

## 4724 Core Mathematics 4

<p>1 (a) <math>2x^2 - 7x - 4 = (2x+1)(x-4)</math> or  <math>3x^2 + x - 2 = (3x-2)(x+1)</math></p> <p><math>\frac{2x+1}{3x-2}</math> as final answer; this answer only</p>	<p><b>B1</b></p> <p><b>B1</b> Do not ISW</p> <p><b>2</b></p>
<p>(b) For correct leading term <math>x</math> in quotient  For evidence of correct division process  Quotient = <math>x - 2</math></p> <p>Remainder = <math>x - 3</math></p>	<p><b>B1</b> <u>Identity method</u></p> <p><b>M1</b> <math>M1: x^3 + 2x^2 - 6x - 5 = Q(x^2 + 4x + 1) + R</math></p> <p><b>A1</b> <math>M1: Q = ax + b</math> or <math>x + b, R = cx + d</math> &amp; <math>\geq 2</math> ops  [N.B. If <math>Q = x + b</math>, this <math>\Rightarrow</math> 1 of the 2 ops ]</p> <p><b>A1</b> <math>A2: a = 1, b = -2, c = 1, d = -3</math> SR: <u>B1</u> for two</p> <p><b>4</b></p>
<p>2 Parts with correct split of <math>u = \ln x, \frac{dv}{dx} = x^4</math></p> <p><math>\frac{x^5}{5} \ln x - \int \frac{x^5}{5} \cdot \frac{1}{x} (dx)</math></p> <p><math>\frac{x^5}{5} \ln x - \frac{x^5}{25}</math></p> <p>Correct method with the limits  <math>\frac{4e^5}{25} + \frac{1}{25}</math> ISW (Not '+c')</p>	<p><b>*M1</b> obtaining result <math>f(x) + /- \int g(x) dx</math></p> <p><b>A1</b></p> <p><b>A1</b></p> <p>dep<b>*M1</b> Decimals acceptable here</p> <p><b>A1</b> Accept equiv fract; like terms amalgamated</p> <p><b>5</b></p>
<p>3 (i) <math>\frac{d}{dx}(x^2 y) = x^2 \frac{dy}{dx} + 2xy</math> or <math>\frac{d}{dx}(xy^2) = 2xy \frac{dy}{dx} + y^2</math></p> <p>Attempt to solve their differentiated equation for <math>\frac{dy}{dx}</math></p> <p><math>\frac{dy}{dx} = \frac{y^2 - 2xy}{x^2 - 2xy}</math> only</p>	<p><b>*B1</b></p> <p>dep<b>*M1</b></p> <p><b>A1</b> WWW <b>AG</b> Must have intermediate line &amp;...  ...could imply "=0" on 1<sup>st</sup> line</p> <p><b>3</b></p>
<p>(ii)(a) Attempt to solve <b>only</b> <math>y^2 - 2xy = 0</math> &amp; derive <math>y = 2x</math>  Clear indication why <math>y = 0</math> is not acceptable</p>	<p><b>B1</b> <b>AG</b> Any effort at solving <math>x^2 - 2xy = 0 \rightarrow B0</math></p> <p><b>B1</b> Substituting <math>y = 2x \rightarrow B0, B0</math></p> <p><b>2</b></p>
<p>(b) Attempt to solve <math>y = 2x</math> simult with <math>x^2 y - xy^2 = 2</math>  Produce <math>-2x^3 = 2</math> or <math>y^3 = -8</math>  <math>(-1, -2)</math> or <math>x = -1, y = -2</math> <b>only</b></p>	<p><b>M1</b></p> <p><b>A1</b> AEF</p> <p><b>A1</b></p> <p><b>3</b></p>

