GCE

Mathematics

Advanced GCE **4724/01**

Core Mathematics 4

Mark Scheme for June 2010

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1	First 2 terms in expansion = $1-5x$	B1	(simp to this, now or later)
	3rd term shown as $\dfrac{-\frac{5}{3}.-\frac{8}{3}}{2}(3x)^2$	M1	$-\dfrac{8}{3}$ can be $-\dfrac{5}{3}-1$ $(3x)^2$ can be $9x^2$ or $3x^2$
	$=+\ 20x^2$	A1	
	4th term shown as $\dfrac{-\frac{5}{3}.-\frac{8}{3}.-\frac{11}{3}}{2.3}(3x)^3$	M1	$-\dfrac{11}{3}$ can be $-\dfrac{5}{3}-2$ $(3x)^3$ can be $27x^3$ or $3x^3$
	$=-\dfrac{220}{3}x^3$ ISW	A1	Accept $-\dfrac{440}{6}x^3$ ISW
	N.B. If 0, SR B2 to be awarded for $1-\frac{5}{3}x+\frac{20}{9}x^2-\frac{220}{81}x^3$. Do not mark $(1+x)^{-5/3}$ as a MR.		
		$\boxed{5}$	
2	Attempt quotient rule [Show fraction with denom $(1-\sin x)^2$ & num $+/-(1-\sin x)+/-\sin x+/-\cos x+/-\cos x$]	M1	
	Numerator = $(1-\sin x).-\sin x-\cos x.-\cos x$ { Product symbols must be clear or implied by further work }	A1	terms in any order
	Reduce correct numerator to $1-\sin x$	B1	or $-\sin x+\sin^2 x+\cos^2 x$
	Simplify to $\dfrac{1}{1-\sin x}$ ISW	A1	Accept $-\dfrac{1}{\sin x-1}$
		$\boxed{4}$	
3	$\dfrac{A}{x-1}+\dfrac{B}{(x-1)^2}+\dfrac{C}{x-2}$	M1	For correct format
	$A(x-1)(x-2)+B(x-2)+C(x-1)^2\equiv x^2$	M1	
	$A=-3$	A1	
	$B=-1$	A1	(B1 if cover-up rule used)
	$C=4$	A1	(B1 if cover-up rule used)
	[NB1: Partial fractions need not be written out; correct format + correct values sufficient. NB2: Having obtained B & C by cover-up rule, candidates may substitute into general expression & algebraically manipulate; the M1 & A1 are then available if deserved.]		
		$\boxed{5}$	

These special cases using different formats are the only other ones to be considered		Max
$\dfrac{A}{x-1}+\dfrac{Bx+C}{(x-1)^2}+\dfrac{D}{x-2}$;	M1 M1; A0 for any values of A, B & C, A1 or B1 for $D=4$	3
$\dfrac{Ax+B}{(x-1)^2}+\dfrac{C}{x-2}$;	M0 M1; A1 for $A=-3$ and $B=2$, A1 or B1 for $C=4$	3

4		Att by diff to connect dx & du or find $\frac{dx}{du}$ or $\frac{du}{dx}$ (not dx=du)	M1	no accuracy; not 'by parts'
		$dx = 2u\,du$ or $\frac{du}{dx} = \frac{1}{2}(x+2)^{-\frac{1}{2}}$ AEF	A1	
		Indefinite integral $\rightarrow \int 2(u^2-2)^2 \left(\frac{u}{u}\right)(du)$	A1	May be implied later
		{If relevant, cancel u/u and} attempt to square out	M1	
		{dep $\int kI(du)$ where $k = 2$ or $\frac{1}{2}$ or 1 and I = $(u^2-2)^2$ or $(2-u^2)^2$ or $(u^2+2)^2$ }		
		Att to change limits if working with f(u) after integration	M1	or re-subst into integral attempt and use −1 & 7
		Indef integ = $\frac{2}{5}u^5 +/- \frac{8}{3}u^3 + 8u$ or $\frac{1}{10}u^5 +/- \frac{2}{3}u^3 + 2u$	A1	or $\frac{1}{5}u^5 +/- \frac{4}{3}u^3 + 4u$
		$\frac{652}{15}$ or $43\frac{7}{15}$ ISW but no '+c'	A1	
			7	
5		$\frac{d}{dx}(xy) = x\frac{dy}{dx} + y$ s.o.i.	B1	Implied by e.g., $4x\frac{dy}{dx} + y$
		$\frac{d}{dx}(y^2) = 2y\frac{dy}{dx}$	B1	
		Diff eqn(=0 can be implied)(solve for $\frac{dy}{dx}$ and) put $\frac{dy}{dx} = 0$	M1	
		Produce only $2x + 4y = 0$ (though AEF acceptable)	*A1	without any error seen
		Eliminate x or y from curve eqn & eqn(s) just produced	M1	
		Produce either $x^2 = 36$ or $y^2 = 9$	dep* A1	Disregard other solutions
		$(\pm 6, \mp 3)$ AEF, as the only answer ISW	dep* A1	Sign aspect must be clear
			7	
6	**(i)**	State/imply scalar product of any two vectors = 0	M1	
		Scalar product of correct two vectors = 4 + 2a −6	A1	($4 + 2a - 6 = 0 \rightarrow$ M1A1)
		$a = 1$	A1 **3**	
	(ii)	**(a)** Attempt to produce at least two relevant equations	M1	e.g. $2t = 3 + 2s$ …..
		Solve two not containing 'a' for s and t	M1	
		Obtain at least one of $s = -\frac{1}{2}$, $t = 1$	A1	
		Substitute in third equation & produce $a = -2$	A1 **4**	
		(b) Method for finding magnitude of any vector	M1	possibly involving 'a'
		Using $\cos\theta = \frac{\mathbf{a.b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}$ for the pair of direction vectors	M1	possibly involving 'a'
		107, 108 (107.548) or 72, 73, 72.4, 72.5 (72.4516) c.a.o.	A1 **3**	1.87, 1.88 (1.87707) or 1.26
			10	

