OCR
RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced GCE **4723**

Core Mathematics 3

Mark Scheme for June 2010

Oxford Cambridge and RSA Examinations

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1 **(i)** Attempt use of product rule — M1 producing … + … form
Obtain $3x^2e^{2x} + 2x^3e^{2x}$ — A1 **2** or equiv

(ii) Attempt use of chain rule to produce $\frac{kx}{3+2x^2}$ form — M1 any constant k
Obtain $\frac{4x}{3+2x^2}$ — A1 **2**

(iii) Attempt use of quotient rule — M1 or equiv; condone u/v confusions
Obtain $\frac{2x+1-2x}{(2x+1)^2}$ or $(2x+1)^{-1} - 2x(2x+1)^{-2}$ — A1 **2** or (unsimplified) equiv

[If $...+c$ included in all three parts and all three parts otherwise correct, award M1A1, M1A1, M1A0; otherwise ignore any inclusion of $...+c$.]

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2 **(i)** Obtain one of $\pm\ln(\pm x \pm 4)$ — M1
Obtain correct equation $y = -\ln(x-4)$ — A1 **2** or equiv; condone use of modulus signs instead of brackets

(ii) State, in any order, S, S and T — M1 or equiv such as S^2 , T or 2S, T
State T, then S, then S — A1 **2** or equiv (note that S, S, T^9 and S, T^3 , S are alternative correct answers)

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3 **(i)** Use $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$ — B1
Attempt to express equation in terms of $\sin\theta$ — M1 using $\cos 2\theta = \pm 1 \pm 2\sin^2\theta$ or equiv
Obtain or clearly imply $6\sin^2\theta - 11\sin\theta - 10 = 0$ — A1 **3** or $-6\sin^2\theta + 11\sin\theta + 10 = 0$

(ii) Attempt solution to obtain at least one value of $\sin\theta$ — M1 should be $s = -\frac{2}{3}, \frac{5}{2}$
Obtain –41.8 — A1 allow –42 or greater accuracy
Obtain –138 — A1 **3** or greater accuracy; and no others between –180 and 180

[Answer(s) only: award 0 out of 3.]

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4 (i) Either: Integrate to obtain $k \ln x$ B1
Use at least one relevant logarithm property M1
Obtain $k \ln 3 = \ln 81$ and hence $k = 4$ A1 **3** AG; accurate work required

Or 1: (where solution involves no use of a logarithm property)
Integrate to obtain $k \ln x$ B1
Obtain correct explicit expression for k and
conclude $k = 4$ with no error seen B2 **3** AG; e.g. $k = \frac{\ln 81}{\ln 6 - \ln 2} = 4$

Or 2: (where solution involves verification of result by initial substitution of 4 for k)
Integrate to obtain $4 \ln x$ B1
Use at least one relevant logarithm property M1
Obtain $\ln 81$ legitimately with no error seen A1 **3** AG; accurate work required

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(ii) State volume involves $\int \pi(\frac{4}{x})^2 \, dx$ B1 possibly implied

Obtain integral of form $k_1 x^{-1}$ M1 any constant k_1 including π or not

Use correct process for finding volume produced from S M1 $\int (k_2 2^2 - k_3 y^2) \, dx$, including π or not with correct limits indicated; or equiv

Obtain $16\pi - \frac{16}{3}\pi$ and hence $\frac{32}{3}\pi$ A1 **4** or exact equiv

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5 (i) Attempt process for finding both critical values M1 squaring both sides to obtain 3 terms on each side or considering 2 different linear eqns/inequalities

Obtain –4 A1

Obtain $\frac{2}{3}$ A1

Attempt process for solving inequality M1 table, sketch, …; needs two critical values; implied by plausible answer

Obtain $-4 \le x \le \frac{2}{3}$ A1 **5** with $\le$ and not $<$

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(ii) Use correct process to find value of $|x+2|$ using any value M1 … whether part of answer to **(i)** or not

Obtain $2\frac{2}{3}$ or $\frac{8}{3}$ A1 **2** dependent on 5 marks awarded in part **(i)**

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6 **(i)** Attempt calculations involving 1.0 and 1.1 — M1 using radians
Obtain -0.57 and 0.76 — A1 or values to 1 dp (rounded or truncated); or equivs (where eqn rearranged)
Refer to sign change (or equiv for rearranged eqn) — A1 **3** AG; following correct work only

