Correct h (soi) for their y-values
		$\approx$ 1.55	A1	**4**	Obtain 1.55
	(ii)	$\int_3^5 \log_{10}(2+x)^{\frac{1}{2}}\,dx = \frac{1}{2}\int_3^5 \log_{10}(2+x)\,dx$	B1√		Divide by 2, or equiv, at any stage to obtain 0.78 or 0.77, following their answer to **(i)**
		$\approx$ ½ × 1.55			
		$\approx$ 0.78	B1	**2**	Explicitly use log $\sqrt{a}$ = ½ log a on a single term
				6	
5		$\int_1^3 \{(11 - 9x^{-2}) - (x^2 + 1)\}dx = [9x^{-1} - \frac{1}{3}x^3 + 10x]_1^3$	M1		Attempt subtraction (correct order) at any point
		= (3 – 9 + 30) – (9 – $^1/_3$ +10)	M1		Attempt integration – inc. in power for at least one term
		= 24 – 18$^2/_3$	A1		Obtain ± (– $^1/_3x^3$ + 10x) or 11x and $^1/_3x^3 + x$
		= 5$^1/_3$	M1		Obtain remaining term of form kx^{-1}
		OR	A1		Obtain ± $9x^{-1}$ or any unsimplified equiv
		$[11x + 9x^{-1}]_1^3 - [\frac{1}{3}x^3 + x]_1^3$	M1		Use limits x = 1, 3 – correct order & subtraction
		= [(33 + 3) – (11 + 9)] – [(9 + 3) – ($^1/_3$ + 1)]	A1	**7**	Obtain 5$^1/_3$, or exact equiv
		= 16 – 10$^2/_3$			
		= 5$^1/_3$			
				7	
6	**(i)**	f(–3) = 0 ⇒ –54 + 9a – 3b + 15 = 0	M1		Attempt f(–3) and equate to 0, or equiv method
		3a – b = 13	A1		Obtain 3a – b = 13, or unsimplified equiv
		f(2) = 35 ⇒ 16 + 4a + 2b + 15 = 35	M1		Attempt f(2) and equate to 35, or equiv method
		2a + b = 2	A1		Obtain 2a + b = 2, or unsimplified equiv
		Hence a = 3, b = –4	M1		Attempt to solve simultaneous eqns
			A1	**6**	Obtain a = 3, b = –4
	(ii)	f(x) = (x + 3)(2x^2 – 3x + 5)	M1		Attempt complete division by (x + 3), or equiv
			A1		Obtain 2x^2 – 3x + c or 2x^2 + bx + 5, from correct f(x)
		ie quotient is (2x^2 – 3x + 5)	A1	**3**	Obtain 2x^2 – 3x + 5 (state or imply as quotient)
				9	

7	(i)	$13^2 = 10^2 + 14^2 - 2 \times 10 \times 14 \times \cos\theta$	M1		Attempt to use correct cosine rule in ΔABC
		$\cos\theta = 0.4536$			
		$\theta = 1.10$ **A.G.**	A1	**2**	Obtain 1.10 radians (allow 1.1 radians) **SR** B1 only for verification of 1.10, unless complete method
	(ii)	arc $EF = 4 \times 1.10 = 4.4$	B1		State or imply $EF = 4.4$cm (allow 4×1.10)
		perimeter $= 4.4 + 10 + 13 + 6$	M1		Attempt perimeter of region - sum of arc and three sides with attempt to subtract 4 from at least one relevant side
		$= 33.4$ cm	A1	**3**	Obtain 33.4 cm
	(iii)	area $AEF = ½ \times 4^2 \times 1.1$	M1		Attempt use of $(½)r^2\theta$, with $r = 4$ and $\theta = 1.10$
		$= 8.8$	A1		Obtain 8.8
		area $ABC = ½ \times 10 \times 14 \times \sin 1.1$	M1		Attempt use of $(½)ab\sin\theta$, sides consistent with angle used
		$= 62.4$	A1		Obtain 62.4 or better (allow 62.38 or 62.39)
		hence total area $= 53.6$ cm^2	A1	**5**	Obtain total area as 53.6 cm^2
				10	

