

**ADVANCED GCE**  
**MATHEMATICS**  
Core Mathematics 4

**4724**

Candidates answer on the Answer Booklet

**OCR Supplied Materials:**

- 8 page Answer Booklet
- List of Formulae (MF1)

**Other Materials Required:**

None

**Friday 5 June 2009**  
**Afternoon**

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 Find the quotient and the remainder when  $3x^4 - x^3 - 3x^2 - 14x - 8$  is divided by  $x^2 + x + 2$ . [4]

2 Use the substitution  $x = \tan \theta$  to find the exact value of

$$\int_1^{\sqrt{3}} \frac{1-x^2}{1+x^2} dx. \quad [7]$$

3 (i) Expand  $(a+x)^{-2}$  in ascending powers of  $x$  up to and including the term in  $x^2$ . [4]

(ii) When  $(1-x)(a+x)^{-2}$  is expanded, the coefficient of  $x^2$  is 0. Find the value of  $a$ . [3]

4 (i) Differentiate  $e^x(\sin 2x - 2 \cos 2x)$ , simplifying your answer. [4]

(ii) Hence find the exact value of  $\int_0^{\frac{1}{4}\pi} e^x \sin 2x dx$ . [3]

5 A curve has parametric equations

$$x = 2t + t^2, \quad y = 2t^2 + t^3.$$

(i) Express  $\frac{dy}{dx}$  in terms of  $t$  and find the gradient of the curve at the point  $(3, -9)$ . [5]

(ii) By considering  $\frac{y}{x}$ , find a cartesian equation of the curve, giving your answer in a form not involving fractions. [4]

6 The expression  $\frac{4x}{(x-5)(x-3)^2}$  is denoted by  $f(x)$ .

(i) Express  $f(x)$  in the form  $\frac{A}{x-5} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$ , where  $A$ ,  $B$  and  $C$  are constants. [4]

(ii) Hence find the exact value of  $\int_1^2 f(x) dx$ . [5]

7 (i) The vector  $\mathbf{u} = \frac{3}{13}\mathbf{i} + b\mathbf{j} + c\mathbf{k}$  is perpendicular to the vector  $4\mathbf{i} + \mathbf{k}$  and to the vector  $4\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$ . Find the values of  $b$  and  $c$ , and show that  $\mathbf{u}$  is a unit vector. [6]

(ii) Calculate, to the nearest degree, the angle between the vectors  $4\mathbf{i} + \mathbf{k}$  and  $4\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$ . [3]

8 (i) Given that  $14x^2 - 7xy + y^2 = 2$ , show that  $\frac{dy}{dx} = \frac{28x - 7y}{7x - 2y}$ . [4]

(ii) The points  $L$  and  $M$  on the curve  $14x^2 - 7xy + y^2 = 2$  each have  $x$ -coordinate 1. The tangents to the curve at  $L$  and  $M$  meet at  $N$ . Find the coordinates of  $N$ . [6]

9 A tank contains water which is heated by an electric water heater working under the action of a thermostat. The temperature of the water,  $\theta$  °C, may be modelled as follows. When the water heater is first switched on,  $\theta = 40$ . The heater causes the temperature to increase at a rate  $k_1$  °C per second, where  $k_1$  is a constant, until  $\theta = 60$ . The heater then switches off.

(i) Write down, in terms of  $k_1$ , how long it takes for the temperature to increase from 40 °C to 60 °C. [1]

The temperature of the water then immediately starts to decrease at a variable rate  $k_2(\theta - 20)$  °C per second, where  $k_2$  is a constant, until  $\theta = 40$ .

(ii) Write down a differential equation to represent the situation as the temperature is decreasing. [1]

(iii) Find the total length of time for the temperature to increase from 40 °C to 60 °C and then decrease to 40 °C. Give your answer in terms of  $k_1$  and  $k_2$ . [8]



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