



# **Mathematics**

Advanced GCE

Unit 4724: Core Mathematics 4

# Mark Scheme for January 2011

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Not just 
$$\sec x = \frac{1}{\cos x}$$
  
Allow  $\frac{u \, dv - v \, du}{v^2}$  & wrong trig signs  
No inaccuracy allowed here  
Or vice versa. Not just = sec x.tan x  
or  $\pm (\cos^2 x - \sin^2 x)$   
 $\sqrt{2 - 2 \sin^2 x}$  needs simplifying  
irrespective of any const multiples  
Condone  $\theta$  for x except final line

M1 or write as 
$$1 - (2y - 4y^2 \text{ or } 2y + 4y^2)$$
  
B1  
A1 $\sqrt{3}$  where b = cf $(x^2)$  in part (i)

Accept  $\ln |x-2|, \ln |2-x|, \ln (2-x)$ 

Negative sign is required

Still accept lns as before

No indication of ln(negative)

A1 3  $-\frac{1}{8}x^2$  without work  $\rightarrow$  M1 A1

Third term 
$$= +\frac{3}{2}y^2$$
 or  $\sqrt{(4b+2)y^2}$   
A1 $\sqrt{3}$  where  $b = cf(x^2)$  in part (i)  
A(x-2)+B = 7-2x  
M1 or  $A(x-2)^2 + B(x-2) = (7-2x)(x-2)$ 

$$A = -2$$
  

$$B = 3$$
(ii) 
$$\int \frac{A}{x-2} dx = \left(A \text{ or } \frac{1}{A}\right) \ln (x-2)$$
  

$$\int \frac{B}{x-2} dx = -\left(B \text{ or } \frac{1}{A}\right) \frac{1}{x-2}$$

$$J(x-2)^{2} (B) x-2$$
  
Correct f.t. of A & B;  $A \ln(x-2) - \frac{B}{x-2}$   
Using limits =  $-2 \ln 3 + 2 \ln 2 + \frac{1}{2}$  ISW

3 (i) State/imply 
$$\frac{d}{dx}(\sec x) = \frac{d}{dx}\left(\frac{1}{\cos x}\right) \operatorname{or} \frac{d}{dx}(\cos x)^{-1}$$

Attempt quotient rule or chain rule to power -1

Obtain 
$$\frac{\sin x}{\cos^2 x}$$
 or  $-.-(\sin x)(\cos x)^{-2}$ 

e.g.  $\frac{1}{\cos x} \cdot \frac{\sin x}{\cos x}$ Simplify with suff evid to AG

(ii) Use 
$$\cos 2x = +/-1 + /-2 \cos^2 x$$
 or  $+/-1 + /-2 \sin^2 x$   
Correct denominator =  $\sqrt{2 \cos^2 x}$ 

Evidence that 
$$\frac{\tan x}{\cos x} = \sec x \tan x$$
 or  $\int \frac{\tan x}{\cos x} dx = \sec x$ 

$$\frac{1}{\sqrt{2}}\sec x \quad (+c) \qquad \qquad \text{A1 4}$$

A1 A1 3

B1

B1

B1√

B1 4

B1

M1

A1

A1 4

M1

A1

B1

1

8

B1

**1** (i) First two terms are  $1 - \frac{1}{2}x$ .....

First two terms are 1-y

 $= -\frac{1}{8}x^2$ 

2 (i) A(x-2)+B=7-2x

Third term =  $\frac{\frac{1}{2} - \frac{1}{2}}{2} [(-x)^2 \text{ or } x^2 \text{ or } -x^2]$ 

(ii) Attempt to replace x by  $2y - 4y^2$  or  $2y + 4y^2$ 

4 (i) Attempt to use 
$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$
 or  $\frac{dy}{dt} \cdot \frac{dt}{dx}$   
 $\frac{4}{2t}$  or  $\frac{2}{t}$ 

