OCR
RECOGNISING ACHIEVEMENT

Thursday 14 June 2012 – Morning

A2 GCE MATHEMATICS

4723 Core Mathematics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4723
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 Solve the inequality $|2x - 5| > |x + 1|$. **[5]**

2 It is given that $p = e^{280}$ and $q = e^{300}$.

(i) Use logarithm properties to show that $\ln\left(\frac{ep^2}{q}\right) = 261$. **[3]**

(ii) Find the smallest integer n which satisfies the inequality $5^n > pq$. **[3]**

3 It is given that θ is the acute angle such that $\sec\theta \sin\theta = 36\cot\theta$.

(i) Show that $\tan\theta = 6$. **[3]**

(ii) Hence, using an appropriate formula in each case, find the exact value of

(a) $\tan(\theta - 45°)$, **[2]**

(b) $\tan 2\theta$. **[2]**

4 **(a)** Show that $\int_0^4 \frac{18}{\sqrt{6x+1}}\,dx = 24$. **[4]**

(b) Find $\int_0^1 (e^x + 2)^2\,dx$, giving your answer in terms of e. **[4]**

5 **(i)** It is given that k is a positive constant. By sketching the graphs of

$$y = 14 - x^2 \text{ and } y = k\ln x$$

on a single diagram, show that the equation

$$14 - x^2 = k\ln x$$

has exactly one real root. **[3]**

(ii) The real root of the equation $14 - x^2 = 3\ln x$ is denoted by α.

(a) Find by calculation the pair of consecutive integers between which α lies. **[3]**

(b) Use the iterative formula $x_{n+1} = \sqrt{14 - 3\ln x_n}$, with a suitable starting value, to find α. Show the result of each iteration, and give α correct to 2 decimal places. **[4]**

6 The volume, $V\,\text{m}^3$, of liquid in a container is given by

$$V = (3h^2 + 4)^{\frac{3}{2}} - 8\,,$$

where h m is the depth of the liquid.

(i) Find the value of $\dfrac{\mathrm{d}V}{\mathrm{d}h}$ when $h = 0.6$, giving your answer correct to 2 decimal places. **[4]**

(ii) Liquid is leaking from the container. It is observed that, when the depth of the liquid is 0.6 m, the depth is decreasing at a rate of 0.015 m per hour. Find the rate at which the volume of liquid in the container is decreasing at the instant when the depth is 0.6 m. **[3]**

7 The function f is defined for all real values of x by $\text{f}(x) = 2x + 5$. The function g is defined for all real values of x and is such that $\text{g}^{-1}(x) = \sqrt[3]{x - a}$, where a is a constant. It is given that $\text{fg}^{-1}(12) = 9$. Find the value of a and hence solve the equation $\text{gf}(x) = 68$. **[7]**

8 **(i)** Express $3\sin\theta + 4\cos\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. **[3]**

(ii) Hence

(a) solve the equation $3\sin\theta + 4\cos\theta + 1 = 0$, giving all solutions for which $-180° < \theta < 180°$, **[4]**

(b) find the values of the positive constants k and c such that

$$-37 \leqslant k(3\sin\theta + 4\cos\theta) + c \leqslant 43$$

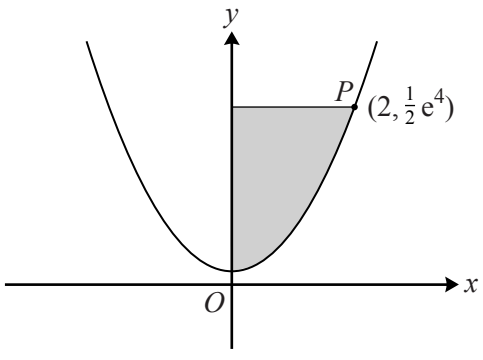
for all values of θ. **[4]**

9 **(i)** Show that the derivative with respect to y of

$$y\ln(2y) - y$$

is $\ln(2y)$. **[3]**

(ii)



The diagram shows the curve with equation $y = \frac{1}{2}\text{e}^{x^2}$. The point $P(2, \frac{1}{2}\text{e}^4)$ lies on the curve. The shaded region is bounded by the curve and the lines $x = 0$ and $y = \frac{1}{2}\text{e}^4$. Find the exact volume of the solid produced when the shaded region is rotated completely about the y-axis. **[6]**

(iii) Hence find the volume of the solid produced when the region bounded by the curve and the lines $x = 0$, $x = 2$ and $y = 0$ is rotated completely about the y-axis. **[2]**

THERE ARE NO QUESTIONS WRITTEN ON THIS PAGE.