4 (i) For (either point) + $t$ (difference between vectors) $\mathbf{r} = (3\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ or $\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}) + t(-2\mathbf{i} + \mathbf{j} + \mathbf{k}$ or $2\mathbf{i} - \mathbf{j} - \mathbf{k})$	<b>M1</b> <b>A1</b>	' $t$ ' can be ' $s$ ', ' $\lambda$ ' etc. ' $\mathbf{r}$ ' must be ' $\mathbf{r}$ ' but need not be bold Check other formats, e.g. $ta + (1-t)b$
<b>2</b>		
(ii) State/imply that their $\mathbf{r}$ and their $-2\mathbf{i} + \mathbf{j} + \mathbf{k}$ are perpendicular Consider scalar product = 0 Obtain $t = -\frac{1}{6}$ or $\frac{1}{6}$ or $-\frac{5}{6}$ or $\frac{5}{6}$ Subst their $t$ into their equation of $AB$ Obtain $\frac{1}{6}(16\mathbf{i} + 13\mathbf{j} + 19\mathbf{k})$ AEF	<b>*M1</b> <b>A1</b> <b>M1</b> <b>A1</b>	N.B. This <b>*M1</b> is dep on <b>M1</b> being earned in (i) dep <b>*M1</b> Accept decimals if clear
<b>5</b>		
(i) $(1-x)^{\frac{1}{2}} = 1 - \frac{1}{2}x - \frac{1}{8}x^2$ ignoring $x^3$ etc $(1+x)^{-\frac{1}{2}} = 1 - \frac{1}{2}x + \frac{3}{8}x^2$ ignoring $x^3$ etc Product = $1 - x + \frac{1}{2}x^2$ ignoring $x^3$ etc	<b>B2</b> <b>B2</b> <b>B1</b>	SR Allow <b>B1</b> for $1 - \frac{1}{2}x + kx^2$ , $k \neq -\frac{1}{8}$ or 0 SR Allow <b>B1</b> for $1 - \frac{1}{2}x + kx^2$ , $k \neq \frac{3}{8}$ or 0 <b>AG</b> ; with (at least) 1 intermediate step (cf $x^2$ )
<b>5</b>		
(ii) $\frac{\sqrt{5}}{9}$ or $\frac{\sqrt{5}}{3}$ seen $\frac{37}{49}$ or $1 - \frac{2}{7} + \frac{1}{2}\left(\frac{2}{7}\right)^2$ seen $\frac{\sqrt{5}}{3} \approx \frac{37}{49} \Rightarrow \sqrt{5} \approx \frac{111}{49}$	<b>B1</b> <b>B1</b> <b>B1</b>	seen seen <b>AG</b>
<b>3</b>		
(i) Produce at least 2 of the 3 relevant equations in $t$ and $s$ Solve for $t$ and $s$ $(t, s) = (4, -3)$ AEF Subst $(4, -3)$ into suitable equation(s) & show consistency	<b>M1</b> <b>M1</b> <b>*A1</b> dep <b>*A1</b>	$1 + 2t = 12 + s$ , $3t = -4s$ , $-5 + 4t = 5 - 2s$ Either into "3 <sup>rd</sup> " eqn or into all 3 coordinates. N.B. Intersection coords not asked for
<b>4</b>		
(ii) Method for finding magnitude of any vector Method for finding scalar product of any 2 vectors Using $\cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{ \mathbf{a}   \mathbf{b} }$ AEF for the correct 2 vectors 137 (136.8359) or 43.2(43.164...)	<b>*M1</b> <b>*M1</b> dep <b>*M1</b> <b>A1</b>	Expect $\sqrt{29}$ and $\sqrt{21}$ Expect $-18$ Should be $-\frac{18}{\sqrt{29}\sqrt{21}}$ 2.39 (2.388236..) or 0.753(0.75335...) rads
<b>4</b>		