- (ii) Subst t = 4 into their (i), invert & change sign Subst t = 4 into (x,y) & use num grad for tgt/normal y = -2x + 52 AEF CAO (no f.t.)
- (iii) Attempt to eliminate t from the 2 given equations

$$x = 2 + \frac{y^2}{16}$$
 or  $y^2 = 16(x-2)$  AEF ISW

5 (i) Attempt to connect dx and du

$$5 - x = 4 - u^2$$

Show 
$$\int \frac{4-u^2}{2+u} \cdot 2u \, du$$
 reduced to  $\int 4u - 2u^2 \, du$  AG

Clear explanation of why limits change

$$\frac{4}{3}$$

(ii)(a) 5-x

(b) Show reduction to  $2 - \sqrt{x-1}$ 

$$\int \sqrt{x-1} \, dx = \frac{2}{3} (x-1)^{\frac{3}{2}}$$
$$\left(10 - \frac{2}{3} \cdot 8\right) - \left(4 - \frac{2}{3}\right) = \frac{4}{3} \text{ or } 4\frac{2}{3} - 3\frac{1}{3} = \frac{4}{3}$$

- 6 (i) Work with correct pair of direction vectors Demonstrate correct method for finding scalar product Demonstrate correct method for finding modulus 24, 24.0 (24.006363..) (degrees) 0.419 (0.41899..) (rad) A1 4 Mark earliest value, allow trunc/rounding
  - (ii) Attempt to set up 3 equations Find correct values of  $(s, t) = (1,0) \operatorname{or} (1,4) \operatorname{or} (5,12)$ Substitute their (s,t) into equation not used Correctly demonstrate failure
  - (iii) Subst their (s,t) from first 2 eqns into new 3<sup>rd</sup> eqn a = 6

- M1 Not just quote formula
- A1 2
- M1
- M1
- A1 3 Only the eqn of normal accepted
- M1
- A1 2 Mark at earliest acceptable form.

### 7

- M1 Including  $\frac{du}{dx} = \text{ or } du = ...dx$ ; not dx = du
- perhaps in conjunction with next line B1
- A1 In a fully satisfactory & acceptable manner
- B1 e.g. when x = 2, u = 1 and when x = 5, u = 2
- B1 5 not dependent on any of first 4 marks
- \*B1 1 Accept 4-x-1=5-x (this is not AG)

#### dep\*B1

- B1 Indep of other marks, seen anywhere in (b)
- B1 3 Working must be shown

## 9

M1

- M1 Of any two 3x3 vectors rel to question
- M1 Of any vector relevant to question
- Of type 3 + 2s = 5, 3s = 3 + t, -2 4s = 2 2tM1
- A1 Or 2 diff values of s (or of t)
- and make a relevant deduction M1
- A1 4 dep on all 3 prev marks
- New  $3^{rd}$  eqn of type a 4s = 2 2tM1
- A1 2

10

3

#### **Mark Scheme**

January 2011

- 7 Attempt parts with  $u = x^2 + 5x + 7$ ,  $dv = \sin x$   $1^{st} \text{ stage} = -(x^2 + 5x + 7)\cos x + \int (2x + 5)\cos x \, dx$   $\int (2x + 5)\cos x \, dx = (2x + 5)\sin x - \int 2\sin x \, dx$   $= (2x + 5)\sin x + 2\cos x$   $I = -(x^2 + 5x + 7)\cos x + (2x + 5)\sin x + 2\cos x$ (Substitute  $x = \pi$ ) -(Substitute x = 0)  $\pi^2 + 5\pi + 10$  WWW AG
- 8 (i)  $\frac{d}{dx}(y^2) = 2y \frac{dy}{dx}$   $\frac{d}{dx}(-5xy) = (-)(5)x \frac{dy}{dx} + (-)(5)y$ LHS completely correct  $4x - 5x \frac{dy}{dx} - 5y + 2y \frac{dy}{dx} (= 0)$ Substitute  $\frac{dy}{dx} = \frac{3}{8}$  or solve for  $\frac{dy}{dx}$  & then equate to  $\frac{3}{8}$ Produce x = 2y WWW AG (Converse acceptable)
  - (ii) Substitute 2y for x or  $\frac{1}{2}x$  for y in curve equation Produce either  $x^2 = 36$  or  $y^2 = 9$ AEF of  $(\pm 6, \pm 3)$

