

**ADVANCED GCE  
MATHEMATICS**  
Core Mathematics 4

**4724**

Candidates answer on the Answer Booklet

**OCR Supplied Materials:**

- 8 page Answer Booklet
- List of Formulae (MF1)

**Other Materials Required:**

None

**Friday 15 January 2010  
Afternoon**

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 Find the quotient and the remainder when  $x^4 + 11x^3 + 28x^2 + 3x + 1$  is divided by  $x^2 + 5x + 2$ . [4]

2 Points  $A$ ,  $B$  and  $C$  have position vectors  $-5\mathbf{i} - 10\mathbf{j} + 12\mathbf{k}$ ,  $\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$  and  $3\mathbf{i} + 6\mathbf{j} + p\mathbf{k}$  respectively, where  $p$  is a constant.

(i) Given that angle  $ABC = 90^\circ$ , find the value of  $p$ . [4]

(ii) Given instead that  $ABC$  is a straight line, find the value of  $p$ . [2]

3 By expressing  $\cos 2x$  in terms of  $\cos x$ , find the exact value of  $\int_{\frac{1}{4}\pi}^{\frac{1}{3}\pi} \frac{\cos 2x}{\cos^2 x} dx$ . [5]

4 Use the substitution  $u = 2 + \ln t$  to find the exact value of

$$\int_1^e \frac{1}{t(2 + \ln t)^2} dt. \quad [6]$$

5 (i) Expand  $(1 + x)^{\frac{1}{3}}$  in ascending powers of  $x$ , up to and including the term in  $x^2$ . [2]

(ii) (a) Hence, or otherwise, expand  $(8 + 16x)^{\frac{1}{3}}$  in ascending powers of  $x$ , up to and including the term in  $x^2$ . [4]

(b) State the set of values of  $x$  for which the expansion in part (ii) (a) is valid. [1]

6 A curve has parametric equations

$$x = 9t - \ln(9t), \quad y = t^3 - \ln(t^3).$$

Show that there is only one value of  $t$  for which  $\frac{dy}{dx} = 3$  and state that value. [6]

7 Find the equation of the normal to the curve  $x^3 + 2x^2y = y^3 + 15$  at the point  $(2, 1)$ , giving your answer in the form  $ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are integers. [8]

8 (i) State the derivative of  $e^{\cos x}$ . [1]

(ii) Hence use integration by parts to find the exact value of

$$\int_0^{\frac{1}{2}\pi} \cos x \sin x e^{\cos x} dx. \quad [6]$$

9 The equation of a straight line  $l$  is  $\mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ .  $O$  is the origin.

(i) The point  $P$  on  $l$  is given by  $t = 1$ . Calculate the acute angle between  $OP$  and  $l$ . [4]

(ii) Find the position vector of the point  $Q$  on  $l$  such that  $OQ$  is perpendicular to  $l$ . [4]

(iii) Find the length of  $OQ$ . [2]

10 (i) Express  $\frac{1}{(3-x)(6-x)}$  in partial fractions. [2]

(ii) In a chemical reaction, the amount  $x$  grams of a substance at time  $t$  seconds is related to the rate at which  $x$  is changing by the equation

$$\frac{dx}{dt} = k(3-x)(6-x),$$

where  $k$  is a constant. When  $t = 0$ ,  $x = 0$  and when  $t = 1$ ,  $x = 1$ .

(a) Show that  $k = \frac{1}{3} \ln \frac{5}{4}$ . [7]

(b) Find the value of  $x$  when  $t = 2$ . [4]

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