

**ADVANCED GCE
MATHEMATICS**

Core Mathematics 4

QUESTION PAPER

4724

Candidates answer on the printed answer book.

OCR supplied materials:

- Printed answer book 4724
- List of Formulae (MF1)

Other materials required:

- Scientific or graphical calculator

**Thursday 16 June 2011
Afternoon**

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets [] at the end of each question or part question on the question paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The printed answer book consists of **16** pages. The question paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this question paper for marking; it should be retained in the centre or destroyed.

1 Simplify $\frac{x^4 - 10x^2 + 9}{(x^2 - 2x - 3)(x^2 + 8x + 15)}$. [4]

2 Find the unit vector in the direction of $\begin{pmatrix} 2 \\ -3 \\ \sqrt{12} \end{pmatrix}$. [3]

3 (i) Find the quotient when $3x^3 - x^2 + 10x - 3$ is divided by $x^2 + 3$, and show that the remainder is x . [4]

(ii) Hence find the exact value of

$$\int_0^1 \frac{3x^3 - x^2 + 10x - 3}{x^2 + 3} dx. \quad [4]$$

4 Use the substitution $x = \frac{1}{3} \sin \theta$ to find the exact value of

$$\int_0^{\frac{1}{6}} \frac{1}{(1 - 9x^2)^{\frac{3}{2}}} dx. \quad [6]$$

5 The lines l_1 and l_2 have equations

$$\mathbf{r} = \begin{pmatrix} 4 \\ 6 \\ 4 \end{pmatrix} + s \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \quad \text{and} \quad \mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

respectively.

(i) Show that l_1 and l_2 are skew. [3]

(ii) Find the acute angle between l_1 and l_2 . [4]

(iii) The point A lies on l_1 and OA is perpendicular to l_1 , where O is the origin. Find the position vector of A . [3]

6 Find the coefficient of x^2 in the expansion in ascending powers of x of

$$\sqrt{\frac{1+ax}{4-x}},$$

giving your answer in terms of a . [8]

7 The gradient of a curve at the point (x, y) , where $x > -2$, is given by

$$\frac{dy}{dx} = \frac{1}{3y^2(x+2)}.$$

The points $(1, 2)$ and $(q, 1.5)$ lie on the curve. Find the value of q , giving your answer correct to 3 significant figures. [7]

8 A curve has parametric equations

$$x = \frac{1}{t+1}, \quad y = t - 1.$$

The line $y = 3x$ intersects the curve at two points.

(i) Show that the value of t at one of these points is -2 and find the value of t at the other point. [2]

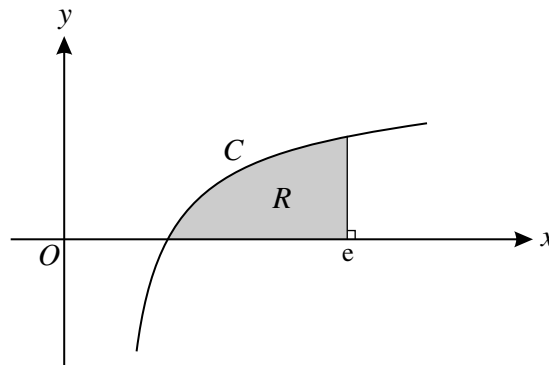
(ii) Find the equation of the normal to the curve at the point for which $t = -2$. [6]

(iii) Find the value of t at the point where this normal meets the curve again. [2]

(iv) Find a cartesian equation of the curve, giving your answer in the form $y = f(x)$. [3]

9 (i) Show that $\frac{d}{dx}(x \ln x - x) = \ln x$. [3]

(ii)



In the diagram, C is the curve $y = \ln x$. The region R is bounded by C , the x -axis and the line $x = e$.

(a) Find the exact volume of the solid of revolution formed by rotating R completely about the x -axis. [6]

(b) The region R is rotated completely about the y -axis. Explain why the volume of the solid of revolution formed is given by

$$\pi e^2 - \pi \int_0^1 e^{2y} dy,$$

and find this volume.

[4]

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