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5 (i)	$\int x dy = \int ((y-3)^2 - 2) dy$	B1	Show $x = y^2 - 6y + 7$ convincingly
	$=\int (y^2 - 6y + 7) dy$ A.G.	B1	State or imply that required area = $\int x dy$
	$3 + \sqrt{(2+2)} = 5$, $3 + \sqrt{(14+2)} = 7$	B1	Use $x = 2$, 14 to show new limits of $y = 5$, 7
(ii)	$\left[\frac{1}{3}y^3 - 3y^2 + 7y\right]_5^7$	M1	Integration attempt, with at least one
term	$= ({}^{343}/_3 - 147 + 49) - ({}^{125}/_3 - 75 + 35)$ = $16^1/_3 - 1^2/_3$ = $14^2/_3$	A1 M1 A1 4	correct All three terms correct Attempt $F(7) - F(5)$ Obtain 14 ² / ₃ , or exact equiv
6 (i)	$ABC = 360 - (150 + 110) = 100^{\circ}$ A.G.	B1	Show convincingly that angle ABC is 100°
(ii)	$CA^{2} = 15^{2} + 27^{2} - 2 \times 15 \times 27 \times \cos 100^{0}$ = 1094.655	<u>M1</u>	Attempt use of correct cosine rule
	CA = 33.1	A1	Obtain 33.1 km
(iii)	$\frac{\sin C}{15} = \frac{\sin 100}{33.1} \qquad \text{or} \qquad \frac{\sin A}{27} = \frac{\sin 100}{33.1}$	M1	Attempt use of sine rule to find angle C or A
	$C = 26.5^{\circ}$ $A = 53.5^{\circ}$ Hence bearing is 263°	A1√ A1 A1√ 4	(or equiv using cosine rule) Correct unsimplified eqn, following their C Obtain $C = 26.5^{\circ}$ or $A = 53.5^{\circ}$ (allow 53.4°) Obtain 263 or 264 (or 290° – their angle C 210 + their angle A)
7 (a)	$\int (x^5 - x^4 + 5x^3) \mathrm{d}x$	M1	Expand brackets and attempt integration, or
	$= \frac{1}{6}x^6 - \frac{1}{5}x^5 + \frac{5}{4}x^4 (+c)$	A1	other valid integration attempt Obtain at least one correct term
	$= \frac{1}{6}x + \frac{1}{5}x + \frac{1}{4}x + \frac{1}{6}x + \frac{1}{6}$	A1 A1	Obtain a fully correct expression
		B1	For $+c$, and no $\int or dx$ (can be given in
		4	(b)(i) if not given here)
(b)	(i) $-6x^{-3}(+c)$	M1 A1 2	Obtain integral of the form kx^{-3} Obtain $-6x^{-3}$ (+ <i>c</i>)
	(ii) $\left[-6x^{-3}\right]_{2}^{\infty}$ = $\frac{3}{4}$	B1* B1d* 2	State or imply that $F(\infty) = 0$ (for kx^n , $n - 1$) Obtain ³ / ₄ (or equiv)

8 (i)		M1 A1 B1 3	Attempt sketch of exponential graph (1 st quad) - if seen in 2 nd quad must be approx correct Correct graph in both quadrants State or imply (0, 2) only
(ii)	$8^{x} = 2 \times 3^{x}$ $\log_{2} 8^{x} = \log_{2} (2 \times 3^{x})$ $x \log_{2} 8 = \log_{2} 2 + x \log_{2} 3$	M1 M1 M1	Form equation in x and take logs (to any consistent base, or no base) – could use log $_8$ Use log $a^b = b \log a$ Use log $ab = \log a + \log b$, or equiv with $\log a/b$
	$3x = 1 + x \log_2 3$ $x (3 - \log_2 3) = 1$, hence $x = \frac{1}{3 - \log_2 3}$ A.G.	M1 A1	Use $\log_2 8 = 3$ Show given answer correctly
