

Mathematics

Advanced GCE

Unit **4723**: Core Mathematics 3

Mark Scheme for June 2013

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

© OCR 2013

Annotations

Annotation in scoris	Meaning
✓ and ✕	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given

Subject-specific Marking Instructions for GCE Mathematics Pure strand

- a. Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

- b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

- c. The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d. When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep *’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question		Answer	Marks	Guidance
1	(i)	Obtain integral of form $k(4-3x)^8$ Obtain $-\frac{1}{24}(4-3x)^8$	M1 A1	any non-zero constant k ; using substitution to obtain ku^8 earns M1 or unsimplified equiv; must be in terms of x
1	(ii)	Obtain integral of form $k \ln(4-3x)$ Obtain $-\frac{1}{3} \ln(4-3x)$ Include $+c$ or $+k$ at least once	M1 A1 B1 [5]	any non-zero constant k ; allow M1 if brackets missing; using substitution to obtain $k \ln u$ earns M1; $\log(4-3x)$ with base e not specified is M1A0 now with either brackets or modulus signs; must be in terms of x ; note that $-\frac{1}{3} \ln(x-\frac{4}{3})$ and $-\frac{1}{3} \ln(\frac{4}{3}-x)$ are correct alternatives anywhere in solution to question 1; this mark available even if no other marks earned
2	(i)	Use $2\cos^2 \alpha - 1$ or $\cos^2 \alpha - \sin^2 \alpha$ or $1 - 2\sin^2 \alpha$ Obtain equation in which $\sin^2 \alpha$ appears once Obtain $\pm \frac{2}{3}$	B1 M1 A1 [3]	condoning sign slips or arithmetic slips; for solution which gives equation involving $\tan^2 \alpha$, M1 is not earned until valid method for reaching $\sin \alpha$ is used; attempt involving $4(1-s^2) = s^2$ is M0 both values needed; ± 0.667 is A0; $\pm \sqrt{\frac{4}{9}}$ is A0; ignore subsequent work to find angle(s)
2	(ii)	<u>Either</u> Attempt use of identity Obtain $2\sec^2 \beta - 9\sec \beta - 5 = 0$ Attempt solution of 3-term quadratic in $\sec \beta$ to obtain at least one value of $\sec \beta$ Obtain 5 with no errors in solution <u>Or</u> Attempt to express equation in terms of $\cos \beta$ Obtain $5\cos^2 \beta + 9\cos \beta - 2 = 0$ Attempt solution of 3-term quadratic and show switch at least once to a secant value Obtain 5 with no errors in solution	M1 A1 M1 A1 [4] M1 A1 M1 A1 [4]	of form $\tan^2 \beta = \pm \sec^2 \beta \pm 1$ condone absence of $= 0$ if factorising, factors must be such that expansion gives their first and third terms; if using formula, this must be correct for their values and, finally, no other value; no need to justify rejection of $-\frac{1}{2}$ using identities which are correct apart maybe for sign slips condone absence of $= 0$ if factorising, factors must be such that expansion gives their first and third terms; if using formula, this must be correct for their values and, finally, no other value; no need to justify rejection of $-\frac{1}{2}$

