4724 Core Mathematics 4

1	**(a)** $2x^2-7x-4=(2x+1)(x-4)$ or $3x^2+x-2=(3x-2)(x+1)$	**B1**	
	$\frac{2x+1}{3x-2}$ as final answer; this answer only	**B1**	Do not ISW
		$\boxed{2}$	
	(b) For correct leading term x in quotient	**B1**	<u>Identity method</u>
	For evidence of correct division process	**M1**	M1: $x^3+2x^2-6x-5=Q(x^2+4x+1)+R$
	Quotient = $x-2$	**A1**	M1: $Q=ax+b$ or $x+b, R=cx+d$ & ≥ 2 ops
			[N.B. If $Q=x+b$, this $\Rightarrow$ 1 of the 2 ops]
	Remainder = $x-3$	**A1**	A2: $a=1, b=-2, c=1, d=-3$ SR: <u>B</u>1 for two
		$\boxed{4}$	
2	Parts with correct split of $u=\ln x, \frac{dv}{dx}=x^4$	***M1**	obtaining result $f(x)+/-\int g(x)\,dx$
	$\frac{x^5}{5}\ln x-\int \frac{x^5}{5}.\frac{1}{x}(dx)$	**A1**	
	$\frac{x^5}{5}\ln x-\frac{x^5}{25}$	**A1**	
	Correct method with the limits	dep***M1**	Decimals acceptable here
	$\frac{4e^5}{25}+\frac{1}{25}$ ISW (Not '+c')	**A1**	Accept equiv fracts; like terms amalgamated
		$\boxed{5}$	
3	**(i)** $\frac{d}{dx}(x^2y)=x^2\frac{dy}{dx}+2xy$ or $\frac{d}{dx}(xy^2)=2xy\frac{dy}{dx}+y^2$	***B1**	
	Attempt to solve their differentiated equation for $\frac{dy}{dx}$	dep***M1**	
	$\frac{dy}{dx}=\frac{y^2-2xy}{x^2-2xy}$ only	**A1**	WWW **AG** Must have intermediate line &… …could imply "=0" on 1st line
		$\boxed{3}$	
	(ii)(a) Attempt to solve **only** $y^2-2xy=0$ & derive $y=2x$	**B1**	**AG** Any effort at solving $x^2-2xy=0 \rightarrow$ B0
	Clear indication why $y=0$ is not acceptable	**B1**	Substituting $y=2x \rightarrow$ B0, B0
		$\boxed{2}$	
	(b) Attempt to solve $y=2x$ simult with $x^2y-xy^2=2$	**M1**	
	Produce $-2x^3=2$ or $y^3=-8$	**A1**	AEF
	$(-1, -2)$ or $x=-1, y=-2$ **only**	**A1**	
		$\boxed{3}$	

4	(i)	For (either point) + t(difference between vectors) $\mathbf{r} = (3\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ or $\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}) + t(-2\mathbf{i} + \mathbf{j} + \mathbf{k}$ or $2\mathbf{i} - \mathbf{j} - \mathbf{k})$	**M1** **A1** $\boxed{2}$	't' can be 's', 'λ' etc. '**r**' must be '**r**' but need not be bold Check other formats, e.g. $ta + (1-t)b$
	(ii)	State/imply that their **r** and their $-2\mathbf{i} + \mathbf{j} + \mathbf{k}$ are perpendicular	***M1**	N.B.This *M1 is dep on M1 being earned in (i)
		Consider scalar product = 0	dep***M1**	
		Obtain $t = -\frac{1}{6}$ or $\frac{1}{6}$ or $-\frac{5}{6}$ or $\frac{5}{6}$	**A1**	
		Subst their t into their equation of AB	**M1**	
		Obtain $\frac{1}{6}(16\mathbf{i} + 13\mathbf{j} + 19\mathbf{k})$ AEF	**A1** $\boxed{5}$	Accept decimals if clear
5	(i)	$(1-x)^{\frac{1}{2}} = 1 - \frac{1}{2}x - \frac{1}{8}x^2$ ignoring x^3 etc	**B2**	SR Allow B1 for $1 - \frac{1}{2}x + kx^2$, $k \neq -\frac{1}{8}$ or 0
		$(1+x)^{-\frac{1}{2}} = 1 - \frac{1}{2}x + \frac{3}{8}x^2$ ignoring x^3 etc	**B2**	SR Allow B1 for $1 - \frac{1}{2}x + kx^2$, $k \neq \frac{3}{8}$ or 0
		Product = $1 - x + \frac{1}{2}x^2$ ignoring x^3 etc	**B1** $\boxed{5}$	**AG**; with (at least) 1 intermediate step (cf x^2)
	(ii)	$\sqrt{\frac{5}{9}}$ or $\frac{\sqrt{5}}{3}$ seen	**B1**	
		$\frac{37}{49}$ or $1 - \frac{2}{7} + \frac{1}{2}\left(\frac{2}{7}\right)^2$ seen	**B1**	
		$\frac{\sqrt{5}}{3} \approx \frac{37}{49} \Rightarrow \sqrt{5} \approx \frac{111}{49}$	**B1** $\boxed{3}$	**AG**
6	(i)	Produce at least 2 of the 3 relevant equations in t and s	**M1**	$1 + 2t = 12 + s$, $3t = -4s$, $-5 + 4t = 5 - 2s$
		Solve for t and s	**M1**	
		$(t, s) = (4, -3)$ AEF	***A1**	
		Subst $(4, -3)$ into suitable equation(s) & show consistency	dep***A1** $\boxed{4}$	Either into "3rd" eqn or into all 3 coordinates. N.B. Intersection coords not asked for
	(ii)	Method for finding magnitude of any vector	***M1**	Expect $\sqrt{29}$ and $\sqrt{21}$
		Method for finding scalar product of any 2 vectors	***M1**	Expect -18
		Using $\cos\theta = \frac{\mathbf{a}.\mathbf{b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}$ AEF for the correct 2 vectors	dep***M1**	Should be $-\frac{18}{\sqrt{29}\sqrt{21}}$
		137 (136.8359) or 43.2(43.164…)	**A1** $\boxed{4}$	2.39 (2.388236..) or 0.753(0.75335…) rads

