Oxford Cambridge and RSA

GCE

Mathematics

Unit **4723**: Core Mathematics 3

Advanced GCE

Mark Scheme for June 2014

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

1. **Annotations and abbreviations**

Annotation in scoris	**Meaning**
BP	Blank Page – this annotation **must** be used on all blank pages within an answer booklet (structured or unstructured) and on each page of an additional object where there is no candidate response.
✓ and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

2. Subject-specific Marking Instructions for GCE Mathematics Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1			Attempt use of product rule to find first derivative	M1	producing form … ± … where one term involves $\ln x$ and the other does not	
			Obtain	A1	or unsimplified equiv	
			Attempt use of correct product rule to find second derivative	M1	with one term involving $\ln x$	
			Obtain $8\ln x+12$	A1	or unsimplified equiv	
			Obtain 28	A1		
				[5]		
2			State or imply $\text{cosec}\,\theta=1\div\sin\theta$	B1	allow $\text{cosec}=1\div\sin$	
			Attempt to express equation in terms of $\sin\theta$ only	M1	using identity of form $\pm1\pm2\sin^2\theta$ for $\cos2\theta$	
			Obtain $10\sin^2\theta+2\sin\theta-5=0$	A1	or unsimplified equiv involving $\sin\theta$ only but with no $\sin\theta$ remaining in denominator	
			Attempt use of formula to find $\sin\theta$ from 3-term quadratic equation involving $\sin\theta$ (using formula or completing square even if their equation can be solved by factorisation)	M1	use implied by at least one correct value of $\sin\theta$ or θ; if correct quadratic formula quoted, condone one sign error for M1; if formula not first quoted, any error leads to M0	if completion of square used to solve equation, this must be correct for M1 to be earned
			Obtain 37.9°	A1	or greater accuracy 37.8896…	
			Obtain 142°	A1	or greater accuracy 142.1103…; and no others between 0 and 180; ignore any answers, right or wrong, outside 0 - 180	no working and answers only (max 2/6): 37.9 (or greater accuracy) B1 142 (or greater accuracy) and no others … B1
				[6]		

Question			Answer	Marks	Guidance	
3	(i)		Attempt calculation $k(y+4y+2y+...)$	M1	any constant k; using y values with coefficients 1, 2, 4 each occurring at least once; brackets may be implied by subsequent calculation	allow M1 for attempt using y values based on wrong x values such as 0, 1, 2, 3, 4; attempt based on $k(y_0+y_4)+4y_1+2y_2+4y_3$ is M0 unless subsequent calculation shows missing brackets are 'present'
			Obtain $k(e^0+4e^{\sqrt{0.5}}+2e+4e^{\sqrt{1.5}}+e^{\sqrt{2}})$	A1	or equiv perhaps involving decimal values 1, 2.02811…, 2.71828…, 3.40329…, 4.11325…	
			Use $k=\frac{1}{3}\times\frac{1}{2}$	A1		
			Obtain 5.38	A1	allow 5.379 but not, in final answer, greater 'accuracy'; answer $5.38+c$ is final A0	answer only: 0/4
				[4]		
3	(ii)		Attempt calculation of form $10\times(\text{answer to part i})+k$	M1	implied by correct answer only or by answer following correctly from their incorrect part **(i)** ; any non-zero constant k	allow attempt involving second use of Simpson's rule: M1 for complete correct expression, A1 for answer
			Obtain 55.8 or greater accuracy based on their part **(i)** – more than 3 s.f. acceptable	A1ft	following their answer to part **(i)** but A0 for $55.8+c$	answer only 54.8 with no working earns M1A0 (as does 10(their ans) + 1); otherwise incorrect answer with no working earns 0/2
				[2]		
4	(i)		Either: State $2x^3+4=-50$	B1		
			State -3 and no other	B1		
			Or: Obtain $\sqrt[3]{\frac{1}{2}(x-4)}$ for inverse of f	B1	or equiv; using any letter	
			State -3 and no other	B1		
				[2]		
4	(ii)		Show composition of functions the right way round	M1		
			Obtain $2x-16$	A1	AG; necessary detail needed	first step $2(x-10)+4$ acceptable but then two more steps needed
				[2]		