Question		Answer	Marks	Guidance
3	(i)	Use α (possibly implicitly) to state that radius of 'base' is $\frac{1}{2}x$ Substitute into formula to obtain $\frac{1}{3}\pi(\frac{1}{2}x)^2x$ or $\frac{1}{3}\pi\frac{1}{4}x^2x$ and obtain $\frac{1}{12}\pi x^3$	*B1 B1 [2]	or to obtain equiv such as $2r = x$ or $\frac{r}{x} = \frac{1}{2}$ or $\frac{x}{r} = 2$ dep *B; AG; necessary detail needed Note: comparing formulae $\frac{1}{3}\pi r^2h$ and $\frac{1}{12}\pi x^3$ to 'deduce' is BOB0
3	(ii)	Differentiate to obtain $\frac{1}{4}\pi x^2$ or equiv Attempt division involving 14 and their value of derivative when $x = 8$ Obtain 0.28	B1 M1 A1 [3]	whatever they call it ie $14 \div \text{deriv}$ or $\text{deriv} \div 14$ with $x = 8$ allow 0.279 but not greater accuracy Alternatives: 1. $14t = \frac{1}{12}\pi x^3$ Obtain $\frac{dt}{dx} = \frac{1}{56}\pi x^2$ <u>B1</u> Sub 8 and invert <u>M1</u> Ans <u>A1</u> 2. $x^3 = \frac{168t}{\pi}$ Obtain $3x^2 \frac{dx}{dt} = \frac{168}{\pi}$ <u>B1</u> Sub 8 <u>M1</u> Ans <u>A1</u>
4		Differentiate first term to obtain form $k(4x-7)^{-\frac{1}{2}}$ Obtain $2(4x-7)^{-\frac{1}{2}}$ Attempt use of quotient rule or, after adjustment, product rule Obtain $\frac{4(2x+1)-8x}{(2x+1)^2}$ or $4(2x+1)^{-1} - 8x(2x+1)^{-2}$ Substitute 4 into expression for first derivative so that (initially at least) exactness is retained Obtain $\frac{58}{81}$	*M1 A1 *M1 A1 M1 A1 [6]	any non-zero constant k ; M0 if this differentiation is carried out in the midst of some incorrect involved expression or (unsimplified) equiv for QR, allow numerator wrong way round but needs – sign in numerator; condone a single error such as absence of square in denominator, absence of brackets, ...; for PR, condone no use of chain rule M0 if this differentiation is carried out in the midst of some incorrect involved expression or (unsimplified) equivs; give A0 if brackets absent unless subsequent calculation indicates their 'presence' dep *M *M answer must be exact Note: using $y = \sqrt{4x-7} + \frac{4}{2x+1}$: do not apply MR

Question		Answer	Marks	Guidance
5	(i)	Refer to translation and stretch	M1	in either order; ignore details here; allow any equiv wording (such as move or shift for translation) to describe geometrical transformation but not statements such as add 3 to x
		<u>Either</u> State translation in negative x -direction by 3 State stretch by factor 2 in y -direction	A1 A1	or state translation by $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$; accept horizontal to indicate direction; term 'translate' or 'translation' needed for award of A1 or parallel to y -axis or vertically; term 'stretch' needed for award of A1; these two transformations can be given in either order SC: if M0 but details of one transformation correct, award B1 for 1/3 (in <u>Either</u> , <u>Or 1</u> , <u>Or 2</u> cases)
		<u>Or 1</u> State stretch by factor $\frac{1}{2}$ in x -direction State translation in negative x -direction by 3	A1 A1 [3]	or parallel to x -axis; term 'stretch' needed for award of A1 or state translation by $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$; term 'translate' or 'translation' needed for award of A1; these two transformations must be in this order – if details correct for M1A1A1 but order wrong, award M1A1A0
		<u>Or 2</u> State translation in negative x -direction by 6 State stretch by factor $\frac{1}{2}$ in x -direction	A1 A1 [3]	or state translation by $\begin{pmatrix} -6 \\ 0 \end{pmatrix}$; term 'translate' or 'translation' needed for award of A1 or parallel to x -axis; term 'stretch' needed for award of A1; these two transformations must be in this order – if details correct for M1A1A1 but order wrong, award M1A1A0
5	(ii)	<u>Either</u> Solve linear eqn/ineq to obtain critical value -6 Attempt solution of linear eqn/ineq where signs of x and $2x$ are different Obtain critical value -2 Attempt solution of inequality Obtain $-6 < x < -2$	B1 M1 A1 M1 A1 [5]	using table, sketch, ...; implied by correct answer or answer of form $a < x < b$ or of form $x < a, x > b$ (where $a < b$); allow \leq here as final answer; must be $<$ not \leq ; allow " $x > -6$ and $x < -2$ "

