

**ADVANCED GCE** 

**MATHEMATICS** Core Mathematics 4 4724

Candidates answer on the Answer Booklet

OCR Supplied Materials:

- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required: None Tuesday 13 January 2009 Morning

Duration: 1 hour 30 minutes



## INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer all the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

## INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- You are reminded of the need for clear presentation in your answers.
- The total number of marks for this paper is 72.
- This document consists of 4 pages. Any blank pages are indicated.

1 Simplify 
$$\frac{20-5x}{6x^2-24x}$$
. [3]

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2 Find 
$$\int x \sec^2 x \, dx$$
. [4]

- 3 (i) Expand  $(1+2x)^{\frac{1}{2}}$  as a series in ascending powers of x, up to and including the term in  $x^3$ . [3]
  - (ii) Hence find the expansion of  $\frac{(1+2x)^{\frac{1}{2}}}{(1+x)^3}$  as a series in ascending powers of x, up to and including the term in  $x^3$ . [5]
  - (iii) State the set of values of x for which the expansion in part (ii) is valid. [1]

4 Find the exact value of 
$$\int_0^{\frac{1}{4}\pi} (1 + \sin x)^2 dx.$$
 [6]

5 (i) Show that the substitution  $u = \sqrt{x}$  transforms  $\int \frac{1}{x(1+\sqrt{x})} dx$  to  $\int \frac{2}{u(1+u)} du$ . [3]

(ii) Hence find the exact value of 
$$\int_{1}^{9} \frac{1}{x(1+\sqrt{x})} dx.$$
 [5]

6 A curve has parametric equations

$$x = t^2 - 6t + 4$$
,  $y = t - 3$ .

Find

- (i) the coordinates of the point where the curve meets the *x*-axis, [2]
- (ii) the equation of the curve in cartesian form, giving your answer in a simple form without brackets, [2]
- (iii) the equation of the tangent to the curve at the point where t = 2, giving your answer in the form ax + by + c = 0, where *a*, *b* and *c* are integers. [5]
- 7 (i) Show that the straight line with equation  $\mathbf{r} = \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix} + t \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix}$  meets the line passing through (9, 7, 5) and (7, 8, 2), and find the point of intersection of these lines. [6]

[4]

(ii) Find the acute angle between these lines.

- 8 The equation of a curve is  $x^3 + y^3 = 6xy$ .
  - (i) Find  $\frac{dy}{dx}$  in terms of x and y. [4]
  - (ii) Show that the point  $(2^{\frac{4}{3}}, 2^{\frac{5}{3}})$  lies on the curve and that  $\frac{dy}{dx} = 0$  at this point. [4]
  - (iii) The point (a, a), where a > 0, lies on the curve. Find the value of a and the gradient of the curve at this point.
- 9 A liquid is being heated in an oven maintained at a constant temperature of 160 °C. It may be assumed that the rate of increase of the temperature of the liquid at any particular time *t* minutes is proportional to  $160 \theta$ , where  $\theta$  °C is the temperature of the liquid at that time.
  - (i) Write down a differential equation connecting  $\theta$  and t. [2]

When the liquid was placed in the oven, its temperature was 20  $^{\circ}$ C and 5 minutes later its temperature had risen to 65  $^{\circ}$ C.

(ii) Find the temperature of the liquid, correct to the nearest degree, after another 5 minutes. [9]



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