Question			Answer	Marks	Guidance	
4	(iii)		Obtain $\sqrt[3]{2x^3-6}$ or $(2x^3-6)^{\frac{1}{3}}$ for $\mathrm{gf}(x)$	B1	or unsimplified equiv	
			Apply chain rule to function which is cube root of a non-linear expression	M1	condone incorrect constant; otherwise use of chain rule for their function must be correct	may use $u = 2x^3 - 6$; M1 earned for expression involving u
			Obtain $2x^2(2x^3-6)^{-\frac{2}{3}}$	A1	or similarly simplified equiv; do not accept final answer with $\frac{6}{3}$ unsimplified	… in terms of x
				[3]		
5	(a)		Differentiate to produce $ke^{-0.33t}$	M1	where constant k is different from 58	method must involve differentiation
			Obtain $-19.14e^{-0.33t}$ or $19.14e^{-0.33t}$	A1	or unsimplified equiv	
			Obtain -5.1 or 5.1	A1	whatever they claim value represents; accept 5.11 but not greater accuracy	
				[3]		
5	(b)		<u>Either</u>:			
			State or imply formula $42e^{kt}$ or $42a^t$	B1	$42e^{-kt}$, $42e^{-kx}$, etc. also acceptable	
			Attempt to find k from $42e^{6k} = 51.8$ or a from $42a^6 = 51.8$	M1	using sound process involving logarithms at least as far as $6k = \ldots$ or $a = \ldots$	
			Obtain $k = 0.035$ or $a = 1.0356$	A1	or greater accuracy 0.03495… or exact equiv $\frac{1}{6}\ln\frac{37}{30}$	
			Substitute 24 to obtain value between 97.1 and 97.3 inclusive	A1	allow greater accuracy than 3 s.f.	
			<u>Or</u>:			
			Use ratio $\frac{51.8}{42}$ in calculation	B1		
			Attempt calculation of form $42 \times r^n$	M1		
			Obtain $42 \times (\frac{51.8}{42})^4$ or $51.8 \times (\frac{51.8}{42})^3$	A1		
			Obtain value between 97.1 and 97.3 inclusive	A1	allow greater accuracy than 3 s.f.	
				[4]		

Question			Answer	Marks	Guidance	
6	(i)		Draw inverted parabola roughly symmetrical about the y-axis and with maximum point more or less on y-axis	M1	drawing enough of the parabola that two intersections occur, ignoring their locations at this stage	
			State $y = 9 - x^2$ and indicate two intersections by marks on diagram or written reference to two intersections	A1	now needs second curve drawn so that right-hand intersection occurs in first quadrant	
				[2]		
6	(ii)	(a)	Calculate values of quartic expression for 2.1 and 2.2	M1	if no explicit working seen, M1 is implied by at least one correct value; but if no explicit working seen and both values wrong, award M0	
			Obtain –1.9... and 1.6… and draw attention to sign change or clear equiv	A1		
				[2]		
6	(ii)	(b)	Obtain correct first iterate	B1	starting anywhere between –1 and 9 and showing at least 3 d.p.	
			Carry out process to produce at least three iterates in all	M1	implied by plausible sequence of values; allow recovery after error	$2.1 \to 2.15056 \to 2.15531 \to 2.15575 \to 2.15579$
						$2.15 \to 2.15526 \to 2.15574 \to 2.15579$
			Obtain at least two more correct iterates	A1	showing at least 3 decimal places	
			Obtain 2.156	A1	final answer needed to exactly 3 d.p.; not given for 2.156 as final iterate in sequence, i.e. needs indication (perhaps just underlining) that value of α found	$2.2 \to 2.15980 \to 2.15616 \to 2.15583 \to 2.15580$ answer only: 0/4
				[4]		

Question		Answer	Marks	Guidance	
7	(i)	Integrate to obtain $k(4x+1)^{\frac{1}{2}}$ or $ku^{\frac{1}{2}}$	*M1	any constant k	
		Obtain correct $\frac{1}{2}\sqrt{3}(4x+1)^{\frac{1}{2}}$ or $\frac{1}{2}\sqrt{3}u^{\frac{1}{2}}$	A1	or exact equiv	
		Apply limits 0 and 20 and attempt subtraction of area of rectangle (or limits 1 and 81 if u involved)	M1	dep *M; or equiv such as including term $-\frac{1}{9}\sqrt{3}$ in the integration or finding $\int \frac{1}{9}\sqrt{3}\,dx$ separately; allow M1 if decimal values used here	Alternative: (region between curve and y-axis) Obtain equation $x=\frac{3}{4}y^{-2}-\frac{1}{4}$ B1 Integrate to obtain form $k_1y^{-1}+k_2y$ *M1 Apply limits $\frac{1}{9}\sqrt{3}$ and $\sqrt{3}$ the right way round M1 d*M Obtain $\frac{6}{\sqrt{3}}-\frac{8}{36}\sqrt{3}$ or better A1
		Obtain $4\sqrt{3}-\frac{20}{9}\sqrt{3}$ and hence $\frac{16}{9}\sqrt{3}$	A1 **[4]**	answer must be exact and a single term; $\frac{16}{9}\sqrt{3}+c$ as answer is final A0	
	(ii)	State volume is $\pi\int\frac{3}{4x+1}\,dx$	B1	no need for limits here; condone absence of dx; condone absence of π here if it appears later in solution	allow B1 for $\int \pi y^2$ <u>and</u> $y^2=\frac{3}{4x+1}$ stated
		Obtain integral of form $k\ln(4x+1)$	M1	any constant k with or without π	if brackets missing, and subsequent calculation does not show their 'presence', marks are max B1M1A0A0M1A0
		Obtain $\frac{3}{4}\pi\ln(4x+1)$ or $\frac{3}{4}\ln(4x+1)$	A1		
		Apply limits to obtain $\frac{3}{4}\pi\ln 81$ or $\frac{3}{4}\ln 81$	A1	or exact equiv perhaps with $\ln 1$ present	
		Attempt to subtract volume of cylinder, using correct radius and 'height'	M1	with exact volume of cylinder attempted	do not treat rotation around y-axis as mis-read: this is 0/6
		Obtain $3\pi\ln 3-\frac{20}{27}\pi$ or $\pi(\frac{3}{4}\ln 81-\frac{20}{27})$	A1 **[6]**	or exact equiv involving two terms	