7 (i)	Correct (calc) method for dealing with $\frac{1}{\sin x}$ or $(\sin x)^{-1}$	M1	
	Obtain $-\frac{\cos x}{\sin^2 x}$ or $-(\sin x)^{-2} \cos x$	A1	
	Show manipulation to $-\operatorname{cosec} x \cot x$ (or vice-versa)	A1	WWW AG with $\geq 1$ line intermed working
<b>3</b>			
(ii)	Separate variables, $\int (-)\frac{1}{\sin x \tan x} dx = \int \cot t dt$	M1	or $\int \frac{1}{\sin x \tan x} dx = \int (-)\cot t dt$
	<u>Style:</u> For the M1 to be awarded, dx and dt must appear on correct sides or there must be $\int$ sign on both sides		
	$\int -\operatorname{cosec} x \cot x dx = \operatorname{cosec} x (+c)$	A1	or $\int \operatorname{cosec} x \cot x dx = -\operatorname{cosec} x$
	$\int \cot t dt = \ln \sin t$ or $\ln  \sin t $ (+c)	B1	or $\int -\cot t dt = -\ln \sin t$ or $-\ln  \sin t $
	Subst $(t, x) = \left(\frac{1}{2}\pi, \frac{1}{6}\pi\right)$ into their equation containing 'c'	M1	and attempt to find 'c'
	$\operatorname{cosec} x = \ln \sin t + 2$ or $\ln  \sin t  + 2$	A1	WWW ISW; $\operatorname{cosec} \frac{\pi}{6}$ to be changed to 2
<b>5</b>			
8 (i)	$A(t+1) + B = 2t$ $A = 2$ $B = -2$	M1 A1 A1	<u>Beware:</u> correct values for A and/or B can be ... ... obtained from a wrong identity <u>Alt method:</u> subst suitable values into given... ...expressions
<b>3</b>			
(ii)	Attempt to connect dx and dt $dx = t dt$ s.o.i. AEF	M1 A1	But not just $dx = dt$ . As AG, look carefully.
	$x + \sqrt{2x-1} \rightarrow \frac{t^2+1}{2} + t = \frac{(t+1)^2}{2}$ s.o.i.	B1	Any wrong working invalidates
	$\int \frac{2t}{(t+1)^2} dt$	A1	AG WWW The 'dt' must be present
<b>4</b>			
(iii)	$\int \frac{1}{t+1} dt = \ln(t+1)$	B1	Or parts $u = 2t, dv = (t+1)^{-2}$ or subst $u = t+1$
	$\int \frac{1}{(t+1)^2} dt = -\frac{1}{t+1}$	B1	
	Attempt to change limits (expect 1 & 3) and use f(t)	M1	<u>or</u> re-substitute and use 1 and 5 on g(x)
	$\ln 4 - \frac{1}{2}$	A1	AEF (like terms amalgamated); if A0 A0 in (i), then final A0
<b>4</b>			

<b>9 (i)</b> $A: \theta = \frac{1}{2}\pi$ (accept $90^\circ$ ) $B: \theta = 2\pi$ (accept $360^\circ$ )	<b>B1</b> <b>B2</b> SR If B0 awarded for point B, allow B1 SR for any angle s.t. $\sin \theta = 0$
<b>3</b>	
<b>(ii)</b> $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$ $\frac{dx}{d\theta} = 2 + 2 \cos 2\theta$ $2 + 2 \cos 2\theta = 4 \cos^2 \theta$ with $\geq 1$ line intermed work $\frac{dy}{dx} = \frac{4 \cos \theta}{2 + 2 \cos 2\theta}$ s.o.i. $= \sec \theta$	<b>M1</b> or $\frac{dy}{d\theta} \cdot \frac{d\theta}{dx}$ Must be used, not just quoted <b>B1</b> <b>*B1</b> <b>A1</b> This & previous line are interchangeable dep* <b>A1</b> WWW <b>AG</b>
<b>5</b>	
<b>(iii)</b> Equating $\sec \theta$ to 2 and producing at least one value of $\theta$ $(x =) -\frac{2}{3}\pi - \frac{\sqrt{3}}{2}$ $(y =) -2\sqrt{3}$	<b>M1</b> degrees or radians <b>A1</b> 'Exact' form required <b>A1</b> 'Exact' form required <b>3</b>