0ĸ	$8^{x} = 2 \times 3^{x}$ $2^{3x} = 2 \times 3^{x}$ $2^{(3x-1)} = 3^{x}$ $\log_{2} 2^{(3x-1)} = \log_{2} 3^{x}$ $(3x-1)\log_{2} 2 = x \log_{2} 3$ $x (3 - \log_{2} 3) = 1, \text{ hence } x = \frac{1}{3 - \log_{2} 3} \text{ A.G.}$	M1 M1 M1 A1 5	Use $8^x = 2^{3x}$ Attempt to rearrange equation to $2^k = 3^x$ Take logs (to any base) Use log $a^b = b \log a$ Show given answer correctly
9 (a)	(i) $2\sin x \cdot \frac{\sin x}{\cos x} - 5 = \cos x$ $2\sin^2 x - 5\cos x = \cos^2 x$	M1	Use $\tan x \equiv \frac{\sin x}{\cos x}$
	$2-2\cos^2 x - 5\cos x = \cos^2 x$ $3\cos^2 x + 5\cos x - 2 = 0$	M1 A1 3	Use $\sin^2 x \equiv 1 - \cos^2 x$ Show given equation convincingly
(ii)	$(3\cos x - 1)(\cos x + 2) = 0$ $\cos x = \frac{1}{3}$ x = 1.23 rad x = 5.05 rad	M1 M1 A1 A1√	Attempt to solve quadratic in cosx Attempt to find x from root(s) of quadratic Obtain 1.23 rad or 70.5° Obtain 5.05 rad or 289° (or $2\pi / 360^{\circ}$ - their solution) SR: B1 B1 for answer(s) only
(b)	0.5x0.25x{cos0+2(cos0.25+cos0.5+cos0.75)+cos1}	4 M1	Attempt <i>y</i> -coords for at least 4 of the correct 5 <i>x</i> -coords
		M1	Use correct trapezium rule, any <i>h</i> , for their <i>y</i> values to find area between $x = 0$ and $x = 1$
	≈ 0.837	M1 A1 4	Correct <i>h</i> (soi) for their <i>y</i> values Obtain 0.837

10 (i)	$u_{15} = 2 + 14 \ge 0.5$	M1	Attempt use of $a + (n-1)d$
	= 9 km	A1	Obtain 9 km
		2	
(ii)	$u_{20} = 2 \ge 1.1^{19} = 12.2$	B1	State, or imply, $r = 1.1$
		M1	Attempt u_{20} , using ar^{n-1}
	$u_{19} = 2 \ge 1.1^{18} = 11.1$	A1	Obtain $u_{20} = 12.2$, and obtain $u_{19} = 11.1$
OR			
		B 1	State, or imply, $r = 1.1$
		M1	Attempt to solve $ar^{n-1} = 12$
		A1	Obtain $n = 20$ (allow $n \ge 20$)
		3	
(iii)	$2(1.1^n - 1) > 200$	B1	State or imply $S_N = \frac{2(1.1^n - 1)}{(1.1 - 1)}$
	$\frac{2(1.1^n - 1)}{(1.1 - 1)} > 200$		(1.1-1)
	$1.1^n > 11$	M1	Link (any sign) their attempt at S_N (of a GP)
			to 200 and attempt to solve
	$n > \frac{\log 11}{\log 1.1}$	A1	Obtain 26, or 25.2 or better
	n > 25.2 ie Day 26	A1	Conclude $n = 26$ only, or equiv eg Day 26
	n 25.2 10 Day 20	4	Conclude n 20 only, of equiver buy 20
(iv)	$swum = 2 \times 30 = 60 \text{ km}$	B1	Obtain 60 km, or 2 x 30km
	$run = \frac{1}{2} \times 30 \times (4 + 29 \times 0.5)$	M1	Attempt sum of AP, $d = 0.5$, $a = 2$, $n = 30$
	= 277.5 km		
	$cycle = 2(1.1^{30} - 1)$	M1	Attempt sum of GP, $r = 1.1$, $a = 2$, $n = 30$
	(1.1-1)		1
	= 329.0 km		
	total = 666 km	A1	Obtain 666 or 667 km
		4	