7	(i)	Differentiate x as a quotient, $\frac{v\,\mathrm{d}u - u\,\mathrm{d}v}{v^2}$ or $\frac{u\,\mathrm{d}v - v\,\mathrm{d}u}{v^2}$	M1	or product clearly defined	
		$\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{1}{(t+1)^2}$ or $\frac{-1}{(t+1)^2}$ or $-(t+1)^{-2}$	A1	WWW → 2	
		$\frac{\mathrm{d}y}{\mathrm{d}t} = -\frac{2}{(t+3)^2}$ or $\frac{-2}{(t+3)^2}$ or $-2(t+3)^{-2}$	B1		
		$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\frac{\mathrm{d}y}{\mathrm{d}t}}{\frac{\mathrm{d}x}{\mathrm{d}t}}$	M1	quoted/implied and used	
		$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2(t+1)^2}{(t+3)^2}$ or $\frac{2(t+3)^{-2}}{(t+1)^{-2}}$ (dep 1st 4 marks)	*A1	ignore ref $t=-1, t=-3$	
		State squares +ve or $(t+1)^2$ & $(t+3)^2$ + ve $\therefore \frac{\mathrm{d}y}{\mathrm{d}x}$ +ve	dep*A1 **6**	or $\left(\frac{t+1}{t+3}\right)^2$ + ve . Ignore ≥ 0	
	(ii)	Attempt to obtain t from either the x or y equation	M1	No accuracy required	
		$t = \frac{2-x}{x-1}$ AEF or $t = \frac{2}{y} - 3$ AEF	A1		
		Substitute in the equation not yet used in this part	M1	or equate the 2 values of t	
		Use correct meth to eliminate ('double-decker') fractions	M1		
		Obtain $2x + y = 2xy + 2$ ISW AEF	A1 **5**	but not involving fractions	**11**
8	(i)	Long division method		Identity method	
		Evidence of division process as far as 1st stage incl sub	M1	$\equiv Q(x-1) + R$	
		(Quotient =) $x - 4$	A1	$Q = x - 4$	
		(Remainder =) 2 ISW	A1 **3**	$R = 2$; N.B. might be B1	
	(ii)	**(a)** Separate variables; $\int \frac{1}{y-5}\,\mathrm{d}y = \int \frac{x^2-5x+6}{x-1}\,\mathrm{d}x$	M1	'$\int$' may be implied later	
		Change $\frac{x^2-5x+6}{x-1}$ into their (Quotient + $\frac{\text{Rem}}{x-1}$)	M1		
		$\ln(y-5) = \sqrt{}$ (integration of their previous result) (+c)ISW	√A1 **3**	f.t. if using Quot + $\frac{\text{Rem}}{x-1}$	
	(ii)	**(b)** Substitute $y = 7$, $x = 8$ into their eqn containing 'c'	M1	& attempt 'c' (-3.2 , $\ln\frac{2}{49}$)	
		Substitute $x = 6$ and their value of 'c'	M1	& attempt to find y	
		$y = 5.00$ (5.002529) Also $5 + \frac{50}{49}e^{-6}$	A2 **4**	Accept 5, 5.0,	
		Beware: any wrong working anywhere → A0 even if answer is one of the acceptable ones.			
					10