7	**(i)**	Correct (calc) method for dealing with $\frac{1}{\sin x}$ or $(\sin x)^{-1}$	**M1**	
		Obtain $-\frac{\cos x}{\sin^2 x}$ or $-(\sin x)^{-2}\cos x$	**A1**	
		Show manipulation to –cosec x cot x (or vice-versa)	**A1**	WWW **AG** with ≥ 1 line intermed working
			$\boxed{3}$	
	(ii)	Separate variables, $\int(-)\frac{1}{\sin x \tan x}\,dx = \int \cot t\,dt$	**M1**	or $\int \frac{1}{\sin x \tan x}\,dx = \int(-)\cot t\,dt$
		Style: For the M1 to be awarded, dx and dt must appear on correct sides or there must be $\int$ sign on both sides		
		$\int -\operatorname{cosec} x \cot x\,dx = \operatorname{cosec} x$ (+c)	**A1**	or $\int \operatorname{cosec} x \cot x\,dx = -\operatorname{cosec} x$
		$\int \cot t\,dt = \ln \sin t$ or $\ln\|\sin t\|$ (+c)	**B1**	or $\int -\cot t\,dt = -\ln \sin t$ or $-\ln\|\sin t\|$
		Subst $(t, x) = \left(\frac{1}{2}\pi, \frac{1}{6}\pi\right)$ into their equation containing ‘c’	**M1**	and attempt to find ‘c’
		$\operatorname{cosec} x = \ln \sin t + 2$ or $\ln\|\sin t\| + 2$	**A1**	WWW ISW; cosec $\frac{\pi}{6}$ to be changed to 2
			$\boxed{5}$	
8	**(i)**	$A(t+1) + B = 2t$	**M1**	Beware: correct values for A and/or B can be ...
		$A = 2$	**A1**	… obtained from a wrong identity
		B = –2	**A1**	Alt method: subst suitable values into given… …expressions
			$\boxed{3}$	
	(ii)	Attempt to connect dx and dt	**M1**	But not just dx = dt. As **AG**, look carefully.
		dx = t dt s.o.i. AEF	**A1**	
		$x + \sqrt{2x-1} \to \frac{t^2+1}{2} + t = \frac{(t+1)^2}{2}$ s.o.i.	**B1**	Any wrong working invalidates
		$\int \frac{2t}{(t+1)^2}\,dt$	**A1**	**AG** WWW The ‘dt’ must be present
			$\boxed{4}$	
	(iii)	$\int \frac{1}{t+1}\,dt = \ln(t+1)$	**B1**	Or parts $u = 2t, dv = (t+1)^{-2}$ or subst $u = t+1$
		$\int \frac{1}{(t+1)^2}\,dt = -\frac{1}{t+1}$	**B1**	
		Attempt to change limits (expect 1 & 3) and use f(t)	**M1**	or re-substitute and use 1 and 5 on g(x)
		$\ln 4 - \frac{1}{2}$	**A1**	AEF (like terms amalgamated); if A0 A0 in (i), then final A0
			$\boxed{4}$	

9	**(i)**	$A:\theta=\frac{1}{2}\pi$ (accept 90°)	**B1**	
		$B:\theta=2\pi$ (accept 360°)	**B2**	SR If B0 awarded for point B, allow B1 SR for any angle s.t. $\sin\theta=0$
			$\boxed{3}$	
	(ii)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\frac{\mathrm{d}y}{\mathrm{d}\theta}}{\frac{\mathrm{d}x}{\mathrm{d}\theta}}$	**M1**	or $\frac{\mathrm{d}y}{\mathrm{d}\theta}.\frac{\mathrm{d}\theta}{\mathrm{d}x}$ Must be used, not just quoted
		$\frac{\mathrm{d}x}{\mathrm{d}\theta}=2+2\cos 2\theta$	**B1**	
		$2+2\cos 2\theta=4\cos^2\theta$ with $\geq$ 1 line intermed work	***B1**	
		$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{4\cos\theta}{2+2\cos 2\theta}$ s.o.i.	**A1**	This & previous line are interchangeable
		$=\sec\theta$	dep***A1**	WWW **AG**
			$\boxed{5}$	
	(iii)	Equating $\sec\theta$ to 2 and producing at least one value of θ	**M1**	degrees or radians
		$(x=)-\frac{2}{3}\pi-\frac{\sqrt{3}}{2}$	**A1**	'Exact' form required
		$(y=)-2\sqrt{3}$	**A1**	'Exact' form required
			$\boxed{3}$	