

**Mark Scheme 4723
June 2007**

1 (i)	Attempt use of product rule	M1		
	Obtain $3x^2(x+1)^5 + 5x^3(x+1)^4$	A1	2 or equiv	
	[Or: (following complete expansion and differentiation term by term)			
	Obtain $8x^7 + 35x^6 + 60x^5 + 50x^4 + 20x^3 + 3x^2$	B2	allow B1 if one term incorrect]	
(ii)	Obtain derivative of form $kx^3(3x^4 + 1)^n$	M1	any constants k and n	
	Obtain derivative of form $kx^3(3x^4 + 1)^{-\frac{1}{2}}$	M1		
	Obtain correct $6x^3(3x^4 + 1)^{-\frac{1}{2}}$	A1	3 or (unsimplified) equiv	
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2	Identify critical value $x = 2$	B1		
	Attempt process for determining both critical values	M1		
	Obtain $\frac{1}{3}$ and 2	A1		
	Attempt process for solving inequality	M1	table, sketch ...; implied by plausible answer	
	Obtain $\frac{1}{3} < x < 2$	A1	5	
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3 (i)	Attempt correct process for composition	M1	numerical or algebraic	
	Obtain (16 and hence) 7	A1	2	
	(ii)	Attempt correct process for finding inverse	M1	maybe in terms of y so far
		Obtain $(x-3)^2$	A1	2 or equiv; in terms of x , not y
	(iii)	Sketch (more or less) correct $y = f(x)$	B1	with 3 indicated or clearly implied on y -axis, correct curvature, no maximum point
		Sketch (more or less) correct $y = f^{-1}(x)$	B1	right hand half of parabola only
		State reflection in line $y = x$	B1	3 or (explicit) equiv; independent of earlier marks
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	4 (i)	Obtain integral of form $k(2x+1)^{\frac{4}{3}}$	M1	or equiv using substitution; any constant k
Obtain correct $\frac{3}{8}(2x+1)^{\frac{4}{3}}$		A1	or equiv	
Substitute limits in expression of form $(2x+1)^n$ and subtract the correct way round		M1	using adjusted limits if subn used	
Obtain 30		A1	4	
(ii)		Attempt evaluation of $k(y_0 + 4y_1 + y_2)$	M1	any constant k
		Identify k as $\frac{1}{3} \times 6.5$	A1	
		Obtain 29.6	A1	3 or greater accuracy (29.554566...)
		[SR: (using Simpson's rule with 4 strips)		
		Obtain $\frac{1}{3} \times 3.25(1 + 4 \times \sqrt[3]{7.5} + 2 \times \sqrt[3]{14} + 4 \times \sqrt[3]{20.5} + 3)$ and hence 29.9	B1	or greater accuracy (29.897...)]

5 (i)	State $e^{-0.04t} = 0.5$ Attempt solution of equation of form $e^{-0.04t} = k$ Obtain 17	B1 M1 A1	or equiv using sound process; maybe implied 3 or greater accuracy (17.328...)
(ii)	Differentiate to obtain form $ke^{-0.04t}$ Obtain $(\pm) 9.6e^{-0.04t}$ Equate attempt at first derivative to $(\pm) 2.1$ and attempt solution Obtain 38	*M1 A1 M1 A1	constant k different from 240 or (unsimplified) equiv dep *M; method maybe implied 4 or greater accuracy (37.9956...)
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6 (i)	Obtain integral of form $k_1e^{2x} + k_2x^2$ Obtain correct $3e^{2x} + \frac{1}{2}x^2$ Obtain $3e^{2a} + \frac{1}{2}a^2 - 3$ Equate definite integral to 42 and attempt rearrangement Confirm $a = \frac{1}{2}\ln(15 - \frac{1}{6}a^2)$	M1 A1 A1 M1 A1	any non-zero constants k_1, k_2 using sound processes 5 AG; necessary detail required
(ii)	Obtain correct first iterate 1.348... Attempt correct process to find at least 2 iterates Obtain at least 3 correct iterates Obtain 1.344	B1 M1 A1 A1	4 answer required to exactly 3 d.p.; allow recovery after error
[1 → 1.34844 → 1.34382 → 1.34389]			
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7 (i)	Show correct general shape (alternating above and below x -axis) Draw (more or less) correct sketch	M1 A1	with no branch reaching x -axis 2 with at least one of 1 and -1 indicated or clearly implied
(ii)	Attempt solution of $\cos x = \frac{1}{3}$ Obtain 1.23 or 0.392π Obtain 5.05 or 1.61π	M1 A1 A1	maybe implied; or equiv or greater accuracy 3 or greater accuracy and no others within $0 \leq x \leq 2\pi$; penalise answer(s) to 2sf only once
(iii)	<u>Either</u> : Obtain equation of form $\tan \theta = k$ Obtain $\tan \theta = 5$ Obtain two values only of form $\theta, \theta + \pi$ Obtain 1.37 and 4.51 (or 0.437π and 1.44π)	M1 A1 M1 A1	any constant k ; maybe implied within $0 \leq x \leq 2\pi$; allow degrees at this stage 4 allow ± 1 in third sig fig; or greater accuracy
<u>Or</u> :	(for methods which involve squaring, etc.) Attempt to obtain eqn in one trig ratio Obtain correct value Attempt solution at least to find one value in first quadrant and one value in third Obtain 1.37 and 4.51 (or equivalents as above)	M1 A1 M1 A1	$\tan^2 \theta = 25, \cos^2 \theta = \frac{1}{26}, \dots$ ignoring values in second and fourth quadrants

