

**Mathematics**

Advanced GCE

Unit **4724**: Core Mathematics 4

**Mark Scheme for June 2011**

---

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of pupils of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, OCR Nationals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by Examiners. It does not indicate the details of the discussions which took place at an Examiners' meeting before marking commenced.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the Report on the Examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

© OCR 2011

Any enquiries about publications should be addressed to:

OCR Publications  
PO Box 5050  
Annesley  
NOTTINGHAM  
NG15 0DL

Telephone: 0870 770 6622  
Facsimile: 01223 552610  
E-mail: [publications@ocr.org.uk](mailto:publications@ocr.org.uk)

- 1** Attempt to factorise **both** numerator & denominator M1 completely or partially  
 Num = e.g.  $(x^2 - 1)(x^2 - 9)$  or  $(x^2 - 2x - 3)(x^2 + 2x - 3)$  B1 or  $(x - 3)(x + 3)(x - 1)(x + 1)$   
 Denominator = e.g.  $(x^2 - 2x - 3)(x + 5)(x + 3)$  B1 or  $(x - 3)(x + 1)(x + 5)(x + 3)$   
 $\frac{x-1}{x+5}$  or  $1 - \frac{6}{x+5}$  WWW A1 **4** ISW but not if any further 'cancellation'
- Alternative start, attempting long division  
 Expand denom as quartic & attempt to divide  $\frac{\text{numerator}}{\text{denominator}}$  M1 but not divide  $\frac{\text{denominator}}{\text{numerator}}$   
 Obtain quotient = 1 & remainder =  $-6x^3 - 6x^2 + 54x + 54$  B1  
 Final B1 A1 available as before
- 4**
- 2**  $2^2 + (-3)^2 + (\sqrt{12})^2$  soi e.g. 25 or 5 M1 Allow  $2^2 - 3^2 + \sqrt{12}^2$   
 5 A1 May be implied by 5 or 1/5 in final answer
- $\frac{1}{5} \begin{pmatrix} 2 \\ -3 \\ \sqrt{12} \end{pmatrix}$  or  $\begin{pmatrix} \frac{2}{5} \\ -\frac{3}{5} \\ \frac{\sqrt{12}}{5} \end{pmatrix}$  AEF  $\sqrt{A1}$  **3** FT their '5'. Accept  $-\frac{1}{5} \begin{pmatrix} \phantom{2} \\ \phantom{-3} \\ \phantom{\sqrt{12}} \end{pmatrix}$  or  $\frac{1}{\pm 5} \begin{pmatrix} \phantom{2} \\ \phantom{-3} \\ \phantom{\sqrt{12}} \end{pmatrix}$
- 3**
- 3** (i) The words quotient and remainder need not be explicit  
Long division For leading term  $3x$  in quotient B1  
 Suff evidence of div process (  $3x$  , mult back, attempt sub) M1  
 (Quotient) =  $3x - 1$  A1  
 (Remainder) =  $x$  **AG** A1 **4** No wrong working, partic on penult line  
Identity  $3x^3 - x^2 + 10x - 3 = Q(x^2 + 3) + R$  \*M1  
 $Q = ax + b, R = cx + d$  & attempt at least 2 operations dep\*M1 If  $a = 3$ , this  $\Rightarrow$  1 operation  
 $a = 3, b = -1$  A1  
 $c = 1, d = 0$  A1 No wrong working anywhere  
Inspection  $3x^3 - x^2 + 10x - 3 = (x^2 + 3)(3x - 1) + x$  B2 or state quotient =  $3x - 1$   
 Clear demonstration of LHS = RHS B2
- (ii) Change integrand to 'their (i) quotient' +  $\frac{x}{x^2 + 3}$  M1  
 Correct FT integration of 'their (i) quotient'  $\sqrt{A1}$   
 $\int \frac{x}{x^2 + 3} dx = \frac{1}{2} \ln(x^2 + 3)$  A1  
 Exact value of integral =  $\frac{1}{2} + \frac{1}{2} \ln 4 - \frac{1}{2} \ln 3$  AEF ISW A1 **4** Answer as decimal value (only)  $\rightarrow$  A0
- 8**