Question		Answer	Marks	Guidance
		<p><u>Or</u> Square both sides to obtain $x^2 > 4(x^2 + 6x + 9)$</p> <p>Attempt solution of 3-term quadratic eqn/ineq</p> <p>Obtain critical values -6 and -2</p> <p>Attempt solution of inequality</p> <p>Obtain $-6 < x < -2$</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1 [5]</p>	<p>or equiv</p> <p>with same guidelines as in Q2(ii) for factorising and formula</p> <p>using table, sketch, ...; implied by correct answer or answer of form $a < x < b$ or of form $x < a, x > b$ (where $a < b$); allow \leq here as final answer; must be $<$ not \leq; allow '$x > -6$ and $x < -2$'</p>
6	(i)	<p>Attempt evaluation involving y values</p> <p>Obtain $k(\ln 3 + 4\ln 7 + 2\ln 19 + 4\ln 39 + \ln 67)$</p> <p>Identify value of k as $\frac{2}{3}$</p> <p>Obtain 22.4</p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>[4]</p>	<p>with coefficients 1, 4 and 2 each occurring at least once; allow for wrong y-values; solution must include sufficient evidence of method</p> <p>any constant k; or decimal equivs; correct use of brackets required unless subsequent working shows their 'presence'</p> <p>as factor for their complete expression</p> <p>allow any value rounding to 22.4; answer only is 0/4</p>
6	(ii)	<p>State $9 + 6x^2 + x^4 = (3 + x^2)^2$</p> <p>Show relevant property $\ln(3 + x^2)^2 = 2\ln(3 + x^2)$ and conclude with value 2A</p>	<p>B1</p> <p>B1</p> <p>[2]</p>	<p>or, if proceeding numerically, demonstrate in at least three cases that $\ln 9 = \ln 3^2, \ln 49 = \ln 7^2, \ln 361 = \ln 19^2, \dots$</p> <p>AG; necessary detail needed; if proceeding numerically, needs all five cases with relevant property</p> <p>Note: using Simpson's rule again here is B0B0</p>
6	(iii)	<p>Recognise $\ln(3e + ex^2)$ as $1 + \ln(3 + x^2)$</p> <p>Indicate in some way that $\int_0^8 1 \, dx$ is 8 and conclude with value A + 8</p>	<p>B1</p> <p>B1</p> <p>[2]</p>	<p>AG; necessary detail needed</p> <p>Note: using Simpson's rule again here is B0B0</p>
7	(i)	<p>State $y > 3$ or $f(x) > 3$ or $f > 3$ or 'greater than 3'</p>	<p>B1</p> <p>[1]</p>	<p>must be $>$ not \geq; allow $3 < y < \infty$</p>

