

**Thursday 13 June 2013 – Morning**

**A2 GCE MATHEMATICS**

**4723/01** Core Mathematics 3

**QUESTION PAPER**

Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

- Printed Answer Book 4723/01
- List of Formulae (MF1)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

**INFORMATION FOR CANDIDATES**

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

**INSTRUCTION TO EXAMS OFFICER/INVIGILATOR**

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1 Find

(i)  $\int (4 - 3x)^7 dx,$

(ii)  $\int (4 - 3x)^{-1} dx.$

[5]

2 Using an appropriate identity in each case, find the possible values of

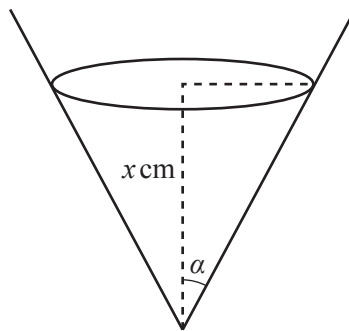
(i)  $\sin \alpha$  given that  $4 \cos 2\alpha = \sin^2 \alpha,$

[3]

(ii)  $\sec \beta$  given that  $2 \tan^2 \beta = 3 + 9 \sec \beta.$

[4]

3



The diagram shows a container in the form of a right circular cone. The angle between the axis and the slant height is  $\alpha$ , where  $\alpha = \tan^{-1}(\frac{1}{2})$ . Initially the container is empty, and then liquid is added at the rate of  $14 \text{ cm}^3$  per minute. The depth of liquid in the container at time  $t$  minutes is  $x$  cm.

(i) Show that the volume,  $V \text{ cm}^3$ , of liquid in the container when the depth is  $x$  cm is given by

$$V = \frac{1}{12} \pi x^3.$$

[The volume of a cone is  $\frac{1}{3} \pi r^2 h.$ ]

[2]

(ii) Find the rate at which the depth of the liquid is increasing at the instant when the depth is 8 cm. Give your answer in cm per minute correct to 2 decimal places.

[3]

4 Find the exact value of the gradient of the curve

$$y = \sqrt{4x - 7} + \frac{4x}{2x + 1}$$

at the point for which  $x = 4.$

[6]

5 (i) Give full details of a sequence of two transformations needed to transform the graph of  $y = |x|$  to the graph of  $y = |2(x + 3)|.$

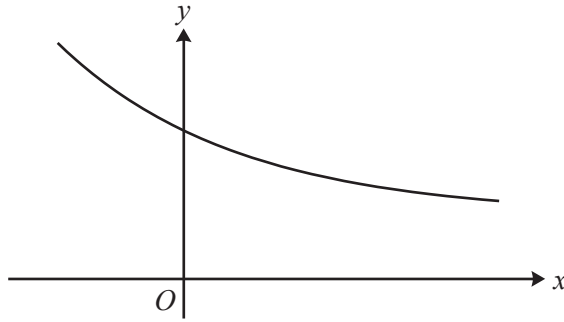
[3]

(ii) Solve the inequality  $|x| > |2(x + 3)|$ , showing all your working.

[5]

- 6 The value of  $\int_0^8 \ln(3 + x^2) dx$  obtained by using Simpson's rule with four strips is denoted by  $A$ .
- (i) Find the value of  $A$  correct to 3 significant figures. [4]
- (ii) Explain why an approximate value of  $\int_0^8 \ln(9 + 6x^2 + x^4) dx$  is  $2A$ . [2]
- (iii) Explain why an approximate value of  $\int_0^8 \ln(3e + ex^2) dx$  is  $A + 8$ . [2]

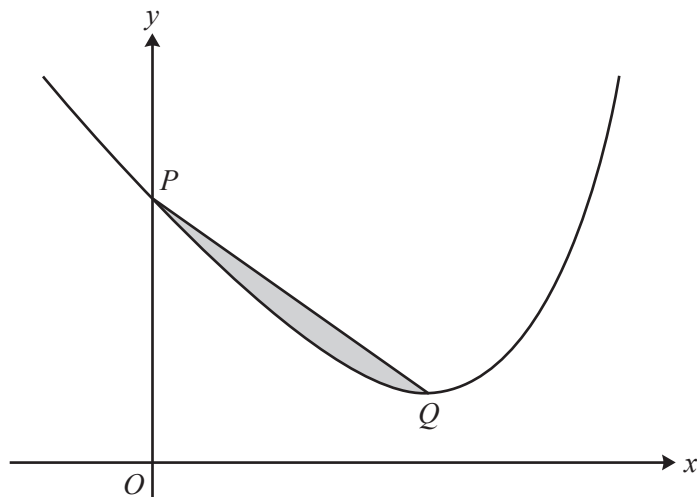
7



The diagram shows the curve  $y = f(x)$ , where  $f$  is the function defined for all real values of  $x$  by

$$f(x) = 3 + 4e^{-x}.$$

- (i) State the range of  $f$ . [1]
- (ii) Find an expression for  $f^{-1}(x)$ , and state the domain and range of  $f^{-1}$ . [4]
- (iii) The straight line  $y = x$  meets the curve  $y = f(x)$  at the point  $P$ . By using an iterative process based on the equation  $x = f(x)$ , with a starting value of 3, find the coordinates of the point  $P$ . Show all your working and give each coordinate correct to 3 decimal places. [4]
- (iv) How is the point  $P$  related to the curves  $y = f(x)$  and  $y = f^{-1}(x)$ ? [1]
- 8 (i) Express  $4 \cos \theta - 2 \sin \theta$  in the form  $R \cos(\theta + \alpha)$ , where  $R > 0$  and  $0^\circ < \alpha < 90^\circ$ . [3]
- (ii) Hence
- (a) solve the equation  $4 \cos \theta - 2 \sin \theta = 3$  for  $0^\circ < \theta < 360^\circ$ , [4]
- (b) determine the greatest and least values of
- $$25 - (4 \cos \theta - 2 \sin \theta)^2$$
- as  $\theta$  varies, and, in each case, find the smallest positive value of  $\theta$  for which that value occurs. [5]



The diagram shows the curve

$$y = e^{2x} - 18x + 15.$$

The curve crosses the  $y$ -axis at  $P$  and the minimum point is  $Q$ . The shaded region is bounded by the curve and the line  $PQ$ .

- (i) Show that the  $x$ -coordinate of  $Q$  is  $\ln 3$ . [3]
- (ii) Find the exact area of the shaded region. [8]

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