- 4 Indefinite integral Attempt to connect  $dx$  and  $d\theta$  M1 Incl  $\frac{dx}{d\theta} =, \frac{d\theta}{dx} =, dx = \dots d\theta$  ; not  $dx = d\theta$
- Denominator  $(1-9x^2)^{\frac{1}{2}}$  becomes  $\cos^3 \theta$  B1
- Reduce original integral to  $\frac{1}{3} \int \frac{1}{\cos^2 \theta} d\theta$  A1 May be implied, seen only as  $\frac{1}{3} \int \sec^2 \theta d\theta$
- Change  $\int \frac{1}{\cos^2 \theta} d\theta$  to  $\tan \theta$  B1 Ignore  $\frac{1}{3}$  at this stage
- Use appropriate limits for  $\theta$  (allow degrees) or  $x$  M1 Integration need not be accurate
- $\frac{\sqrt{3}}{9}$  AEF, exact answer required, ISW A1 **6**

**6**

- 5 (i) Attempt to set up 3 equations M1 of type  $4 + 3s = 1, 6 + 2s = t, 4 + s = -t$
- $(s, t) = (-1, 4)$  or  $(-1, -3)$  or  $(-\frac{10}{3}, -\frac{2}{3})$  \*A1 or  $s = -1$  &  $-\frac{10}{3}$  or  $t =$  two of  $(4, -3, -\frac{2}{3})$
- Show clear contradiction e.g.  $3 \neq -4, 4 \neq -3, -6 \neq 1$  dep\*A1 **3** Allow  $\checkmark$  unsimpl contradictions. No ISW.
- SC If  $s = -\frac{10}{3}$  found from 2<sup>nd</sup> & 3<sup>rd</sup> eqns and contradiction shown in 1<sup>st</sup> eqn, all 3 marks may be awarded.

- (ii) Work with  $\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$  and  $\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$  M1

Clear method for scalar product of any 2 vectors M1

Clear method for modulus of any vector M1

79.1<sup>(°)</sup> or better (79.1066..) 1.38 (rad) (1.38067..) ISW A1 **4** (From  $\frac{1}{\sqrt{14} \cdot \sqrt{2}}$ )

- (iii) Use  $\begin{pmatrix} 4 + 3s \\ 6 + 2s \\ 4 + s \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = 0$  M1

Obtain  $s = -2$  A1 from  $12 + 9s + 12 + 4s + 4 + s = 0$

A is  $\begin{pmatrix} -2 \\ 2 \\ 2 \end{pmatrix}$  or  $-2\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$  final answer B1 **3** Accept  $(-2, 2, 2)$

**10**

6  $(1+ax)^{1/2} = 1 + \frac{1}{2}ax + \dots + \frac{\frac{1}{2} \cdot \frac{-1}{2}}{2} (ax)^2$  B1, B1 N.B. third term =  $-\frac{1}{8}a^2x^2$

Change  $(4-x)^{-1/2}$  into  $k(1-\frac{x}{4})^{-1/2}$ , where  $k$  is likely to be  $\frac{1}{2}/2/4/-2$ , & work out expansion of  $(1-\frac{x}{4})^{-1/2}$

$(1-\frac{x}{4})^{-1/2} = 1 + \frac{1}{8}x + \dots + \frac{\frac{1}{2} \cdot \frac{-3}{2}}{2} (\frac{-x}{4})^2$  B1, B1 N.B. third term =  $\frac{3}{128}x^2$

OR Change  $\{4-x\}^{1/2}$  into  $l(1-\frac{x}{4})^{1/2}$ , where  $l$  is likely to be  $\frac{1}{2}/2/4/-2$ , & work out expansion of  $(1-\frac{x}{4})^{1/2}$

$(1-\frac{x}{4})^{1/2} = 1 - \frac{1}{8}x - \frac{1}{128}x^2$  B1 (for all 3 terms simplified)

$k = \frac{1}{2}$  (with possibility of M1 + A1 + A1 to follow) B1  $l = 2$  (with no further marks available)

Multiply  $(1+ax)^{1/2}$  by  $(4-x)^{-1/2}$  or  $(1-\frac{x}{4})^{-1/2}$  M1 Ignore irrelevant products