Question		Answer	Marks	Guidance
7	(ii)	Obtain expression or eqn involving $\ln(\frac{y-3}{4})$ or $\ln(\frac{x-3}{4})$ Obtain $\ln(\frac{4}{x-3})$ or $-\ln(\frac{x-3}{4})$ State domain is $x > 3$ or equiv State range is all real numbers or equiv	M1 A1 B1FT B1 [4]	or equivs such as $\ln(\frac{4}{y-3})$ or $\ln(\frac{4}{x-3})$ or equiv following answer to part (i) (but with adjustment so that reference is to x)
7	(iii)	Obtain correct first iterate Show correct iteration process Obtain at least 3 correct iterates Obtain (3.168, 3.168) $[3 \rightarrow 3.199148.. \rightarrow 3.163187.. \rightarrow 3.169162.. \rightarrow 3.168155.. \rightarrow 3.168324..]$	B1 M1 A1 A1 [4]	showing at least 3 dp; B0 if initial value not 3 but then M1A1A1 available showing at least 3 iterates in all; may be implied by plausible converging values; M1 available if based on equation with just a slip in $x = f(x)$ but M0 if based on clearly different equation allowing recovery after error; iterates to only 3 dp acceptable; values may be rounded or truncated each coordinate required to exactly 3 dp; award A0 if fewer than 4 iterates shown; part (iii) consisting of answer only gets 0 out of 4
7	(iv)	State P is point where the curves meet	B1 [1]	or equiv
8	(i)	Obtain $R = \sqrt{20}$ or $R = 4.47$ Attempt to find value of α Obtain 26.6	B1 M1 A1 [3]	implied by correct value or its complement; allow sin/cos muddles; allow use of radians for M1; condone use of $\cos \alpha = 4, \sin \alpha = 2$ here but not for A1 or greater accuracy 26.565...; with no wrong working seen
8	(ii)	(a) Show correct process for finding one answer Obtain 21.3 Show correct process for finding second answer Obtain 286 or 285.6	M1 A1FT M1 A1FT [4]	allowing for case where the answer is negative or greater accuracy 21.3045...; or anything rounding to 21.3 with no obvious error; following a wrong value of α but not wrong R ie attempting fourth quadrant value minus α value or greater accuracy 285.5653...; or anything rounding to 286 with no obvious error; following a wrong value of α but not wrong R ; and no others between 0° and 360°

Question		Answer	Marks	Guidance
8	(ii) (b)	State greatest value is 25 Obtain value 63.4 clearly associated with correct greatest value State least value is 5 Attempt to find θ from $\cos(\theta + \text{their } \alpha) = -1$ Obtain 153 or 153.4	B1 B1FT B1 M1 A1FT [5]	allow if α incorrect or greater accuracy 63.4349...; following a wrong value of α allow if α incorrect and clearly associated with correct least value or greater accuracy 153.4349...; following a wrong value of α
9	(i)	Differentiate to obtain $2e^{2x} - 18$ Equate first derivative to zero and use legitimate method to reach equation without e involved Confirm $x = \ln 3$	B1 M1 A1 [3]	AG; necessary detail needed (in particular, for solutions concluding $x = \frac{1}{2} \ln 9 = \ln 3$ or equiv award A0)
9	(ii)	Attempt integration Obtain $\frac{1}{2}e^{2x} - 9x^2 + 15x$ Apply limits 0 and $\ln 3$ to obtain exact unsimplified expression Obtain $4 - 9(\ln 3)^2 + 15\ln 3$ Attempt area of trapezium or equiv, retaining exactness throughout Obtain $\frac{1}{2} \ln 3 \times (16 + 24 - 18\ln 3)$ Subtract areas the right way round, retaining exactness Obtain $5\ln 3 - 4$	*M1 A1 M1 A1 M1 A1 M1 A1 [8]	confirmed by at least one correct term or equiv dep *M or exact (maybe unsimplified) equiv perhaps still involving e using $\frac{1}{2} \ln 3 \times (y_1 + y_2)$ where y_1 is 15 or 16 and y_2 is attempt at y-coordinate of Q ; if using alternative approach involving rectangle and triangle, complete attempt needs to be seen for M1; another alternative approach involves equation of PQ ($y = \frac{8-18\ln 3}{\ln 3}x + 16$) with integration: M1 for attempting equation and integration, A1 for correct answer or equiv perhaps still including e dep on award of all three M marks or similarly simplified exact equiv

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

Education and Learning

Telephone: 01223 553998

Facsimile: 01223 552627

Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored

Oxford Cambridge and RSA Examinations
is a Company Limited by Guarantee
Registered in England
Registered Office; 1 Hills Road, Cambridge, CB1 2EU
Registered Company Number: 3484466
OCR is an exempt Charity

OCR (Oxford Cambridge and RSA Examinations)
Head office
Telephone: 01223 552552
Facsimile: 01223 552553

© OCR 2013

