

**ADVANCED GCE**  
**MATHEMATICS**  
Core Mathematics 3

**4723**

Candidates answer on the Answer Booklet

**OCR Supplied Materials:**

- 8 page Answer Booklet
- List of Formulae (MF1)

**Other Materials Required:**

None

**Monday 1 June 2009**  
**Morning**

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1

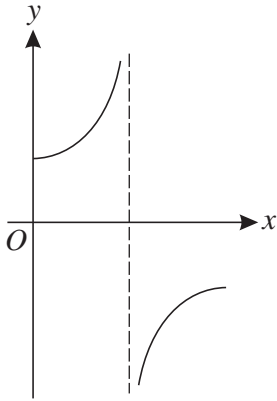


Fig. 1

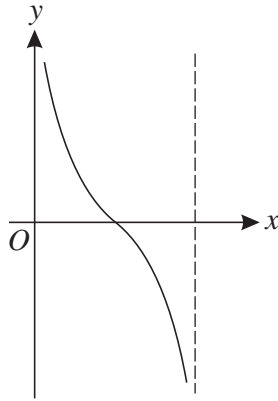


Fig. 2

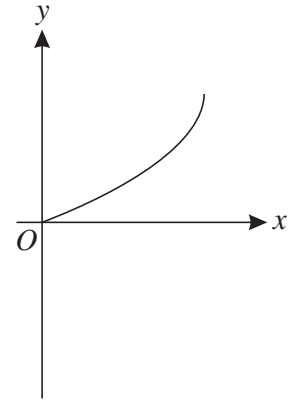


Fig. 3

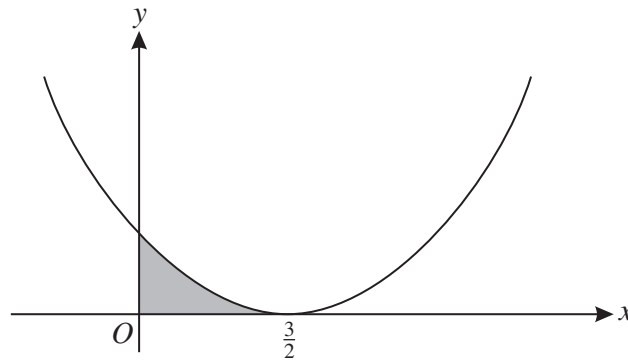
Each diagram above shows part of a curve, the equation of which is one of the following:

$$y = \sin^{-1} x, \quad y = \cos^{-1} x, \quad y = \tan^{-1} x, \quad y = \sec x, \quad y = \operatorname{cosec} x, \quad y = \cot x.$$

State which equation corresponds to

- (i) Fig. 1, [1]  
 (ii) Fig. 2, [1]  
 (iii) Fig. 3. [1]

2



The diagram shows the curve with equation  $y = (2x - 3)^2$ . The shaded region is bounded by the curve and the lines  $x = 0$  and  $y = 0$ . Find the exact volume obtained when the shaded region is rotated completely about the  $x$ -axis. [5]

3 The angles  $\alpha$  and  $\beta$  are such that

$$\tan \alpha = m + 2 \quad \text{and} \quad \tan \beta = m,$$

where  $m$  is a constant.

- (i) Given that  $\sec^2 \alpha - \sec^2 \beta = 16$ , find the value of  $m$ . [3]  
 (ii) Hence find the exact value of  $\tan(\alpha + \beta)$ . [3]

4 It is given that  $\int_a^{3a} (e^{3x} + e^x) dx = 100$ , where  $a$  is a positive constant.

(i) Show that  $a = \frac{1}{9} \ln(300 + 3e^a - 2e^{3a})$ . [5]

(ii) Use an iterative process, based on the equation in part (i), to find the value of  $a$  correct to 4 decimal places. Use a starting value of 0.6 and show the result of each step of the process. [4]