Question			Answer	Marks	Guidance	
8	(i)		Attempt use of quotient rule or equiv	M1	condone one slip only but must be subtraction in numerator; condone absence of necessary brackets; or equiv	
			Obtain $\dfrac{2(x^2+5)-2x(2x+4)}{(x^2+5)^2}$	A1	or correct equiv; now with brackets as necessary	correct numerator but error in denominator: max M1A0A1M1A1A1; numerator wrong way round: max M0A0A0M1A1A1
			Obtain $-2x^2-8x+10=0$	A1	or equiv involving three terms	
			Attempt solution of three-term quadratic equation based on numerator of derivative (even if their equation has no real roots)	M1	implied by no working but 2 correct values obtained	M1 for factorisation awarded if attempt is such that x^2 term and one other term correct upon expansion; if formula used, M1 awarded as per Qn 2
			Obtain –5 and 1	A1		
			Obtain $(-5, -\frac{1}{5})$ and (1, 1)	A1	Allow $-\frac{6}{30}$	
				[6]		
	(ii)	(a)	Sketch (more or less) correct curve	B1	showing negative part reflected in x-axis and positive part unchanged; ignore intercept values on axes, right or wrong	
			State values between 0 and their y-value of maximum point lying in first quadrant	M1	accept $\le$ or $<$ signs here	
			State correct $0 \le y \le 1$	A1ft	following their y-value of maximum point in first quadrant; now with $\le$ signs; or equiv perhaps involving g or $g(x)$	for "$y \ge 0$ and $y \le 1$", award M1A1; for separate statements $y \ge 0$, $y \le 1$, award M1A0
				[3]		
	(ii)	(b)	Indicate, in some way, values between y-coordinates of maximum point and reflected minimum point (provided their y-coordinate of minimum point is negative)	M1	allow $\le$ sign(s) here; could be clear indication on graph	for "$k > \frac{1}{5}$ and $k < 1$", award M1A1; for separate statements, award M1A0
			State $\frac{1}{5} < k < 1$	A1	or correct equiv; not $\le$ now; correct answer only earns M1A1	
				[2]		

Question			Answer	Marks	Guidance	
9	(i)		Simplify to obtain $\frac{11}{2}\cos\theta + \frac{5\sqrt{3}}{2}\sin\theta$	B1	or equiv with two terms perhaps with $\sin 60$ retained	accept decimal values
			Attempt correct process to find R	M1	for expression of form $a\cos\theta + b\sin\theta$	obtained after initial simplification
			Attempt correct process to find α	M1	for expression of form $a\cos\theta + b\sin\theta$; condone $\sin\alpha = \frac{11}{2}$, $\cos\alpha = \frac{5}{2}\sqrt{3}$	obtained after initial simplification
			Obtain $7\sin(\theta + 51.8)$	A1	or greater accuracy 51.786…	
				[4]		
	(ii)	(a)	State stretch and translation in either order	M1	or equiv but using correct terminology, not move, squash, …	SC: if M0 but one transformation completely correct, award B1 for 1/3
			State stretch parallel to y-axis with factor $\frac{1}{7}$	A1ft	following their R and clearly indicating correct direction	
			State translation parallel to θ-axis or x-axis by 51.8 in positive direction or state translation by vector $\begin{pmatrix} 51.8 \\ 0 \end{pmatrix}$	A1ft	following their α and clearly indicating correct direction; or equiv such as 308.2 parallel to x-axis in negative direction	
				[3]		
		(b)	State left-hand side (their R) $\sin(\frac{1}{3}\beta + \gamma)$ where $\gamma \neq \pm$(their α), $\gamma \neq \pm 40$, $\gamma \neq \pm 20$	M1	or equiv such as stating $\theta = \frac{1}{3}\beta + 20$	
			Obtain (their R) $\sin(\frac{1}{3}\beta +$ their $\alpha + 20) = 3$	A1ft	(and, in this case, allowing A1ft provided value of $\frac{1}{3}\beta$ attempted later)	
			Attempt correct process to find any value of $\frac{1}{3}\beta$	M1	for equation of form $\sin(\frac{1}{3}\beta + \gamma) = k$ where $\lvert k \rvert < 1$, $k \neq 0$	
			Attempt complete process to find positive value of β	M1	including choosing second quadrant value of their $\sin^{-1}\frac{3}{7}$	
			Obtain 248 or 249 or 248.5	A1	or greater accuracy 248.508…	
				[5]		

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