

4724 Core Mathematics 4

1	<p><u>Long Division</u> For leading term $3x^2$ in quotient B1</p> <p>Suff evid of div process (ax^2, mult back, attempt sub) M1</p> <p>(Quotient) = $3x^2 - 4x - 5$ A1</p> <p>(Remainder) = $-x + 2$ A1</p> <p><u>Identity</u> $3x^4 - x^3 - 3x^2 - 14x - 8 = Q(x^2 + x + 2) + R$ *M1</p> <p>$Q = ax^2 + bx + c, R = dx + e$ & attempt ≥ 3 ops. dep*M1 If $a = 3$, this \Rightarrow 1 operation</p> <p>$a = 3, b = -4, c = -5$ A1 dep*M1; $Q = ax^2 + bx + c$</p> <p>$d = -1, e = 2$ A1</p> <p><u>Inspection</u> Use 'Identity' method; if $R = e$, check cf(x) correct before awarding 2nd M1</p>	
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2	<p><u>Indefinite Integral</u> Attempt to connect dx & $d\theta$ *M1</p> <p>Reduce to $\int 1 - \tan^2 \theta (d\theta)$ A1</p> <p>Use $\tan^2 \theta = (1, -1) + (\sec^2 \theta, -\sec^2 \theta)$ dep*M1</p> <p>Produce $\int 2 - \sec^2 \theta (d\theta)$ A1</p> <p>Correct $\sqrt{\quad}$ integration of function of type $d + e \sec^2 \theta$ $\sqrt{A1}$</p> <p>EITHER Attempt limits change (allow degrees here) M1</p> <p>OR Attempt integ, re-subst & use original ($\sqrt{3}, 1$)</p> <p>$\frac{1}{6}\pi - \sqrt{3} + 1$ isw Exact answer required A1</p>	<p>Incl $\frac{dx}{d\theta}$ or $\frac{d\theta}{dx}$; not $dx = d\theta$</p> <p>A0 if $\frac{d\theta}{dx} = \sec^2 \theta$; but allow all following</p> <p>A marks</p> <p>including $d = 0$</p> <p>(This is 'limits' aspect; the integ need not be accurate)</p>
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- 3 (i) $\left(1 + \frac{x}{a}\right)^{-2} = 1 + (-2)\frac{x}{a} + \frac{-2 \cdot -3}{2}\left(\frac{x}{a}\right)^2 + \dots$ M1 Check 3rd term; accept $\frac{x^2}{a}$
- $= 1 - \frac{2x}{a} + \dots$ or $1 + \left(-\frac{2x}{a}\right)$ B1 or $1 - 2xa^{-1}$ (Ind of M1)
- $\dots + \frac{3x^2}{a^2} + \dots$ (or $3\left(\frac{x}{a}\right)^2$ or $3x^2a^{-2}$) A1 Accept $\frac{6}{2}$ for 3
- $(a+x)^{-2} = \frac{1}{a^2} \left\{ \text{their expansion of } \left(1 + \frac{x}{a}\right)^{-2} \right\}$ mult out $\sqrt{A1}$ 4 $\frac{1}{a^2} - \frac{2x}{a^3} + \frac{3x^2}{a^4}$; accept eg a^{-2}

- (ii) Mult out $(1-x)$ (their exp) to produce all terms/cfs(x^2) M1 Ignore other terms
- Produce $\frac{3}{a^2} + \frac{2}{a} (=0)$ or $\frac{3}{a^4} + \frac{2}{a^3} (=0)$ or AEF A1 Accept x^2 if in both terms
- $a = -\frac{3}{2}$ www seen anywhere in (i) or (ii) A1 3 Disregard any ref to $a = 0$

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- 4 (i) Differentiate as a product, $u dv + v du$ M1 or as 2 separate products
- $\frac{d}{dx}(\sin 2x) = 2 \cos 2x$ or $\frac{d}{dx}(\cos 2x) = -2 \sin 2x$ B1
- $e^x(2 \cos 2x + 4 \sin 2x) + e^x(\sin 2x - 2 \cos 2x)$ A1 terms may be in diff order
- Simplify to $5e^x \sin 2x$ www A1 4 Accept $10e^x \sin x \cos x$

- (ii) Provided result (i) is of form $k e^x \sin 2x$, k const

$$\int e^x \sin 2x dx = \frac{1}{k} e^x (\sin 2x - 2 \cos 2x) \quad B1$$

$$\left[e^x (\sin 2x - 2 \cos 2x) \right]_0^{\frac{1}{4}\pi} = e^{\frac{1}{4}\pi} + 2 \quad B1$$

$$\frac{1}{5} \left(e^{\frac{1}{4}\pi} + 2 \right) \quad B1 \quad 3 \quad \text{Exact form to be seen}$$

SR Although 'Hence', award M2 for double integration by parts and solving + A1 for correct answer.

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- 5 (i) $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ aef used M1
- $$= \frac{4t + 3t^2}{2 + 2t} \quad \text{A1}$$
- Attempt to find t from one/both equations M1 or diff (ii) cartesian eqn \rightarrow M1
- State/imply $t = -3$ is only solution of both equations A1 subst(3,-9), solve for $\frac{dy}{dx} \rightarrow$ M1
- Gradient of curve = $-\frac{15}{4}$ or $\frac{-15}{4}$ or $\frac{15}{-4}$ A1 **5** grad of curve = $-\frac{15}{4} \rightarrow$ A1
- [**SR** If $t = 1$ is given as solution & not disqualified, award A0 + $\sqrt{}$ A1 for grad = $-\frac{15}{4}$ & $\frac{7}{4}$;
If $t = 1$ is given/used as only solution, award A0 + $\sqrt{}$ A1 for grad = $\frac{7}{4}$]