8 (i)	Attempt use of quotient rule	M1	allow for numerator 'wrong way round'; or equiv
	Obtain $\frac{(4 \ln x + 3)\frac{4}{x} - (4 \ln x - 3)\frac{4}{x}}{(4 \ln x + 3)^2}$	A1	or equiv
	Confirm $\frac{24}{x(4 \ln x + 3)^2}$	A1	3 AG; necessary detail required
(ii)	Identify $\ln x = \frac{3}{4}$	B1	or equiv
	State or imply $x = e^{\frac{3}{4}}$	B1	
	Substitute e^k completely in expression for derivative	M1	and deal with $\ln e^k$ term
	Obtain $\frac{2}{3}e^{-\frac{3}{4}}$	A1	4 or exact (single term) equiv
(iii)	State or imply $\int \frac{4\pi}{x(4 \ln x + 3)^2} dx$	B1	
	Obtain integral of form $k \frac{4 \ln x - 3}{4 \ln x + 3}$		
	or $k(4 \ln x + 3)^{-1}$	*M1	any constant k
	Substitute both limits and subtract right way round	M1	dep *M
	Obtain $\frac{4}{21}\pi$	A1	4 or exact equiv
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9 (i)	Attempt use of either of $\tan(A \pm B)$ identities	M1	
	Substitute $\tan 60^\circ = \sqrt{3}$ or $\tan^2 60^\circ = 3$	B1	
	Obtain $\frac{\tan \theta + \sqrt{3}}{1 - \sqrt{3} \tan \theta} \times \frac{\tan \theta - \sqrt{3}}{1 + \sqrt{3} \tan \theta}$	A1	or equiv (perhaps with $\tan 60^\circ$ still involved)
	Obtain $\frac{\tan^2 \theta - 3}{1 - 3 \tan^2 \theta}$	A1	4 AG
(ii)	Use $\sec^2 \theta = 1 + \tan^2 \theta$	B1	
	Attempt rearrangement and simplification of equation involving $\tan^2 \theta$	M1	or equiv involving $\sec \theta$
	Obtain $\tan^4 \theta = \frac{1}{3}$	A1	or equiv $\sec^2 \theta = 1.57735\dots$
	Obtain 37.2	A1	or greater accuracy
	Obtain 142.8	A1	5 or greater accuracy; and no others between 0 and 180
(iii)	Attempt rearrangement of $\frac{\tan^2 \theta - 3}{1 - 3 \tan^2 \theta} = k^2$ to form		
	$\tan^2 \theta = \frac{f(k)}{g(k)}$	M1	
	Obtain $\tan^2 \theta = \frac{k^2 + 3}{1 + 3k^2}$	A1	
	Observe that RHS is positive for all k , giving one value in each quadrant	A1	3 or convincing equiv