9 (i)	Attempt to multiply out $(x+\cos 2x)^2$	M1	Min of 2 correct terms
	Finding $\int 2x\cos 2x\,\mathrm{d}x$		
	Use $u=2x, \mathrm{d}v=\cos 2x$	M1	1st stage $\mathrm{f}(x) +/- \int \mathrm{g}(x)\,\mathrm{d}x$
	1st stage $x\sin 2x - \int \sin 2x\,\mathrm{d}x$	A1	
	$\therefore \int 2x\cos 2x\,\mathrm{d}x = x\sin 2x + \frac{1}{2}\cos 2x$	A1	
	Finding $\int \cos^2 2x\,\mathrm{d}x$		
	Change to $k\int +/-1+/-\cos 4x\,\mathrm{d}x$	M1	where $k=\frac{1}{2}, 2$ or 1
	Correct version $\frac{1}{2}\int 1+\cos 4x\,\mathrm{d}x$	A1	
	$\int \cos 4x\,\mathrm{d}x = \frac{1}{4}\sin 4x$	B1	seen anywhere in this part
	Result = $\frac{1}{2}x+\frac{1}{8}\sin 4x$	A1	
	(i) ans $=\frac{1}{3}x^3 + x\sin 2x + \frac{1}{2}\cos 2x + \frac{1}{2}x + \frac{1}{8}\sin 4x$ (+ c)	A1 **9**	Fully correct
(ii)	V = $\pi\int_0^{\frac{1}{2}\pi}(x+\cos 2x)^2\ (\mathrm{d}x)$	M1	
	Use limits 0 & $\frac{1}{2}\pi$ correctly on their **(i)** answer	M1	
	(i) correct value = $\frac{1}{24}\pi^3 - \frac{1}{2} + \frac{1}{4}\pi - \frac{1}{2}$	A1	
	Final answer = $\pi\left(\frac{1}{24}\pi^3 + \frac{1}{4}\pi - 1\right)$	A1 **4**	**c.a.o.** No follow-through
		$\boxed{13}$	

Alternative methods

2 If $y=\dfrac{\cos x}{1-\sin x}$ is changed into $y(1-\sin x)=\cos x$, award

M1 for clear use of the product rule (though possibly trig differentiation inaccurate)

A1 for $-y\cos x + (1-\sin x)\frac{\mathrm{d}y}{\mathrm{d}x} = -\sin x$ AEF

B1 for reducing to a fraction with $1-\sin x$ or $-\sin x + \sin^2 x + \cos^2 x$ in the numerator

A1 for correct final answer of $\dfrac{1}{1-\sin x}$ or $(1-\sin x)^{-1}$

If $y=\dfrac{\cos x}{1-\sin x}$ is changed into $y=\cos x(1-\sin x)^{-1}$, award

M1 for clear use of the product rule (though possibly trig differentiation inaccurate)

A1 for $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)= \cos^2 x(1-\sin x)^{-2} + (1-\sin x)^{-1}.-\sin x$ AEF

B1 for reducing to a fraction with $1-\sin x$ or $-\sin x+\sin^2 x+\cos^2 x$ in the numerator

A1 for correct final answer of $\frac{1}{1-\sin x}$ or $(1-\sin x)^{-1}$

6(ii)(a) If candidates use some long drawn-out method to find '*a*' instead of the direct route, allow

M1 as before, for producing the 3 equations

M1 for any satisfactory method which will/does produce 'a', however involved

A2 for $a=-2$

7(ii) Marks for obtaining this Cartesian equation are not available in part (i).

If part (ii) is done first and then part (i) is attempted using the Cartesian equation, award marks as follow:

Method 1 where candidates differentiate implicitly

M1 for attempt at implicit differentiation

A1 for $\frac{dy}{dx}=\frac{2y-2}{1-2x}$ AEF

M1 for substituting parametric values of x and y

A2 for simplifying to $\frac{2(t+1)^2}{(t+3)^2}$

A1 for finish as in original method

Method 2 where candidates manipulate the Cartesian equation to find x = or y=

M1 for attempt to re-arrange so that either $y=\mathrm{f}(x)$ or $x=\mathrm{g}(y)$

A1 for correct $y=\frac{2-2x}{1-2x}$ AEF or $x=\frac{2-y}{2-2y}$ AEF

M1 for differentiating as a quotient

A2 for obtaining $\frac{dy}{dx}=\frac{2}{(1-2x)^2}$ or $\frac{(2-2y)^2}{2}$

A1 for finish as in original method

8(ii)(b) If definite integrals are used , then

M2 for $[\ ]_y^7=[\ ]_6^8$ or equivalent or M1 for $[\ ]_7^y=[\ ]_6^8$ or equivalent

A2 for 5, 5.0, 5.00 (5.002529) with caveat as in main scheme dep M2

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