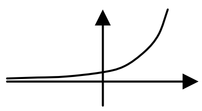
4722 Core Mathematics 2

1 (i)	$\cos\theta = \frac{6.4^2+7.0^2-11.3^2}{2\times 6.4\times 7.0}$	M1		Attempt use of cosine rule (any angle)
	$= -0.4211$	A1		Obtain one of 115°, 34.2°, 30.9°, 2.01, 0.597, 0.539
	$\theta = 115^\circ$ or 2.01 rads	A1	**3**	Obtain 115° or 2.01 rads, or better
(ii)	area = $\frac{1}{2}\times 7\times 6.4\times \sin 115$	M1		Attempt triangle area using (½)*ab*sin*C*, or equiv
	= 20.3 cm^2	A1	**2**	Obtain 20.3 (cao)
			5	
2 (i)	$a + 9d = 2(a + 3d)$	M1*		Attempt use of $a + (n-1)d$ or $a + nd$ at least once for u_4, u_{10} or u_{20}
	$a = 3d$	A1		Obtain $a = 3d$ (or unsimplified equiv) and $a + 19d = 44$
	$a + 19d = 44 \Rightarrow 22d = 44$	M1dep*		Attempt to eliminate one variable from two simultaneous equations in a and d, from u_4, u_{10}, u_{20} and no others
	$d = 2$, $a = 6$	A1	**4**	Obtain $d = 2$, $a = 6$
(ii)	$S_{50} = {}^{50}/_2\ (2\text{x}6 + 49\text{x}2)$	M1		Attempt S_{50} of AP, using correct formula, with $n = 50$, allow $25(2a + 24d)$
	= 2750	A1	**2**	Obtain 2750
			6	
3	$\log 7^x = \log 2^{x+1}$	M1		Introduce logarithms throughout, or equiv with base 7 or 2
	$x\log 7 = (x+1)\log 2$	M1		Drop power on at least one side
		A1		Obtain correct linear equation (allow with no brackets)
	$x(\log 7 - \log 2) = \log 2$	M1		**Either** expand bracket and attempt to gather x terms, **or** deal correctly with algebraic fraction
	$x = 0.553$	A1	**5**	Obtain $x = 0.55$, or rounding to this, with no errors seen
			5	
4 (i)	$(x^2-5)^3 = (x^2)^3 + 3(x^2)^2(-5) + 3(x^2)(-5)^2 + (-5)^3$	M1*		Attempt expansion, with product of powers of x^2 and $\pm$ 5, at least 3 terms
	$= x^6 - 15x^4 + 75x^2 - 125$	M1*		Use at least 3 of binomial coeffs of 1, 3, 3, 1
		A1dep*		Obtain at least two correct terms, coeffs simplified
		A1	**4**	Obtain fully correct expansion, coeffs simplified
	OR			
	$(x^2-5)^3 = (x^2-5)(x^4-10x^2+25)$	M2		Attempt full expansion of all 3 brackets
	$= x^6 - 15x^4 + 75x^2 - 125$	A1		Obtain at least two correct terms
		A1		Obtain full correct expansion
(ii)	$\int (x^2-5)^3\,dx = \frac{1}{7}x^7 - 3x^5 + 25x^3 - 125x + c$	M1		Attempt integration of terms of form kx^n
		A1√		Obtain at least two correct terms, allow unsimplified coeffs
		A1		Obtain $\frac{1}{7}x^7 - 3x^5 + 25x^3 - 125x$
		B1	**4**	+ c, and no dx or ∫ sign
			8	

