

**ADVANCED GCE UNIT
MATHEMATICS**

Core Mathematics 4
THURSDAY 14 JUNE 2007

4724/01

Afternoon

Time: 1 hour 30 minutes

Additional Materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

- 1 The equation of a curve is $y = f(x)$, where $f(x) = \frac{3x + 1}{(x + 2)(x - 3)}$.
- (i) Express $f(x)$ in partial fractions. [2]
- (ii) Hence find $f'(x)$ and deduce that the gradient of the curve is negative at all points on the curve. [3]
- 2 Find the exact value of $\int_0^1 x^2 e^x dx$. [6]
- 3 Find the exact volume generated when the region enclosed between the x -axis and the portion of the curve $y = \sin x$ between $x = 0$ and $x = \pi$ is rotated completely about the x -axis. [6]
- 4 (i) Expand $(2 + x)^{-2}$ in ascending powers of x up to and including the term in x^3 , and state the set of values of x for which the expansion is valid. [5]
- (ii) Hence find the coefficient of x^3 in the expansion of $\frac{1 + x^2}{(2 + x)^2}$. [2]
- 5 A curve C has parametric equations
- $$x = \cos t, \quad y = 3 + 2 \cos 2t, \quad \text{where } 0 \leq t \leq \pi.$$
- (i) Express $\frac{dy}{dx}$ in terms of t and hence show that the gradient at any point on C cannot exceed 8. [4]
- (ii) Show that all points on C satisfy the cartesian equation $y = 4x^2 + 1$. [3]
- (iii) Sketch the curve $y = 4x^2 + 1$ and indicate on your sketch the part which represents C . [2]
- 6 The equation of a curve is $x^2 + 3xy + 4y^2 = 58$. Find the equation of the normal at the point $(2, 3)$ on the curve, giving your answer in the form $ax + by + c = 0$, where a , b and c are integers. [8]
- 7 (i) Find the quotient and the remainder when $2x^3 + 3x^2 + 9x + 12$ is divided by $x^2 + 4$. [4]
- (ii) Hence express $\frac{2x^3 + 3x^2 + 9x + 12}{x^2 + 4}$ in the form $Ax + B + \frac{Cx + D}{x^2 + 4}$, where the values of the constants A , B , C and D are to be stated. [1]
- (iii) Use the result of part (ii) to find the exact value of $\int_1^3 \frac{2x^3 + 3x^2 + 9x + 12}{x^2 + 4} dx$. [5]

- 8 The height, h metres, of a shrub t years after planting is given by the differential equation

$$\frac{dh}{dt} = \frac{6-h}{20}.$$

A shrub is planted when its height is 1 m.

- (i) Show by integration that $t = 20 \ln\left(\frac{5}{6-h}\right)$. [6]
- (ii) How long after planting will the shrub reach a height of 2 m? [1]
- (iii) Find the height of the shrub 10 years after planting. [2]
- (iv) State the maximum possible height of the shrub. [1]
- 9 Lines L_1 , L_2 and L_3 have vector equations

$$L_1: \mathbf{r} = (5\mathbf{i} - \mathbf{j} - 2\mathbf{k}) + s(-6\mathbf{i} + 8\mathbf{j} - 2\mathbf{k}),$$

$$L_2: \mathbf{r} = (3\mathbf{i} - 8\mathbf{j}) + t(\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}),$$

$$L_3: \mathbf{r} = (2\mathbf{i} + \mathbf{j} + 3\mathbf{k}) + u(3\mathbf{i} + c\mathbf{j} + \mathbf{k}).$$

- (i) Calculate the acute angle between L_1 and L_2 . [4]
- (ii) Given that L_1 and L_3 are parallel, find the value of c . [2]
- (iii) Given instead that L_2 and L_3 intersect, find the value of c . [5]

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