

4723 Core Mathematics 3

- 1 (i) State $y = \sec x$ B1
 (ii) State $y = \cot x$ B1
 (iii) State $y = \sin^{-1} x$ B1 3

3

- 2 Either: State or imply $\int \pi(2x-3)^4 dx$ B1 or unsimplified equiv
 Obtain integral of form $k(2x-3)^5$ M1 any constant k involving π or not
 Obtain $\frac{1}{10}(2x-3)^5$ or $\frac{1}{10}\pi(2x-3)^5$ A1
 Attempt evaluation using 0 and $\frac{3}{2}$ M1 subtraction correct way round
 Obtain $\frac{243}{10}\pi$ A1 5 or exact equiv

- Or: State or imply $\int \pi(2x-3)^4 dx$ B1 or unsimplified equiv
 Expand and obtain integral of order 5 M1 with at least three terms correct
 Ob'n $\frac{16}{5}x^5 - 24x^4 + 72x^3 - 108x^2 + 81x$ A1 with or without π
 Attempt evaluation using (0 and) $\frac{3}{2}$ M1
 Obtain $\frac{243}{10}\pi$ A1 (5) or exact equiv

5

- 3 (i) Attempt use of identity for $\sec^2 \alpha$ M1 using $\pm \tan^2 \alpha \pm 1$
 Obtain $1 + (m+2)^2 - (1+m^2)$ A1 absent brackets implied by subsequent correct working
 Obtain $4m + 4 = 16$ and hence $m = 3$ A1 3

- (ii) Attempt subn in identity for $\tan(\alpha + \beta)$ M1 using $\frac{\pm \tan \alpha \pm \tan \beta}{1 \pm \tan \alpha \tan \beta}$
 Obtain $\frac{5+3}{1-15}$ or $\frac{m+2+m}{1-m(m+2)}$ A1✓ following their m
 Obtain $-\frac{4}{7}$ A1 3 or exact equiv

6

- 4 (i) Obtain $\frac{1}{3}e^{3x} + e^x$ B1
 Substitute to obtain $\frac{1}{3}e^{9a} + e^{3a} - \frac{1}{3}e^{3a} - e^a$ B1 or equiv
 Equate definite integral to 100 and attempt rearrangement M1 as far as $e^{9a} = \dots$
 Introduce natural logarithm M1 using correct process
 Obtain $a = \frac{1}{9} \ln(300 + 3e^a - 2e^{3a})$ A1 5 AG; necessary detail needed

- (ii) Obtain correct first iterate B1 allow for 4 dp rounded or truncated
 Show correct iteration process M1 with at least one more step
 Obtain at least three correct iterates in all A1 allowing recovery after error
 Obtain 0.6309 A1 4 following at least three correct steps;
 answer required to exactly 4 dp

[0.6 \rightarrow 0.631269 \rightarrow 0.630884 \rightarrow 0.630889]

9

- 5 (i) Either: Show correct process for comp'n M1 correct way round and in terms of x
 Obtain $y = 3(3x + 7) - 2$ A1 or equiv
 Obtain $x = -\frac{19}{9}$ A1 3 or exact equiv; condone absence of $y = 0$
- Or: Use $fg(x) = 0$ to obtain $g(x) = \frac{2}{3}$ B1
 Attempt solution of $g(x) = \frac{2}{3}$ M1
 Obtain $x = -\frac{19}{9}$ A1 (3) or exact equiv; condone absence of $y = 0$
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- (ii) Attempt formation of one of the equations
 $3x + 7 = \frac{x-7}{3}$ or $3x + 7 = x$ or $\frac{x-7}{3} = x$ M1 or equiv
 Obtain $x = -\frac{7}{2}$ A1 or equiv
 Obtain $y = -\frac{7}{2}$ A1√ 3 or equiv; following their value of x
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- (iii) Attempt solution of modulus equation M1 squaring both sides to obtain 3-term quadratics or forming linear equation with signs of $3x$ different on each side
 Obtain $-12x + 4 = 42x + 49$ or $3x - 2 = -3x - 7$ A1 or equiv
 Obtain $x = -\frac{5}{6}$ A1 or exact equiv; as final answer
 Obtain $y = \frac{9}{2}$ A1 4 or equiv; and no other pair of answers
- 10**

- 6 (i) Obtain derivative $k(37 + 10y - 2y^2)^{-\frac{1}{2}} f(y)$ M1 any constant k ; any linear function for f
 Obtain $\frac{1}{2}(10 - 4y)(37 + 10y - 2y^2)^{-\frac{1}{2}}$ A1 2 or equiv
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- (ii) Either: Sub'te $y = 3$ in expression for $\frac{dx}{dy}$ *M1
 Take reciprocal of expression/value *M1 and without change of sign
 Obtain -7 for gradient of tangent A1
 Attempt equation of tangent M1 dep *M *M
 Obtain $y = -7x + 52$ A1 5 and no second equation
- Or: Sub'te $y = 3$ in expression for $\frac{dx}{dy}$ M1
 Attempt formation of eq'n $x = m'y + c$ M1 where m' is attempt at $\frac{dx}{dy}$
 Obtain $x - 7 = -\frac{1}{7}(y - 3)$ A1 or equiv
 Attempt rearrangement to required form M1
 Obtain $y = -7x + 52$ A1 (5) and no second equation
- 7**

7 (i)	State $R = 10$	B1	or equiv
	Attempt to find value of α	M1	implied by correct answer or its complement; allow sin/cos muddles
	Obtain 36.9 or $\tan^{-1} \frac{3}{4}$	A1 3	or greater accuracy 36.8699...

