4723 Core Mathematics 3

1 (i)	State $y = \sec x$	B1		
(ii)	State $y = \cot x$	B1		
(iii)	State $y = \sin^{-1} x$	B1	3	
			$\boxed{3}$	
2	Either: State or imply $\int \pi(2x-3)^4 \, dx$	B1		or unsimplified equiv
	Obtain integral of form $k(2x-3)^5$	M1		any constant k involving π or not
	Obtain $\frac{1}{10}(2x-3)^5$ or $\frac{1}{10}\pi(2x-3)^5$	A1		
	Attempt evaluation using 0 and $\frac{3}{2}$	M1		subtraction correct way round
	Obtain $\frac{243}{10}\pi$	A1	5	or exact equiv
	Or: State or imply $\int \pi(2x-3)^4 \, dx$	B1		or unsimplified equiv
	Expand and obtain integral of order 5	M1		with at least three terms correct
	Ob'n $\frac{16}{5}x^5 - 24x^4 + 72x^3 - 108x^2 + 81x$	A1		with or without π
	Attempt evaluation using (0 and) $\frac{3}{2}$	M1		
	Obtain $\frac{243}{10}\pi$	A1	(5)	or exact equiv
			$\boxed{5}$	
3 (i)	Attempt use of identity for $\sec^2 \alpha$	M1		using $\pm\tan^2\alpha \pm 1$
	Obtain $1 + (m+2)^2 - (1+m^2)$	A1		absent brackets implied by subsequent correct working
	Obtain $4m + 4 = 16$ and hence $m = 3$	A1	3	
(ii)	Attempt subn in identity for $\tan(\alpha + \beta)$	M1		using $\frac{\pm\tan\alpha \pm \tan\beta}{1 \pm \tan\alpha\tan\beta}$
	Obtain $\frac{5+3}{1-15}$ or $\frac{m+2+m}{1-m(m+2)}$	A1√		following their m
	Obtain $-\frac{4}{7}$	A1	3	or exact equiv
			$\boxed{6}$	
4 (i)	Obtain $\frac{1}{3}e^{3x} + e^x$	B1		
	Substitute to obtain $\frac{1}{3}e^{9a} + e^{3a} - \frac{1}{3}e^{3a} - e^a$	B1		or equiv
	Equate definite integral to 100 and attempt rearrangement	M1		as far as $e^{9a} = \ldots$
	Introduce natural logarithm	M1		using correct process
	Obtain $a = \frac{1}{9}\ln(300 + 3e^a - 2e^{3a})$	A1	5	AG; necessary detail needed
(ii)	Obtain correct first iterate	B1		allow for 4 dp rounded or truncated
	Show correct iteration process	M1		with at least one more step
	Obtain at least three correct iterates in all	A1		allowing recovery after error
	Obtain 0.6309	A1	4	following at least three correct steps; answer required to exactly 4 dp
	[0.6 → 0.631269 → 0.630884 → 0.630889]			
			$\boxed{9}$	

5 (i)	Either: Show correct process for comp'n	M1		correct way round and in terms of x
	Obtain $y = 3(3x+7) - 2$	A1		or equiv
	Obtain $x = -\frac{19}{9}$	A1	3	or exact equiv; condone absence of $y = 0$
	Or: Use fg(x) = 0 to obtain $g(x) = \frac{2}{3}$	B1		
	Attempt solution of $g(x) = \frac{2}{3}$	M1		
	Obtain $x = -\frac{19}{9}$	A1	(3)	or exact equiv; condone absence of $y = 0$
(ii)	Attempt formation of one of the equations $3x + 7 = \dfrac{x-7}{3}$ or $3x + 7 = x$ or $\dfrac{x-7}{3} = x$	M1		or equiv
	Obtain $x = -\frac{7}{2}$	A1		or equiv
	Obtain $y = -\frac{7}{2}$	A1√	3	or equiv; following their value of x
(iii)	Attempt solution of modulus equation	M1		squaring both sides to obtain 3-term quadratics or forming linear equation with signs of $3x$ different on each side
	Obtain $-12x + 4 = 42x + 49$ or $3x - 2 = -3x - 7$	A1		or equiv
	Obtain $x = -\frac{5}{6}$	A1		or exact equiv; as final answer
	Obtain $y = \frac{9}{2}$	A1	4	or equiv; and no other pair of answers
			10	

6 (i)	Obtain derivative $k(37 + 10y - 2y^2)^{-\frac{1}{2}} f(y)$	M1		any constant k; any linear function for f
	Obtain $\frac{1}{2}(10 - 4y)(37 + 10y - 2y^2)^{-\frac{1}{2}}$	A1	2	or equiv
(ii)	Either: Sub'te $y = 3$ in expression for $\dfrac{dx}{dy}$	*M1		
	Take reciprocal of expression/value	*M1		and without change of sign
	Obtain -7 for gradient of tangent	A1		
	Attempt equation of tangent	M1		dep *M *M
	Obtain $y = -7x + 52$	A1	5	and no second equation
	Or: Sub'te $y = 3$ in expression for $\dfrac{dx}{dy}$	M1		
	Attempt formation of eq'n $x = m'y + c$	M1		where m' is attempt at $\dfrac{dx}{dy}$
	Obtain $x - 7 = -\frac{1}{7}(y - 3)$	A1		or equiv
	Attempt rearrangement to required form	M1		
	Obtain $y = -7x + 52$	A1	(5)	and no second equation
			7	