8	(i)	$u_5 = 8 + 4 \times 3$	M1		Attempt $a + (n-1)d$ or equiv inc list of terms
		$= 20$ **A.G.**	A1	**2**	Obtain 20
	(ii)	$u_n = 3n + 5$ ie $p = 3$, $q = 5$	B1		Obtain correct expression, poss unsimplified, eg $8 + 3(n-1)$
			B1	**2**	Obtain correct $3n + 5$, or $p = 3$, $q = 5$ stated
	(iii)	arithmetic progression	B1	**1**	Any mention of arithmetic
	(iv)	$\frac{2N}{2}(16 + (2N-1)3) - \frac{N}{2}(16 + (N-1)3) = 1256$	M1		Attempt S_N, using any correct formula (inc $\sum(3n+5)$)
			M1		Attempt S_{2N} , using any correct formula, with $2N$ consistent (inc $\sum(3n+5)$)
		$26N + 12N^2 - 13N - 3N^2 = 2512$			
		$9N^2 + 13N - 2512 = 0$	M1*		Attempt subtraction (correct order) and equate to 1256
		$(9N + 157)(N - 16) = 0$	M1dep*		Attempt to solve quadratic in N
		$N = 16$	A1	**5**	Obtain $N = 16$ only, from correct working
					OR: alternative method is to use $^n/_2\,(a + l) = 1256$
			M1		Attempt given difference as single summation with N terms
			M1		Attempt $a = u_{N+1}$
			M1		Attempt $l = u_{2N}$
			M1		Equate to 1256 and attempt to solve quadratic
			A1		Obtain $N = 16$ only, from correct working
				10	

9	**(i)** 	M1 A1 B1	**3**	Reasonable graph in both quadrants Correct graph in both quadrants State or imply (0, 6)
	(ii) $9^x = 150$	M1		Introduce logarithms throughout, or equiv with $\log_9$
	$x \log 9 = \log 150$	M1		Use $\log a^b = b \log a$ and attempt correct method to find x
	$x = 2.28$	A1	**3**	Obtain $x = 2.28$
	(iii) $6 \times 5^x = 9^x$	M1		Form eqn in x and take logs throughout (any base)
	$\log_3 (6 \times 5^x) = \log_3 9^x$	M1		Use $\log a^b = b \log a$ correctly on $\log 5^x$ or $\log 9^x$ or legitimate combination of these two
	$\log_3 6 + x \log_3 5 = x \log_3 9$	M1		Use $\log ab = \log a + \log b$ correctly on $\log (6 \times 5^x)$ or $\log 6$
	$\log_3 3 + \log_3 2 + x \log_3 5 = 2x$	M1		Use $\log_3 9 = 2$ or equiv (need base 3 throughout that line)
	$x(2 - \log_3 5) = 1 + \log_3 2$ $x = \frac{1+\log_3 2}{2-\log_3 5}$ **A.G.**	A1	**5**	Obtain $x = \frac{1+\log_3 2}{2-\log_3 5}$ convincingly (inc base 3 throughout)
			11	

4723 Core Mathematics 3

1	Obtain integral of form $k(2x-7)^{-1}$	M1		any constant k
	Obtain correct $-5(2x-7)^{-1}$	A1		or equiv
	Include … $+c$	B1	**3**	at least once; following any integral
			3	

2	**(i)** Use $\sin 2\theta = 2\sin\theta\cos\theta$	B1		
	Attempt value of $\sin\theta$ from $k\sin\theta\cos\theta = 5\cos\theta$	M1		any constant k; or equiv
	Obtain $\frac{5}{12}$	A1	**3**	or exact equiv; ignore subsequent work
	(ii) Use $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$ or $\operatorname{cosec}^2\theta = 1+\cot^2\theta$	B1		or equiv
	Attempt to produce equation involving $\cos\theta$ only	M1		using $\sin^2\theta = \pm 1 \pm \cos^2\theta$ or equiv
	Obtain $3\cos^2\theta + 8\cos\theta - 3 = 0$	A1		or equiv
	Attempt solution of 3-term quadratic equation	M1		using formula or factorisation or equiv
	Obtain $\frac{1}{3}$ as only final value of $\cos\theta$	A1	**5**	or exact equiv; ignore subsequent work
			8	

3	**(i)** Obtain or clearly imply $60\ln x$	B1		
	Obtain ($60\ln 20 - 60\ln 10$ and hence) $60\ln 2$	B1	**2**	with no error seen
	(ii) Attempt calculation of form $k(y_0 + 4y_1 + y_2)$	M1		any constant k; using y-value attempts
	Identify k as $\frac{5}{3}$	A1		
	Obtain $\frac{5}{3}(6+4\times 4+3)$ and hence $\frac{125}{3}$ or 41.7	A1	**3**	or equiv
	(iii) Equate answers to parts **(i)** and **(ii)**	M1		provided $\ln 2$ involved
	Obtain $60\ln 2 = \frac{125}{3}$ and hence $\frac{25}{36}$	A1	**2**	AG; necessary detail required including clear use of an exact value from **(ii)**
			7	

4	**(i)** Attempt correct process for composition	M1		numerical or algebraic
	Obtain (7 and hence) 0	A1	**2**	
	(ii) Attempt to find x-intercept	M1		
	Obtain $x \le 7$	A1	**2**	or equiv; condone use of <
	(iii) Attempt correct process for finding inverse	M1		
	Obtain $\pm(2-y)^3 - 1$ or $\pm(2-x)^3 - 1$	A1		
	Obtain correct $(2-x)^3 - 1$	A1	**3**	or equiv in terms of x
	(iv) Refer to reflection in $y = x$	B1	**1**	or clear equiv
			8	