5 The functions  $f$  and  $g$  are defined for all real values of  $x$  by

$$f(x) = 3x - 2 \quad \text{and} \quad g(x) = 3x + 7.$$

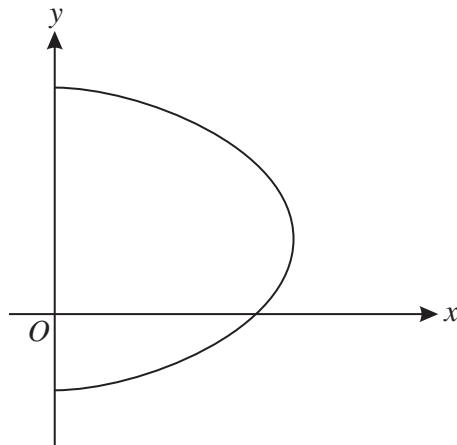
Find the exact coordinates of the point at which

(i) the graph of  $y = fg(x)$  meets the  $x$ -axis, [3]

(ii) the graph of  $y = g(x)$  meets the graph of  $y = g^{-1}(x)$ , [3]

(iii) the graph of  $y = |f(x)|$  meets the graph of  $y = |g(x)|$ . [4]

6



The diagram shows the curve with equation  $x = (37 + 10y - 2y^2)^{\frac{1}{2}}$ .

(i) Find an expression for  $\frac{dx}{dy}$  in terms of  $y$ . [2]

(ii) Hence find the equation of the tangent to the curve at the point  $(7, 3)$ , giving your answer in the form  $y = mx + c$ . [5]

7 (i) Express  $8 \sin \theta - 6 \cos \theta$  in the form  $R \sin(\theta - \alpha)$ , where  $R > 0$  and  $0^\circ < \alpha < 90^\circ$ . [3]

(ii) Hence

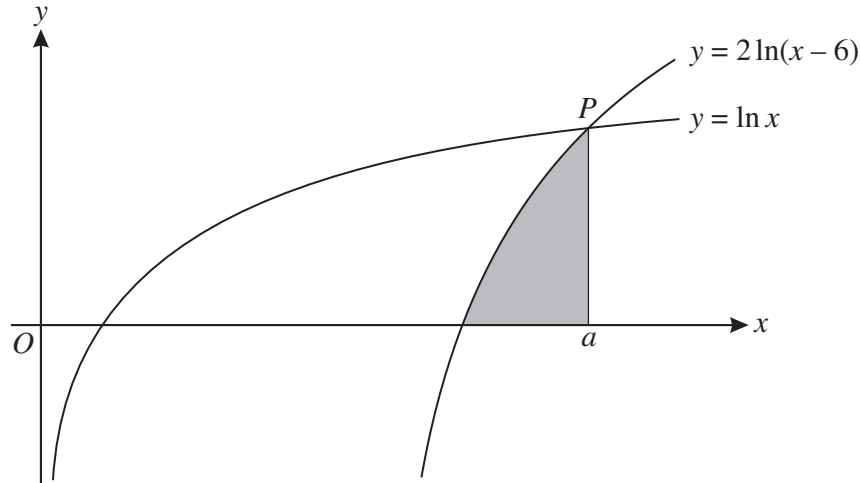
(a) solve, for  $0^\circ < \theta < 360^\circ$ , the equation  $8 \sin \theta - 6 \cos \theta = 9$ , [4]

(b) find the greatest possible value of

$$32 \sin x - 24 \cos x - (16 \sin y - 12 \cos y)$$

as the angles  $x$  and  $y$  vary. [3]

8



The diagram shows the curves  $y = \ln x$  and  $y = 2 \ln(x - 6)$ . The curves meet at the point  $P$  which has  $x$ -coordinate  $a$ . The shaded region is bounded by the curve  $y = 2 \ln(x - 6)$  and the lines  $x = a$  and  $y = 0$ .

(i) Give details of the pair of transformations which transforms the curve  $y = \ln x$  to the curve  $y = 2 \ln(x - 6)$ . [3]

(ii) Solve an equation to find the value of  $a$ . [4]

(iii) Use Simpson's rule with two strips to find an approximation to the area of the shaded region. [3]

9 (a) Show that, for all non-zero values of the constant  $k$ , the curve

$$y = \frac{kx^2 - 1}{kx^2 + 1}$$

has exactly one stationary point. [5]

(b) Show that, for all non-zero values of the constant  $m$ , the curve

$$y = e^{mx}(x^2 + mx)$$

has exactly two stationary points. [7]

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