

**ADVANCED GCE**  
**MATHEMATICS**  
Core Mathematics 3

**4723**

Candidates answer on the Answer Booklet

**OCR Supplied Materials:**

- 8 page Answer Booklet
- List of Formulae (MF1)

**Other Materials Required:**

- Scientific or graphical calculator

**Wednesday 9 June 2010**  
**Afternoon**

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 Find  $\frac{dy}{dx}$  in each of the following cases:

(i)  $y = x^3 e^{2x}$ , [2]

(ii)  $y = \ln(3 + 2x^2)$ , [2]

(iii)  $y = \frac{x}{2x+1}$ . [2]

2 The transformations R, S and T are defined as follows.

R : reflection in the  $x$ -axis

S : stretch in the  $x$ -direction with scale factor 3

T : translation in the positive  $x$ -direction by 4 units

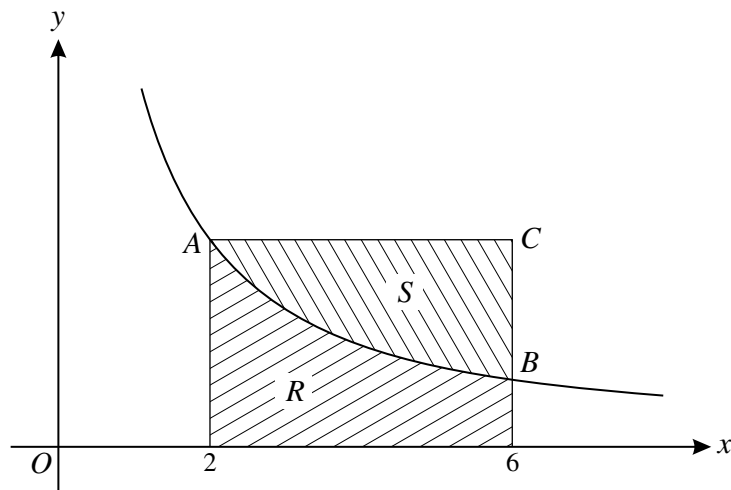
(i) The curve  $y = \ln x$  is transformed by R followed by T. Find the equation of the resulting curve. [2]

(ii) Find, in terms of S and T, a sequence of transformations that transforms the curve  $y = x^3$  to the curve  $y = (\frac{1}{9}x - 4)^3$ . You should make clear the order of the transformations. [2]

3 (i) Express the equation  $\operatorname{cosec} \theta(3 \cos 2\theta + 7) + 11 = 0$  in the form  $a \sin^2 \theta + b \sin \theta + c = 0$ , where  $a$ ,  $b$  and  $c$  are constants. [3]

(ii) Hence solve, for  $-180^\circ < \theta < 180^\circ$ , the equation  $\operatorname{cosec} \theta(3 \cos 2\theta + 7) + 11 = 0$ . [3]

4



The diagram shows part of the curve  $y = \frac{k}{x}$ , where  $k$  is a positive constant. The points A and B on the curve have  $x$ -coordinates 2 and 6 respectively. Lines through A and B parallel to the axes as shown meet at the point C. The region R is bounded by the curve and the lines  $x = 2$ ,  $x = 6$  and  $y = 0$ . The region S is bounded by the curve and the lines AC and BC. It is given that the area of the region R is  $\ln 81$ .

(i) Show that  $k = 4$ . [3]

(ii) Find the exact volume of the solid produced when the region S is rotated completely about the  $x$ -axis. [4]

- 5 (i) Solve the inequality  $|2x + 1| \leq |x - 3|$ . [5]
- (ii) Given that  $x$  satisfies the inequality  $|2x + 1| \leq |x - 3|$ , find the greatest possible value of  $|x + 2|$ . [2]

- 6 (i) Show by calculation that the equation

$$\tan^2 x - x - 2 = 0,$$

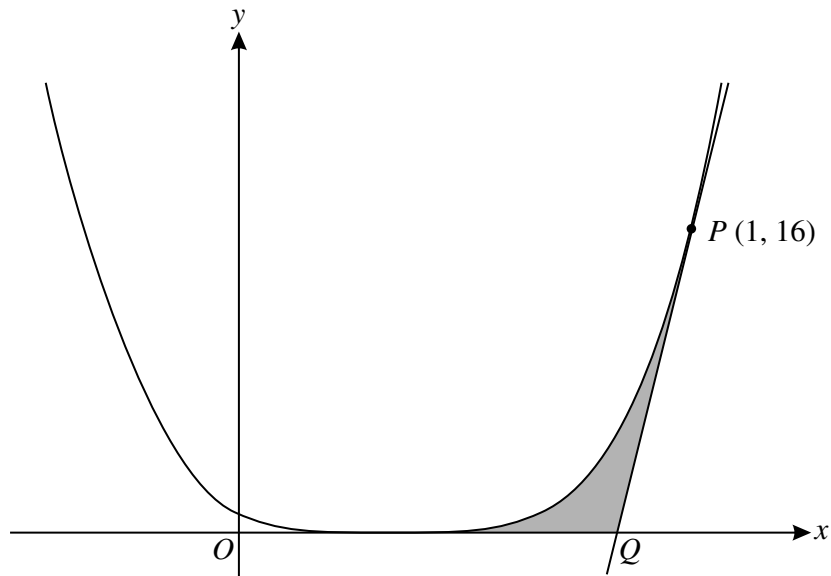
where  $x$  is measured in radians, has a root between 1.0 and 1.1. [3]

- (ii) Use the iteration formula  $x_{n+1} = \tan^{-1} \sqrt{2 + x_n}$  with a suitable starting value to find this root correct to 5 decimal places. You should show the outcome of each step of the process. [4]

- (iii) Deduce a root of the equation

$$\sec^2 2x - 2x - 3 = 0. [3]$$

7



The diagram shows the curve with equation  $y = (3x - 1)^4$ . The point  $P$  on the curve has coordinates  $(1, 16)$  and the tangent to the curve at  $P$  meets the  $x$ -axis at the point  $Q$ . The shaded region is bounded by  $PQ$ , the  $x$ -axis and that part of the curve for which  $\frac{1}{3} \leq x \leq 1$ . Find the exact area of this shaded region. [10]

- 8 (i) Express  $3 \cos x + 3 \sin x$  in the form  $R \cos(x - \alpha)$ , where  $R > 0$  and  $0 < \alpha < \frac{1}{2}\pi$ . [3]
- (ii) The expression  $T(x)$  is defined by  $T(x) = \frac{8}{3 \cos x + 3 \sin x}$ .
- (a) Determine a value of  $x$  for which  $T(x)$  is not defined. [2]
- (b) Find the smallest positive value of  $x$  satisfying  $T(3x) = \frac{8}{9}\sqrt{6}$ , giving your answer in an exact form. [4]

[Question 9 is printed overleaf.]

9 The functions  $f$  and  $g$  are defined for all real values of  $x$  by

$$f(x) = 4x^2 - 12x \quad \text{and} \quad g(x) = ax + b,$$

where  $a$  and  $b$  are non-zero constants.

- (i) Find the range of  $f$ . [3]
- (ii) Explain why the function  $f$  has no inverse. [2]
- (iii) Given that  $g^{-1}(x) = g(x)$  for all values of  $x$ , show that  $a = -1$ . [4]
- (iv) Given further that  $gf(x) < 5$  for all values of  $x$ , find the set of possible values of  $b$ . [4]



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