(ii)(a)	Show correct process for finding one angle	M1	
	Obtain $(64.16 + 36.87)$ and hence 101	A1	or greater accuracy 101.027...
	Show correct process for finding second angle	M1	
	Obtain $(115.84 + 36.87)$ and hence 153	A1√ 4	following their value of α ; or greater accuracy 152.711...; and no other between 0 and 360

(b)	Recognise link with part (i)	M1	signalled by $40 \dots - 20 \dots$
	Use fact that maximum and minimum values of sine are 1 and -1	M1	may be implied; or equiv
	Obtain 60	A1 3	
		10	

8 (i)	Refer to translation and stretch	M1	in either order; allow here equiv informal terms such as 'move', ...
	State translation in x direction by 6	A1	or equiv; now with correct terminology
	State stretch in y direction by 2	A1 3	or equiv; now with correct terminology
	[SC: if M0 but one transformation completely correct, give B1]		

(ii)	State $2 \ln(x-6) = \ln x$	B1	or $2 \ln(a-6) = \ln a$ or equiv
	Show correct use of logarithm property	*M1	
	Attempt solution of 3-term quadratic	M1	dep *M
	Obtain 9 only	A1 4	following correct solution of equation

(iii)	Attempt evaluation of form $k(y_0 + 4y_1 + y_2)$	M1	any constant k ; maybe with $y_0 = 0$ implied
	Obtain $\frac{1}{3} \times (2 \ln 1 + 8 \ln 2 + 2 \ln 3)$	A1	or equiv
	Obtain 2.58	A1 3	or greater accuracy 2.5808...
		10	

9 (a)	Attempt use of quotient rule	*M1	or equiv; allow numerator wrong way round and denominator errors
	Obtain $\frac{(kx^2 + 1)2kx - (kx^2 - 1)2kx}{(kx^2 + 1)^2}$	A1	or equiv; with absent brackets implied by subsequent correct working
	Obtain correct simplified numerator $4kx$	A1	
	Equate numerator of first derivative to zero	M1	dep *M
	State $x = 0$ <u>or</u> refer to $4kx$ being linear <u>or</u> observe that, with $k \neq 0$, only one sol'n	A1√ 5	AG or equiv; following numerator of form $k'kx = 0$, any constant k'

(b)	Attempt use of product rule	*M1	
	Obtain $me^{mx}(x^2 + mx) + e^{mx}(2x + m)$	A1	or equiv
	Equate to zero and either factorise with factor e^{mx} or divide through by e^{mx}	M1	dep *M
	Obtain $mx^2 + (m^2 + 2)x + m = 0$ or equiv and observe that e^{mx} cannot be zero	A1	
	Attempt use of discriminant	M1	using correct $b^2 - 4ac$ with their a, b, c
	Simplify to obtain $m^4 + 4$	A1	or equiv
	Observe that this is positive for all m and hence two roots	A1	7 or equiv; AG
			12

4724 Core Mathematics 4

1	<p><u>Long Division</u> For leading term $3x^2$ in quotient B1</p> <p>Suff evid of div process (ax^2, mult back, attempt sub) M1</p> <p>(Quotient) = $3x^2 - 4x - 5$ A1</p> <p>(Remainder) = $-x + 2$ A1</p> <p><u>Identity</u> $3x^4 - x^3 - 3x^2 - 14x - 8 = Q(x^2 + x + 2) + R$ *M1</p> <p>$Q = ax^2 + bx + c, R = dx + e$ & attempt ≥ 3 ops. dep*M1 If $a = 3$, this \Rightarrow 1 operation</p> <p>$a = 3, b = -4, c = -5$ A1 dep*M1; $Q = ax^2 + bx + c$</p> <p>$d = -1, e = 2$ A1</p> <p><u>Inspection</u> Use 'Identity' method; if $R = e$, check cf(x) correct before awarding 2nd M1</p>	
4		
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2	<p><u>Indefinite Integral</u> Attempt to connect dx & $d\theta$ *M1</p> <p>Reduce to $\int 1 - \tan^2 \theta (d\theta)$ A1</p> <p>Use $\tan^2 \theta = (1, -1) + (\sec^2 \theta, -\sec^2 \theta)$ dep*M1</p> <p>Produce $\int 2 - \sec^2 \theta (d\theta)$ A1</p> <p>Correct $\sqrt{\quad}$ integration of function of type $d + e \sec^2 \theta$ $\sqrt{A1}$</p> <p>EITHER Attempt limits change (allow degrees here) M1</p> <p>OR Attempt integ, re-subst & use original ($\sqrt{3}, 1$)</p> <p>$\frac{1}{6}\pi - \sqrt{3} + 1$ isw Exact answer required A1</p>	<p>Incl $\frac{dx}{d\theta}$ or $\frac{d\theta}{dx}$; not $dx = d\theta$</p> <p>A0 if $\frac{d\theta}{dx} = \sec^2 \theta$; but allow all following</p> <p>A marks</p> <p>including $d = 0$</p> <p>(This is 'limits' aspect; the integ need not be accurate)</p>
7		