5	**(i)**	$2x = 30^o, 150^o$	M1		Attempt $\sin^{-1}$ 0.5, then divide or multiply by 2
		$x = 15^o, 75^o$	A1		Obtain 15^o (allow $^\pi/_{12}$ or 0.262)
			A1	**3**	Obtain 75^o (not radians), and no extra solutions in range
	(ii)	$2(1 - \cos^2 x) = 2 - \sqrt{3}\cos x$	M1		Use $\sin^2 x = 1 - \cos^2 x$
		$2\cos^2 x - \sqrt{3}\cos x = 0$	A1		Obtain $2\cos^2 x - \sqrt{3}\cos x = 0$ or equiv (no constant terms)
		$\cos x\,(2\cos x - \sqrt{3}) = 0$	M1		Attempt to solve quadratic in $\cos x$
		$\cos x = 0$, $\cos x = \frac{1}{2}\sqrt{3}$	A1		Obtain 30^o (allow $^\pi/_6$ or 0524), and no extra solns in range
		$x = 90^o$, $x = 30^o$	B1	**5**	Obtain 90^o (allow $^\pi/_2$ or 1.57), from correct quadratic only
				SR	answer only B1 one correct solution
					B1 second correct solution, and no others
				8	
6		$\int (3x^2 + a)\ dx = x^3 + ax + c$	M1		Attempt to integrate
			A1		Obtain at least one correct term, allow unsimplified
			A1		Obtain $x^3 + ax$
		$(-1, 2) \Rightarrow -1 - a + c = 2$	M1		Substitute at least one of (–1, 2) or (2, 17) into integration attempt involving a and c
		$(2, 17) \Rightarrow 8 + 2a + c = 17$	A1		Obtain two correct equations, allow unsimplified
			M1		Attempt to eliminate one variable from two equations in a and c
		$a = 2$, $c = 5$	A1		Obtain $a = 2$, $c = 5$, from correct equations
		Hence $y = x^3 + 2x + 5$	A1	**8**	State $y = x^3 + 2x + 5$
				8	
7	**(i)**	$f(-2) = -16 + 36 - 22 - 8$	M1		Attempt f(-2), or equiv
		$= -10$	A1	**2**	Obtain -10
	(ii)	$f(\frac{1}{2}) = \frac{1}{4} + 2\frac{1}{4} + 5\frac{1}{2} - 8 = 0$ AG	M1		Attempt f(½) (no other method allowed)
			A1	**2**	Confirm f(½) = 0, extra line of working required
	(iii)	$f(x) = (2x - 1)(x^2 + 5x + 8)$	M1		Attempt complete division by $(2x - 1)$ or $(x - \frac{1}{2})$ or equiv
			A1		Obtain $x^2 + 5x + c$ or $2x^2 + 10x + c$
			A1	**3**	State $(2x - 1)(x^2 + 5x + 8)$ or $(x - \frac{1}{2})(2x^2 + 10x + 16)$
	(iv)	$f(x)$ has one real root ($x = \frac{1}{2}$)	B1√		State 1 root, following their quotient, ignore reason
		because $b^2 - 4ac = 25 - 32 = -7$ hence quadratic has no real roots as $-7 < 0$,	B1√	**2**	Correct calculation, eg discriminant or quadratic formula, following their quotient, or cubic has max at (-2.15, -9.9)
				9	

8 **(i)**	$\frac{1}{2} \times r^2 \times 1.2 = 60$	M1		Attempt $(\frac{1}{2}) r^2\theta = 60$
	$r = 10$	A1		Obtain $r = 10$
	$r\theta = 10 \times 1.2 = 12$	B1√		State or imply arc length is $1.2r$, following their r
	perimeter = 10 + 10 + 12 = 32 cm	A1	**4**	Obtain 32
(ii)(a)	$u_5 = 60 \times 0.6^4$	M1		Attempt u_5 using ar^4, or list terms
	$= 7.78$	A1	**2**	Obtain 7.78, or better
(b)	$S_{10} = \dfrac{60(1-0.6^{10})}{1-0.6}$	M1		Attempt use of correct sum formula for a GP, or sum terms
	$= 149$	A1	**2**	Obtain 149, or better (allow 149.0 – 149.2 inclusive)
(c)	common ratio is less than 1, so series is convergent and hence sum to infinity exists	B1		series is convergent or $-1 < r < 1$ (allow $r < 1$) or reference to areas getting smaller / adding on less each time
	$S_\infty = \dfrac{60}{1-0.6}$	M1		Attempt S_∞ using $\dfrac{a}{1-r}$
	$= 150$	A1	**3**	Obtain $S_\infty = 150$
				SR B1 only for 150 with no method shown
			11	
9 **(i)**		B1		Sketch graph showing exponential growth (both quadrants)
		B1	**2**	State or imply (0, 4)
(ii)	$4k^x = 20k^2$			
	$k^x = 5k^2$	M1		Equate $4k^x$ to $20k^2$ and take logs (any, or no, base)
	$x = \log_k 5k^2$			
	$x = \log_k 5 + \log_k k^2$	M1		Use $\log ab = \log a + \log b$
	$x = 2\log_k k + \log_k 5$	M1		Use $\log a^b = b \log a$
	$x = 2 + \log_k 5$ AG	A1	**4**	Show given answer correctly
OR	$4k^x = 20k^2$			
	$k^x = 5k^2$	M1		Attempt to rewrite as single index
	$k^{x-2} = 5$	A1		Obtain $k^{x-2} = 5$ or equiv eg $4k^{x-2} = 20$
	$x - 2 = \log_k 5$	M1		Take logs (to any base)
	$x = 2 + \log_k 5$ AG	A1		Show given answer correctly
(iii) (a)	area $\approx \frac{1}{2} \times \frac{1}{2} \times \left(4k^0 + 8k^{\frac{1}{2}} + 4k^1\right)$	M1		Attempt y-values at $x = 0$, ½ and 1, and no others
		M1		Attempt to use correct trapezium rule, 3 y-values, $h = \frac{1}{2}$
	$\approx 1 + 2k^{\frac{1}{2}} + k$	A1	**3**	Obtain a correct expression, allow unsimplified
(b)	$1 + 2k^{\frac{1}{2}} + k = 16$	M1		Equate attempt at area to 16
	$\left(k^{\frac{1}{2}} + 1\right)^2 = 16$	M1		Attempt to solve 'disguised' 3 term quadratic
	$k^{\frac{1}{2}} = 3$			
	$k = 9$	A1	**3**	Obtain $k = 9$ only
			12	