9 (i) Attempt to sep variables in the form  $\int \frac{p}{(x-8)^{\frac{1}{3}}} dx = \int q dt M1$ 

 $\int \frac{1}{(x-8)^{\frac{1}{3}}} dx = k(x-8)^{\frac{2}{3}}$ A1 All correct (+ c) A1

For equation containing 'c'; substitute t = 0, x = 72

Correct corresponding value of *c* from correct eqn Subst their c & x = 35 back into eqn

$$t = \frac{21}{8}$$
 or 2.63 / 2.625 [C.A.O]

(ii) State/imply in some way that x = 8 when flow stops B1 Substitute x = 8 back into equation containing numeric 'c' M1 t = 6 A1 3

- as far as  $f(x) + /-\int g(x) dx$ M1 A1 signs need not be amalgamated at this stage indep of previous A1 being awarded **B**1 B1 A1 WWW An attempt at subst x = 0 must be seen M1 A1 7 7 B1 M1 i.e. reasonably clear use of product rule A1 Accept " $\frac{dy}{dy}$  = " provided it is not used M1 Accuracy not required for "solve for  $\frac{dy}{dx}$ " A1 5 Expect 17x = 34y and/or  $\frac{dy}{dx} = \frac{5y - 4x}{2y - 5x}$ M1
- A1 3 ISW Any correct format acceptable 8 M1 Or invert as  $\frac{dt}{dx} = \frac{r}{(x-8)^{\frac{1}{3}}}$ ; *p*,*q*,*r* consts

A1

M1

A1

M1

A1 7

10

k const

M2 for  $\int_{1}^{35} = \int_{1}^{t}$  or  $\int_{1}^{72} = \int_{1}^{t}$ 

A2: 
$$t = \frac{21}{8}$$
 or 2.63 / 2.625 WWW

#### 4724

#### Mark Scheme

- 1 When an acceptable answer has been obtained, ignore subsequent working (ISW) unless stated otherwise.
- 2 Ignore working which has no relevance to question as set; e.g. in Qu.1, ignore all terms in  $x^3$  etc.
- 3 The 'M' marks are awarded if it is clear that candidate is attempting to do what he/she should be doing.
- 4 If an ans is given (AG), working must be checked minutely as answer shown will nearly always be 'correct'. More reasoning/explanation is generally required than when the answer is not given.

#### Comments or Alternative methods

#### Question 1(ii)

Beware: there are often double mistakes leading to the correct terms - errors invalidate marks.

#### Question 2(ii)

For the first 2 marks, we're really testing  $\int \frac{1}{x-2} dx$  and  $\int \frac{1}{(x-2)^2} dx$ ; this is why we accept  $\frac{1}{A}$  and/or  $-\frac{1}{B}$ .

For the 1<sup>st</sup> & 3<sup>rd</sup> marks, accept  $\ln(2-x)$  as these are the indef integ stages. At final, definite, stage, it will be penalised.. 'Exact value' is required; so 0.0945... without equivalent log version  $\rightarrow B0$  2ln2-3ln3 need not be simplified.

#### Question 4

Allow marks for part (iii) to be awarded at any stage of question.

So, if the Cartesian equation is worked out first of all, then award marks in part (i) as follow:

if cart. eqn is found in the form x = f(y), award M1 for finding  $\frac{dx}{dy}$ , inverting & subst y = 4t (in either order)

if cart. eqn is found in the form y = g(x), award M1 for finding  $\frac{dy}{dx}$  and substituting  $x = 2 + t^2$ 

and, finally, A1 as in main scheme.