- (ii) $\frac{y}{x} = t$ B1
- Substitute into either parametric eqn M1
- Final answer $x^3 = 2xy + y^2$ A2 **4**
- [**SR** Any correct unsimplified form (involving fractions or common factors) \rightarrow A1]

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- 6 (i) $4x \equiv A(x-3)^2 + B(x-3)(x-5) + C(x-5)$ M1
- $A = 5$ A1 'cover-up' rule, award B1
- $B = -5$ A1
- $C = -6$ A1 **4** 'cover-up' rule, award B1
- Cands adopting other alg. manip. may be awarded M1 for a full satis method + 3 @ A1

- (ii) $\int \frac{A}{x-5} dx = A \ln(5-x)$ or $A \ln|5-x|$ or $A \ln|x-5|$ $\sqrt{}$ B1 but not $A \ln(x-5)$
- $\int \frac{B}{x-3} dx = B \ln(3-x)$ or $B \ln|3-x|$ or $B \ln|x-3|$ $\sqrt{}$ B1 but not $B \ln(x-3)$
- If candidate is awarded B0,B0, then award **SR** $\sqrt{}$ B1 for $A \ln(x-5)$ **and** $B \ln(x-3)$
- $\int \frac{C}{(x-3)^2} dx = -\frac{C}{x-3}$ $\sqrt{}$ B1
- $5 \ln \frac{3}{4} + 5 \ln 2$ aef, isw $\sqrt{}$ $A \ln \frac{3}{4} - B \ln 2$ $\sqrt{}$ B1 Allow if **SR** B1 awarded
- -3 $\sqrt{}$ $\frac{1}{2}C$ $\sqrt{}$ B1 **5**
- [Mark at earliest correct stage & isw; no ln 1] 9

- 7 (i) Attempt scalar prod $\{\mathbf{u} \cdot (4\mathbf{i} + \mathbf{k})$ or $\mathbf{u} \cdot (4\mathbf{i} + 3\mathbf{j} + 2\mathbf{k})\} = 0$ M1 where \mathbf{u} is the given vector
- Obtain $\frac{12}{13} + c = 0$ or $\frac{12}{13} + 3b + 2c = 0$ A1
- $c = -\frac{12}{13}$ A1
- $b = \frac{4}{13}$ A1 cao No ft
- Evaluate $\left(\frac{3}{13}\right)^2 + (\text{their } b)^2 + (\text{their } c)^2$ M1 Ignore non-mention of $\sqrt{\quad}$
- Obtain $\frac{9}{169} + \frac{144}{169} + \frac{16}{169} = 1$ AG A1 6 Ignore non-mention of $\sqrt{\quad}$

- (ii) Use $\cos \theta = \frac{\mathbf{x} \cdot \mathbf{y}}{|\mathbf{x}| |\mathbf{y}|}$ M1
- Correct method for finding scalar product M1
- 36° (35.837653...) Accept 0.625 (rad) A1 3 From $\frac{18}{\sqrt{17}\sqrt{29}}$
- SR If $4\mathbf{i} + \mathbf{k} = (4, 1, 0)$ in (i) & (ii), mark as scheme but allow final A1 for 31° (31.160968) or 0.544

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- 8 (i) $\frac{d}{dx}(y^2) = 2y \frac{dy}{dx}$ B1
- $\frac{d}{dx}(uv) = u \, dv + v \, du$ used on $(-7)xy$ M1
- $\frac{d}{dx}(14x^2 - 7xy + y^2) = 28x - 7x \frac{dy}{dx} - 7y + 2y \frac{dy}{dx}$ A1 (= 0)
- $2y \frac{dy}{dx} - 7x \frac{dy}{dx} = 7y - 28x \rightarrow \frac{dy}{dx} = \frac{28x - 7y}{7x - 2y}$ www AG A1 4 As AG, intermed step nec

- (ii) Subst $x = 1$ into eqn curve & solve quadratic eqn in y M1 ($y = 3$ or 4)
- Subst $x = 1$ and (one of) their y -value(s) into given $\frac{dy}{dx}$ M1 $\left(\frac{dy}{dx} = 7 \text{ or } 0\right)$
- Find eqn of tgt, with their $\frac{dy}{dx}$, going through (1, their y) *M1 using (one of) y value(s)
- Produce either $y = 7x - 4$ or $y = 4$ A1
- Solve simultaneously their two equations dep*M1 provided they have two
- Produce $x = \frac{8}{7}$ A1 6

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- 9 (i) $\frac{20}{k_1}$ (seconds) B1 1
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- (ii) $\frac{d\theta}{dt} = -k_2(\theta - 20)$ B1 1
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- (iii) Separate variables or invert each side M1 Correct eqn or very similar
 Correct int of each side (+ c) A1,A1 for each integration
 Subst $\theta = 60$ when $t = 0$ into eqn containing 'c' M1 or $\theta = 60$ when $t =$ their (i)
 c (or $-c$) = $\ln 40$ or $\frac{1}{k_2} \ln 40$ or $\frac{1}{k_2} \ln 40k_2$ A1 Check carefully their 'c'
 Subst their value of c and $\theta = 40$ back into equation M1 Use scheme on LHS
 $t = \frac{1}{k_2} \ln 2$ A1 Ignore scheme on LHS
 Total time = $\frac{1}{k_2} \ln 2 +$ their (i) (seconds) $\sqrt{A1}$ 8

SR If the negative sign is omitted in part (ii), allow all marks in (iii) with $\ln 2$ replaced by $\ln \frac{1}{2}$.

SR If definite integrals used, allow M1 for eqn where $t = 0$ and $\theta = 60$ correspond; a second M1 for eqn where $t = t$ and $\theta = 40$ correspond & M1 for correct use of limits. Final answer scores 2.

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