The required three terms (with/without  $x^2$ ) identified as

$-\frac{1}{16}a^2 + \frac{1}{32}a + \frac{3}{256}$  or  $\frac{-16a^2+8a+3}{256}$  AEF ISW A1+A1 8 A1 for one correct term + A1 for other two

SC B1 for  $\frac{1}{4}(1-\frac{x}{4})^{-1}$ ; B1 for  $(1-\frac{x}{4})^{-1} = 1 + \frac{x}{4} + \frac{x^2}{16}$ ; M1 for multiplying  $(1+ax)$  by their  $(4-x)^{-1}$ .

If result is  $p+qx+rx^2$ , then to find  $(p+qx+rx^2)^{1/2}$  award B1 for  $p^{1/2}(\dots)$ ,

B1 correct 1<sup>st</sup> & 2<sup>nd</sup> terms of expansion, B1 correct 3<sup>rd</sup> term; A1, A1 as before, for correct answers.

**8**

7 Attempt to sep variables in format  $\int py^2 (dy) = \int \frac{q}{x+2} (dx)$  M1 where constants  $p$  and/or  $q$  may be wrong

Either  $y^3$  &  $\ln(x+2)$  or  $\frac{1}{3}y^3$  &  $\frac{1}{3}\ln(x+2)$  A1+A1 Accept  $\frac{1}{3}\ln(3x+6)$  for  $\frac{1}{3}\ln(x+2)$  &  $|$  for ( )

If indefinite integrals are being used (most likely scenario)

Substitute  $x=1, y=2$  into an eqn containing 'const' M1

Sub  $y=1.5$  and their value of 'const' & solve for  $x$  or  $q$  M1

$x$  or  $q = -1.97$  only A2

[SC  $x$  or  $q = -1.970$  or  $-1.971$  or  $-1.9705$  or  $-1.9706$  A1] 7

If definite integrals are used (less likely scenario)

Use  $\int_{1.5}^2 \dots dy = \int_q^1 \dots dx$  where 2 corresponds with 1..... M2 & 1.5 corresp with  $q$  (at top/bottom or v.v.)

Then A2 or SC A1 as above

Use  $\int_{1.5}^2 \dots dy = \int_1^q \dots dx$  where 2 corresponds with  $q$ ..... M1 & 1.5 corresp with 1 (at top/bottom or v.v.)

Then A1 for 1.97 only

**7**

## 8 Cartesian equation may be used in parts (i) - (iii) and corresponding marks awarded

(i) Sub parametric eqns into  $y = 3x$  & produce  $t = -2$ OR sub  $t = -2$  into para eqs, obtain  $(-1, -3)$  & state  $y = 3x$ OR other similar methods producing (or verifying)  $t = -2$  B1Value of  $t$  at other point is 2B1 2  $t = \pm 2$  is sufficient for B1+B1(ii) Use (not just quote)  $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$  M1

$$= -(t+1)^2$$

A1 or  $\frac{-1}{x^2}$  or  $\frac{-(2+y)}{x}$ Attempt to use  $-\frac{1}{\frac{dy}{dx}}$  for gradient of normal M1

Gradient normal = 1 cao A1

Subst  $t = -2$  into the parametric eqns. M1

to find pt at which normal is drawn

Produce  $y = x - 2$  as equation of the normal WWW A1 6

'A' marks in (ii) are dep on prev 'A'

(iii) Substitute the parametric values into their eqn of normal M1

Produce  $t = 0$  as final answer cao

A1 2 This is dep on final A1 in (ii)

N.B. If  $y = x - 2$  is found fortuitously in (ii) (&  $\therefore$  given A0 in (ii)), you must award A0 here in (iii).(iv) Attempt to eliminate  $t$  from the parametric equations M1Produce any correct equationA1 e.g.  $x = \frac{1}{y+2}$ Produce  $y = \frac{1}{x} - 2$  or  $y = \frac{1-2x}{x}$  ISW

A1 3 Must be seen in (iv)

{N.B. Candidate producing only  $y = \frac{1}{x} - 2$  is awarded both A1 marks.}**13**