#### Question 5(i)

The problem here will centre on how the candidate manipulates the equation  $u = \sqrt{x-1}$  to get x in terms of u. He/she could get  $x = u^2 + 1$  (correct) or, perhaps,  $x = u^2 - 1$  or  $x = 1 - u^2$  (incorrect) or some other incorrect version. The 1<sup>st</sup>, 4<sup>th</sup> & 5<sup>th</sup> marks in part (i) are unaffected by the correctness or otherwise of this manipulation. However, <u>any</u> <u>error seen</u> must destroy the 2<sup>nd</sup> and 3<sup>rd</sup> marks – but candidates can still score 3 of the 5 marks.

For the A1, there must be some evidence of reduction to the given answer; the one main case that we are <u>not accepting</u> is where  $\frac{8u-2u^3}{2+u}$  is said to be  $4u-2u^2$  without any supporting evidence; long division will suffice; <u>or</u> if  $8u-2u^3$  is said to be  $(2+u)(4u-2u^2)$ , then we will accept (as multiplication can easily be checked in the head whereas division is not reckoned to be). Note that '2' into '8u' gives '4u' and 'u' into '-2u<sup>3</sup>' gives '-2u<sup>2</sup>'.

#### Question 5(ii)(a)

This is just a '1' mark part so we give 1 or 0 purely dependent on the answer and we ignore any sloppy working. A candidate writing 4-x-1=3-x will be awarded 0 marks; however, another candidate writing 4-x-1=5-x will be awarded the B1 mark. This is not an AG so the candidate does not know the required answer.

#### Question 6(i)

For demonstrating correct method for finding scalar product, I expect to see at least 2/3 of the working correct.

Likewise for modulus: examine either vector,  $\sqrt{2^2 + 3^2 - 4^2}$  will score M1 { $\frac{2}{3}$  correct, prob  $\sqrt{29}$  will follow anyway}

### Question 6(ii)

Occasionally candidates do not follow a 'sensible' method. However, the first M1 is always standard. The remaining 3 marks must be awarded for convincing arguments and/for accurate results.

#### Question 7

This is a question where signs are crucial and where the given answer may be obtained even with errors in the working; also the fact that the answer is **AG** means that many candidates will state it on the final line.

Using the standard method, 3 marks out of the 7 are fixed (the 2 @ M1 and the final A1) but the other 4 marks depend on the capability of the candidate to integrate sin *x* and cos *x*.

If he/she uses  $\cos x$  for the integral of  $\sin x$ , candidate should get -(our version of 1st main stage), so that's A0 but he/she still has to integrate  $(2x+5)\cos x$  for the 2<sup>nd</sup> stage. Admittedly he/she may then make a further mistake when integrating  $\cos x$  but the 2 @ B1 are available. These 2 marks are an independent pair and only depend on the integral of  $(2x+5)\cos x$  being attempted. Whether it's the integral of  $(2x+5)\cos x$  or of  $-(2x+5)\cos x$  is immaterial. This gives a maximum of 4 out of 7 if  $\sin x$  is incorrectly integrated.

Even though I have bracketed the 3 terms as  $(x^2 + 5x + 7)$ , we can expect some candidates to multiply out as 3 separate

integrals.,	$\int x^2 \sin x  \mathrm{d}x$	and	$\int 5x \sin x  \mathrm{d}x$	and	$\int 7 \sin x  dx$	
Their equiv	valent 1 <sup>st</sup> stages are:					
$-x^2 \cos x$	+ $\int 2x \cos x  \mathrm{d}x;$		$-5x\cos x + \int 5\cos x$	x dx;	$-7\cos x$	M1 + A1
Their equiv	valent 2 <sup>nd</sup> stages are:					
$2x\sin x + 2$	$2\cos x$ <b>B1</b>		$5 \sin x$ <b>B1</b>			

To obtain the corresponding marks, all components must be correct.

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