4723 Core Mathematics 3

1 (i)	State $y = \sec x$	B1		
(ii)	State $y = \cot x$	B1		
(iii)	State $y = \sin^{-1} x$	B1	3	
			[3]	
2	<u>Either</u>: State or imply $\int \pi(2x-3)^4 \, dx$	B1		or unsimplified equiv
	Obtain integral of form $k(2x-3)^5$	M1		any constant k involving π or not
	Obtain $\frac{1}{10}(2x-3)^5$ or $\frac{1}{10}\pi(2x-3)^5$	A1		
	Attempt evaluation using 0 and $\frac{3}{2}$	M1		subtraction correct way round
	Obtain $\frac{243}{10}\pi$	A1	5	or exact equiv
	<u>Or</u>: State or imply $\int \pi(2x-3)^4 \, dx$	B1		or unsimplified equiv
	Expand and obtain integral of order 5	M1		with at least three terms correct
	Ob'n $\frac{16}{5}x^5 - 24x^4 + 72x^3 - 108x^2 + 81x$	A1		with or without π
	Attempt evaluation using (0 and) $\frac{3}{2}$	M1		
	Obtain $\frac{243}{10}\pi$	A1	(5)	or exact equiv
			[5]	
3 (i)	Attempt use of identity for $\sec^2 \alpha$	M1		using $\pm\tan^2\alpha \pm 1$
	Obtain $1 + (m+2)^2 - (1+m^2)$	A1		absent brackets implied by subsequent correct working
	Obtain $4m + 4 = 16$ and hence $m = 3$	A1	3	
(ii)	Attempt subn in identity for $\tan(\alpha + \beta)$	M1		using $\dfrac{\pm\tan\alpha \pm \tan\beta}{1 \pm \tan\alpha\tan\beta}$
	Obtain $\dfrac{5+3}{1-15}$ or $\dfrac{m+2+m}{1-m(m+2)}$	A1√		following their m
	Obtain $-\frac{4}{7}$	A1	3	or exact equiv
			[6]	
4 (i)	Obtain $\frac{1}{3}e^{3x} + e^x$	B1		
	Substitute to obtain $\frac{1}{3}e^{9a} + e^{3a} - \frac{1}{3}e^{3a} - e^a$	B1		or equiv
	Equate definite integral to 100 and attempt rearrangement	M1		as far as $e^{9a} = \ldots$
	Introduce natural logarithm	M1		using correct process
	Obtain $a = \frac{1}{9}\ln(300 + 3e^a - 2e^{3a})$	A1	5	AG; necessary detail needed
(ii)	Obtain correct first iterate	B1		allow for 4 dp rounded or truncated
	Show correct iteration process	M1		with at least one more step
	Obtain at least three correct iterates in all	A1		allowing recovery after error
	Obtain 0.6309	A1	4	following at least three correct steps; answer required to exactly 4 dp
	[0.6 → 0.631269 → 0.630884 → 0.630889]			
			[9]	