- 9 (i) Treat  $x \ln x$  as a product M1 If  $\int \ln x$ , use parts  $u = \ln x$ ,  $dv = 1$
- Obtain  $x \cdot \frac{1}{x} + \ln x$  A1  $x \ln x - \int 1 dx = x \ln x - x$
- Show  $x \cdot \frac{1}{x} + \ln x - 1 = \ln x$  WWW AG A1 3 And state given result

(ii)(a) Part (a) is mainly based on the indef integral  $\int (\ln x)^2 dx$

[A candidate stating e.g.  $\int (\ln x)^2 dx = \int 2 \ln x dx$  or  $= \int (\ln x - x)^2 dx$  is awarded 0 for (ii)(a)]

Correct use of  $\int \ln x dx = x \ln x - x$  anywhere in this part B1 Quoted from (i) or derived

Use integ by parts on  $\int (\ln x)^2 dx$  with  $u = \ln x$ ,  $dv = \ln x$  M1 or  $u = (\ln x)^2$ ,  $dv = 1$

[For 'integration by parts, candidates must get to a 1<sup>st</sup> stage with format  $f(x) + / - \int g(x) dx$ ]

1<sup>st</sup> stage =  $\ln x(x \ln x - x) - \int \frac{1}{x}(x \ln x - x) dx$  soi A1  $x(\ln x)^2 - \int x \cdot \frac{2}{x} \ln x dx$

2<sup>nd</sup> stage =  $x(\ln x)^2 - 2x \ln x + 2x$  AEF (unsimplified) A1

$\therefore$  Value of definite integral between 1 & e =  $e - 2$  cao A1 Use limits on 2<sup>nd</sup> stage & produce cao

Volume =  $\pi(e - 2)$  ISW A1 6 Answer as decimal value (only)  $\rightarrow$  A0

Alternative method when subst.  $u = \ln x$  used

Attempt to connect  $dx$  and  $du$  M1

Becomes  $\int u^2 e^u du$  A1

First stage  $u^2 e^u - \int 2u e^u du$  A1

Third stage  $(u^2 - 2u + 2)e^u$  A1

Final A1 A1 available as before

(b) Indication that reqd vol = vol cylinder - vol inner solid M1

Clear demonstration of either vol of cylinder being  $\pi e^2$   
(including reason for height =  $\ln e$ ) or rotation of  $x = e$

about the y-axis (including upper limit of  $y = \ln e$ ) A1 Could appear as  $\pi \int_0^1 e^2 dy$

$(\pi) \int x^2 dy = (\pi) \int e^{2y} dy$  B1

$\frac{\pi(e^2 + 1)}{2}$  or 13.2 or 13.18 or better B1 4 May be from graphical calculator

**13**

Possible helpful points

1. M is Method; does the candidate know what he/she should be doing? It does not ask how accurate it is..  
e.g. in Qu.4, a candidate saying  $\frac{dx}{d\theta} = -\frac{1}{3} \cos \theta$  is awarded M1.
2. When checking if decimal places are acceptable, accept both rounding & truncation.
3. In general we ISW unless otherwise stated.
4. The symbol  $\surd$  is sometimes used to indicate 'follow-through' in this scheme.

**OCR (Oxford Cambridge and RSA Examinations)**  
**1 Hills Road**  
**Cambridge**  
**CB1 2EU**

**OCR Customer Contact Centre**

**14 – 19 Qualifications (General)**

Telephone: 01223 553998

Facsimile: 01223 552627

Email: [general.qualifications@ocr.org.uk](mailto:general.qualifications@ocr.org.uk)

**[www.ocr.org.uk](http://www.ocr.org.uk)**

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored

**Oxford Cambridge and RSA Examinations**  
**is a Company Limited by Guarantee**  
**Registered in England**  
**Registered Office; 1 Hills Road, Cambridge, CB1 2EU**  
**Registered Company Number: 3484466**  
**OCR is an exempt Charity**



**OCR (Oxford Cambridge and RSA Examinations)**  
**Head office**  
**Telephone: 01223 552552**  
**Facsimile: 01223 552553**

© OCR 2011