

**Thursday 14 June 2012 – Morning**

**A2 GCE MATHEMATICS**

**4723** Core Mathematics 3

**QUESTION PAPER**

Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

- Printed Answer Book 4723
- List of Formulae (MF1)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

**INFORMATION FOR CANDIDATES**

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

**INSTRUCTION TO EXAMS OFFICER/INVIGILATOR**

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- 1 Solve the inequality  $|2x - 5| > |x + 1|$ . [5]
- 2 It is given that  $p = e^{280}$  and  $q = e^{300}$ .
- (i) Use logarithm properties to show that  $\ln\left(\frac{ep^2}{q}\right) = 261$ . [3]
- (ii) Find the smallest integer  $n$  which satisfies the inequality  $5^n > pq$ . [3]
- 3 It is given that  $\theta$  is the acute angle such that  $\sec\theta \sin\theta = 36 \cot\theta$ .
- (i) Show that  $\tan\theta = 6$ . [3]
- (ii) Hence, using an appropriate formula in each case, find the exact value of
- (a)  $\tan(\theta - 45^\circ)$ , [2]
- (b)  $\tan 2\theta$ . [2]
- 4 (a) Show that  $\int_0^4 \frac{18}{\sqrt{6x+1}} dx = 24$ . [4]
- (b) Find  $\int_0^1 (e^x + 2)^2 dx$ , giving your answer in terms of  $e$ . [4]
- 5 (i) It is given that  $k$  is a positive constant. By sketching the graphs of
- $$y = 14 - x^2 \quad \text{and} \quad y = k \ln x$$
- on a single diagram, show that the equation
- $$14 - x^2 = k \ln x$$
- has exactly one real root. [3]
- (ii) The real root of the equation  $14 - x^2 = 3 \ln x$  is denoted by  $\alpha$ .
- (a) Find by calculation the pair of consecutive integers between which  $\alpha$  lies. [3]
- (b) Use the iterative formula  $x_{n+1} = \sqrt{14 - 3 \ln x_n}$ , with a suitable starting value, to find  $\alpha$ . Show the result of each iteration, and give  $\alpha$  correct to 2 decimal places. [4]

- 6 The volume,  $V \text{ m}^3$ , of liquid in a container is given by

$$V = (3h^2 + 4)^{\frac{3}{2}} - 8,$$

where  $h \text{ m}$  is the depth of the liquid.

- (i) Find the value of  $\frac{dV}{dh}$  when  $h = 0.6$ , giving your answer correct to 2 decimal places. [4]

- (ii) Liquid is leaking from the container. It is observed that, when the depth of the liquid is  $0.6 \text{ m}$ , the depth is decreasing at a rate of  $0.015 \text{ m}$  per hour. Find the rate at which the volume of liquid in the container is decreasing at the instant when the depth is  $0.6 \text{ m}$ . [3]

- 7 The function  $f$  is defined for all real values of  $x$  by  $f(x) = 2x + 5$ . The function  $g$  is defined for all real values of  $x$  and is such that  $g^{-1}(x) = \sqrt[3]{x - a}$ , where  $a$  is a constant. It is given that  $fg^{-1}(12) = 9$ . Find the value of  $a$  and hence solve the equation  $gf(x) = 68$ . [7]

- 8 (i) Express  $3 \sin \theta + 4 \cos \theta$  in the form  $R \sin(\theta + \alpha)$ , where  $R > 0$  and  $0^\circ < \alpha < 90^\circ$ . [3]

(ii) Hence

- (a) solve the equation  $3 \sin \theta + 4 \cos \theta + 1 = 0$ , giving all solutions for which  $-180^\circ < \theta < 180^\circ$ , [4]

- (b) find the values of the positive constants  $k$  and  $c$  such that

$$-37 \leq k(3 \sin \theta + 4 \cos \theta) + c \leq 43$$

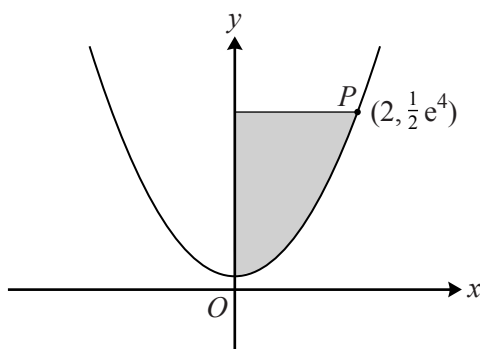
for all values of  $\theta$ . [4]

- 9 (i) Show that the derivative with respect to  $y$  of

$$y \ln(2y) - y$$

is  $\ln(2y)$ . [3]

(ii)



The diagram shows the curve with equation  $y = \frac{1}{2}e^{x^2}$ . The point  $P(2, \frac{1}{2}e^4)$  lies on the curve. The shaded region is bounded by the curve and the lines  $x = 0$  and  $y = \frac{1}{2}e^4$ . Find the exact volume of the solid produced when the shaded region is rotated completely about the  $y$ -axis. [6]

- (iii) Hence find the volume of the solid produced when the region bounded by the curve and the lines  $x = 0$ ,  $x = 2$  and  $y = 0$  is rotated completely about the  $y$ -axis. [2]

**THERE ARE NO QUESTIONS WRITTEN ON THIS PAGE.**



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