(ii) Obtain correct first iterate — B1 using value x_1 such that $1.0 \le x_1 \le 1.1$
Carry out iteration process — M1 obtaining at least 3 iterates in all so far
Obtain at least 3 correct iterates — A1 showing at least 3 dp
Obtain 1.05083 — A1 **4** answer required to exactly 5 d.p.
[1 → 1.047198 → 1.050571 → 1.050809 → 1.050826 → 1.050827;
1.05 → 1.050769 → 1.050823 → 1.050827 → 1.050827;
1.1 → 1.054268 → 1.051070 → 1.050844 → 1.050829 → 1.050827]

(iii) State or imply $\sec^2 2x = 1 + \tan^2 2x$ — B1
Relate to earlier equation — M1 by halving or doubling answer to **(ii)** or carrying out equivalent iteration process
Deduce $2x = 1.05083$ and hence 0.525 — A1√ **3** following their answer to **(ii)**; or greater accuracy

[SC: Rearrange to obtain $x = \frac{1}{2}\cos^{-1}(2x+3)^{-\frac{1}{2}}$ — B1
Use iterative process to obtain 0.525 — B1 **2** or greater accuracy]

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7 Differentiate to obtain $k_1(3x-1)^3$ — M1 any constant k_1
Obtain correct $12(3x-1)^3$ — A1 or (unsimplified) equiv
Substitute 1 to obtain 96 — A1
Attempt to find x-coordinate of Q — M1 using tangent with $y = 0$ or using gradient
Obtain $\frac{5}{6}$ — A1 or exact equiv

Integrate to obtain $k_2(3x-1)^5$ — M1 any constant k_2
Obtain correct $\frac{1}{15}(3x-1)^5$ — A1 or (unsimplified) equiv
Use limits $\frac{1}{3}$ and 1 to obtain $\frac{32}{15}$ — A1
Attempt to find shaded area by correct process — M1 integral – triangle or equiv
Obtain $(\frac{32}{15} - \frac{1}{2}\times\frac{1}{6}\times 16$ and hence) $\frac{4}{5}$ — A1 or equiv

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8 **(i)** Obtain $R = 3\sqrt{2}$ or $R = \sqrt{18}$ or $R = 4.24$ — B1 or equiv
Attempt to find value of α — M1 condone sin/cos muddles and degrees
Obtain $\frac{1}{4}\pi$ or 0.785 — A1 **3** in radians now

(ii) a Equate $x - \alpha$ to $\frac{1}{2}\pi$ or attempt solution of $3\cos x + 3\sin x = 0$ — M1 condone degrees here
Obtain $\frac{3}{4}\pi$ — A1 **2** or $\ldots, -\frac{5}{4}\pi, -\frac{1}{4}\pi, \frac{7}{4}\pi, \ldots$; in radians now

b Attempt correct process to find value of $3x - \alpha$ — *M1 with attempt at rearranging $T(3x) = \frac{8}{9}\sqrt{6}$
Obtain at least one correct exact value of $3x - \alpha$ — A1 $\pm\frac{1}{6}\pi, \pm\frac{11}{6}\pi, \ldots$
Attempt at least one positive value of x — M1 dep *M
Obtain $\frac{1}{36}\pi$ — A1 **4**

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9 **(i)** Attempt to find x-coord of staty point or complete square M1
Obtain $(\frac{3}{2},-9)$ or $4(x-\frac{3}{2})^2-9$ or -9 A1 or equiv
State $f(x)\geq -9$ A1 **3** using any notation; with $\geq$

(ii) Make one correct (perhaps general) relevant statement B1 not 1 –1 , f is many-one, … ; maybe implied if attempt is specific to this f
Conclude with correct evidence related to this f B1 **2** AG; (more or less) correct sketch; correct relevant calculations, …

(iii) Either: Attempt to find expression for g^{-1} *M1 or equiv
Obtain $\frac{1}{a}(x-b)$ A1 or equiv
Compare $\frac{1}{a}(x-b)$ and $ax+b$ M1 dep *M; by equating either coefficients of x or constant terms (or both); or substituting two non-zero values of x and solving eqns for a
Obtain at least $-\frac{b}{a}=b$ and hence $a=-1$ A1 **4** AG; necessary detail required; or equiv
[SC1: first two steps as above, then substitute $a=-1$: max possible M1A1B1]
[SC2: substitute $a=-1$ at start: Attempt to find inverse M1 Obtain $-x+b$ and conclude A1 **2**]
Or: State or imply that $y=g^{-1}(x)$ is reflection of $y=g(x)$ in line $y=x$ B1
State that line unchanged by this reflection is perpendicular to $y=x$ M2
Conclude that a is –1 A1 **4**

(iv) State or imply that $gf(x)=-(4x^2-12x)+b$ B1
Attempt use of discriminant or relate to range of f M1 or equiv
Obtain $64+16b<0$ or $9+b<5$ A1 or equiv
Obtain $b<-4$ A1 **4**
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