7 (i)	State $R = 10$	B1		or equiv
	Attempt to find value of α	M1		implied by correct answer or its complement; allow sin/cos muddles
	Obtain 36.9 or $\tan^{-1}\frac{3}{4}$	A1	3	or greater accuracy 36.8699…
(ii)(a)	Show correct process for finding one angle	M1		
	Obtain (64.16 + 36.87 and hence) 101	A1		or greater accuracy 101.027…
	Show correct process for finding second angle	M1		
	Obtain (115.84 + 36.87 and hence) 153	A1√	4	following their value of α; or greater accuracy 152.711…; and no other between 0 and 360
(b)	Recognise link with part **(i)**	M1		signalled by 40 … – 20 …
	Use fact that maximum and minimum values of sine are 1 and –1	M1		may be implied; or equiv
	Obtain 60	A1	3	
			10	
8 (i)	Refer to translation and stretch	M1		in either order; allow here equiv informal terms such as 'move', …
	State translation in x direction by 6	A1		or equiv; now with correct terminology
	State stretch in y direction by 2	A1	3	or equiv; now with correct terminology
	[SC: if M0 but one transformation completely correct, give B1]			
(ii)	State $2\ln(x-6) = \ln x$	B1		or $2\ln(a-6) = \ln a$ or equiv
	Show correct use of logarithm property	*M1		
	Attempt solution of 3-term quadratic	M1		dep *M
	Obtain 9 only	A1	4	following correct solution of equation
(iii)	Attempt evaluation of form $k(y_0 + 4y_1 + y_2)$	M1		any constant k; maybe with $y_0 = 0$ implied
	Obtain $\frac{1}{3}\times 1(2\ln 1 + 8\ln 2 + 2\ln 3)$	A1		or equiv
	Obtain 2.58	A1	3	or greater accuracy 2.5808…
			10	
9 (a)	Attempt use of quotient rule	*M1		or equiv; allow numerator wrong way round and denominator errors
	Obtain $\dfrac{(kx^2+1)2kx-(kx^2-1)2kx}{(kx^2+1)^2}$	A1		or equiv; with absent brackets implied by subsequent correct working
	Obtain correct simplified numerator $4kx$	A1		
	Equate numerator of first derivative to zero	M1		dep *M
	State $x = 0$ <u>or</u> refer to $4kx$ being linear <u>or</u> observe that, with $k \neq 0$, only one sol'n	A1√	5	AG or equiv; following numerator of form $k'kx = 0$, any constant k'

(b)	Attempt use of product rule	*M1		
	Obtain $me^{mx}(x^2+mx)+e^{mx}(2x+m)$	A1		or equiv
	Equate to zero and either factorise with factor e^{mx} or divide through by e^{mx}	M1		dep *M
	Obtain $mx^2+(m^2+2)x+m=0$ or equiv and observe that e^{mx} cannot be zero	A1		
	Attempt use of discriminant	M1		using correct b^2-4ac with their a, b, c
	Simplify to obtain m^4+4	A1		or equiv
	Observe that this is positive for all m and hence two roots	A1	7	or equiv; AG
			12	

4724 Core Mathematics 4

1	Long Division For leading term $3x^2$ in quotient	B1	
	Suff evid of div process (ax^2 , mult back, attempt sub)	M1	
	(Quotient) = $3x^2 - 4x - 5$	A1	
	(Remainder) = $-x + 2$	A1	
	Identity $3x^4 - x^3 - 3x^2 - 14x - 8 = Q(x^2 + x + 2) + R$	*M1	
	$Q = ax^2 + bx + c, R = dx + e$ & attempt ≥ 3 ops.	dep*M1	If $a = 3$,this $\Rightarrow$ 1 operation
	$a = 3, b = -4, c = -5$	A1	dep*M1; Q = $ax^2 + bx + c$
	$d = -1, e = 2$	A1	
	Inspection Use 'Identity' method; if $R = e$, check cf(x) correct before awarding 2nd M1		
		$\boxed{4}$	
2	Indefinite Integral Attempt to connect $\mathrm{d}x$ & $\mathrm{d}\theta$	*M1	Incl $\frac{\mathrm{d}x}{\mathrm{d}\theta}$ or $\frac{\mathrm{d}\theta}{\mathrm{d}x}$; not $\mathrm{d}x = \mathrm{d}\theta$
	Reduce to $\int 1 - \tan^2\theta \, (\mathrm{d}\theta)$	A1	A0 if $\frac{\mathrm{d}\theta}{\mathrm{d}x} = \sec^2\theta$; but allow all following A marks
	Use $\tan^2\theta = (1,-1) + (\sec^2\theta, -\sec^2\theta)$	dep*M1	
	Produce $\int 2 - \sec^2\theta \, (\mathrm{d}\theta)$	A1	
	Correct √ integration of function of type $d + e\sec^2\theta$	√A1	including $d = 0$
	EITHER Attempt limits change (allow degrees here)	M1	(This is 'limits' aspect; the
	OR Attempt integ, re-subst & use original ($\sqrt{3}$,1)		integ need not be accurate)
	$\frac{1}{6}\pi - \sqrt{3} + 1$ isw Exact answer required	A1	
		$\boxed{7}$	