RECOGNISING ACHIEVEMENT

ADVANCED GCE **4724/01**

MATHEMATICS

Core Mathematics 4

WEDNESDAY 21 MAY 2008 Afternoon

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

1 **(a)** Simplify $\dfrac{(2x^2 - 7x - 4)(x+1)}{(3x^2 + x - 2)(x - 4)}$. [2]

(b) Find the quotient and remainder when $x^3 + 2x^2 - 6x - 5$ is divided by $x^2 + 4x + 1$. [4]

2 Find the exact value of $\displaystyle\int_1^{e} x^4 \ln x \, dx$. [5]

3 The equation of a curve is $x^2y - xy^2 = 2$.

(i) Show that $\dfrac{dy}{dx} = \dfrac{y^2 - 2xy}{x^2 - 2xy}$. [3]

(ii) **(a)** Show that, if $\dfrac{dy}{dx} = 0$, then $y = 2x$. [2]

(b) Hence find the coordinates of the point on the curve where the tangent is parallel to the x-axis. [3]

4 Relative to an origin O, the points A and B have position vectors $3\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ and $\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ respectively.

(i) Find a vector equation of the line passing through A and B. [2]

(ii) Find the position vector of the point P on AB such that OP is perpendicular to AB. [5]

5 **(i)** Show that $\sqrt{\dfrac{1-x}{1+x}} \approx 1 - x + \frac{1}{2}x^2$, for $|x| < 1$. [5]

(ii) By taking $x = \frac{2}{7}$, show that $\sqrt{5} \approx \frac{111}{49}$. [3]

6 Two lines have equations

$$\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ -5 \end{pmatrix} + t\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} \quad \text{and} \quad \mathbf{r} = \begin{pmatrix} 12 \\ 0 \\ 5 \end{pmatrix} + s\begin{pmatrix} 1 \\ -4 \\ -2 \end{pmatrix}.$$

(i) Show that the lines intersect. [4]

(ii) Find the angle between the lines. [4]

7 **(i)** Show that, if $y = \operatorname{cosec} x$, then $\dfrac{dy}{dx}$ can be expressed as $-\operatorname{cosec} x \cot x$. [3]

(ii) Solve the differential equation

$$\frac{dx}{dt} = -\sin x \tan x \cot t,$$

given that $x = \frac{1}{6}\pi$ when $t = \frac{1}{2}\pi$. [5]

8 **(i)** Given that $\frac{2t}{(t+1)^2}$ can be expressed in the form $\frac{A}{t+1} + \frac{B}{(t+1)^2}$, find the values of the constants A and B. [3]

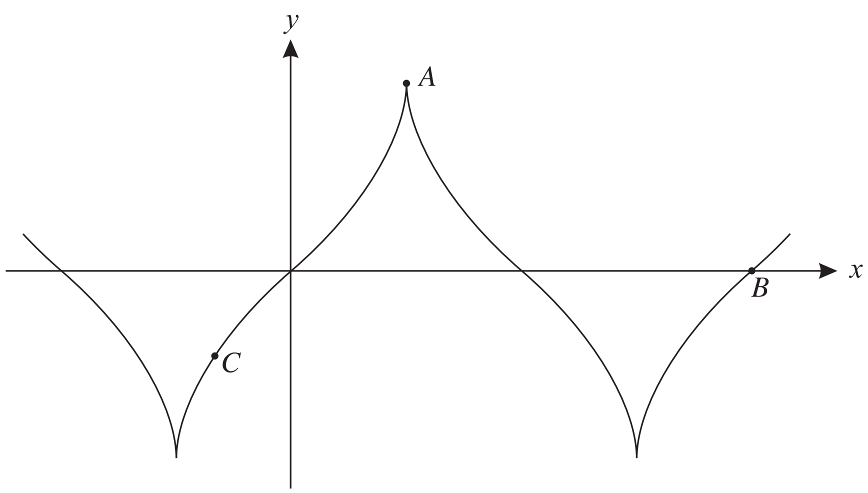
(ii) Show that the substitution $t = \sqrt{2x-1}$ transforms $\int \frac{1}{x+\sqrt{2x-1}}\,dx$ to $\int \frac{2t}{(t+1)^2}\,dt$. [4]

(iii) Hence find the exact value of $\int_1^5 \frac{1}{x+\sqrt{2x-1}}\,dx$. [4]

9 The parametric equations of a curve are

$$x = 2\theta + \sin 2\theta, \quad y = 4\sin\theta,$$

and part of its graph is shown below.



(i) Find the value of θ at A and the value of θ at B. [3]

(ii) Show that $\frac{dy}{dx} = \sec\theta$. [5]

(iii) At the point C on the curve, the gradient is 2. Find the coordinates of C, giving your answer in an exact form. [3]