RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced GCE

Unit **4724:** Core Mathematics 4

Mark Scheme for June 2012

Oxford Cambridge and RSA Examinations

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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Annotations

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
B2	Independent mark awarded 2
A1 dep*	Accuracy mark dependent on a previous mark, indicated by *

Subject-specific Marking Instructions for GCE Mathematics Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work
If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.
If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.
NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance
1	(i)		$x^2-3x+2=\ (x-1)(x-2)$ or $(1-x)(2-x)$ oe	B1	
			Obtain $-\frac{1}{x-2}$ or $\frac{1}{2-x}$ or $\frac{-1}{x-2}$ or $\frac{1}{-(x-2)}$ ISW	B1	Not $\frac{-1}{-(2-x)}$ Accept WW
			If Partial Fractions are used, apply normal mark scheme.	**[2]**	
1	(ii)		Attempt single fraction or 2 fractions with same relevant denom	M1	e.g. $(x-1)(x-4)[(x-3)\text{or}\,(x-3)^2]$
			Fully correct fraction(s) before any simplification	A1	
			Relevant numerator = $3x-9$ or $3x^2-18x+27$	B1	Can award if no denominator
			Final answer = $\frac{3}{(x-1)(x-4)}$ or $\frac{3}{x^2-5x+4}$ ISW	A1	
				[4]	
			S.R. If partial fractions are used on each fraction	(M1)	
			$-\frac{1}{x-1}+\frac{2}{x-3}$	(A1)	
			$\frac{2}{x-3}-\frac{1}{x-4}$	(A1)	
			$-\frac{1}{x-1}+\frac{1}{x-4}$ ISW	(A1)	
2			Write (or imply as) $\int 1.\ln(x+2)(\mathrm{d}x)$ (ln x+ ln 2 $\rightarrow$ M0)	M1	OR: t = ln(x+2) and attempt to connect dx and dt
			Correct 'by parts' 1st stage $x\ln(x+2)-\int\frac{x}{x+2}(\mathrm{d}x)$	A1	$\int te^t\,(dt)$
			Any suitable <u>starting idea</u> for integrating $\frac{x}{x+2}$	M1	Attempt by parts with u = t, $\frac{\mathrm{d}v}{\mathrm{d}t}=e^t$
			[e.g. change num to $x+2-2$ or use substitution $x+2=u$]		
			$\int\frac{x}{x+2}(\mathrm{d}x)==\ x-2\ln(x+2)$ or $x+2-2\ln(x+2)$	A1	te^t-e^t
			Overall result = $x\ln(x+2)-x+2\ln(x+2)$ [(+c) or (-2+c)] ISW	A1	
				[5]	
			SR: Correct answer with no working	(B2)	

Question			Answer	Marks	Guidance
3	(i)		**The first 5 marks are awarded for expansions of either** $(1+4x)^{-\frac{1}{2}}$ **or** $(1+4x)^{\frac{1}{2}}$		
			Expansion of $(1+4x)^{-\frac{1}{2}}$; First 2 terms $= 1-2x$	B1	Or $(1+4x)^{\frac{1}{2}} = 1+2x$.......
			3rd term = $\frac{-\frac{1}{2}.(-\frac{1}{2}-1)}{2}.16x^2$ [Accept $4x^2$ for $16x^2$]	M1	3rd term = $\frac{\frac{1}{2}.-\frac{1}{2}}{2}.16x^2$ [ditto]
			$= +\ 6x^2$	A1	$= -2x^2$
			4th term $= \frac{-\frac{1}{2}.(-\frac{1}{2}-1).(-\frac{1}{2}-2)}{2.3}.64x^3$ [Accept $4x^3$ for $64x^3$]	M1	4th tm $= \frac{\frac{1}{2}.-\frac{1}{2}.-\frac{3}{2}}{2.3}.64x^3$ [ditto]
			$= -20x^3$	A1	$= +\ 4x^3$
			$1-2x+7x^2-22x^3$; $1+ax+(b+1)x^2+(a+c)x^3$	A1 ft	ft only $(1+4x)^{-\frac{1}{2}} = 1+ax+bx^2+cx^3$ provided a, b and c attempted and at least one @ M1 obtained
				[6]	
3	(ii)		$\lvert x\rvert < \frac{1}{4}$; $-\frac{1}{4} < x < \frac{1}{4}$; $\{-\frac{1}{4} < x\ ,\ x < \frac{1}{4}\}$ no equality	B1	But not $\{-\frac{1}{4} < x$ **OR** $x < \frac{1}{4}\}$ If choice mark what appears to be the final answer.
				[1]	
4			+/– $\int e^{2y}\,(dy)$ and +/– $\int \tan x\,(dx)$ seen	M1	may be implied later
			$\int e^{2y}(dy) = \frac{1}{2}e^{2y}$	B1	
			$\int \tan x\,(dx) = \ln\lvert\sec x\rvert$ or $-\ln\lvert\cos x\rvert$	B1	Accept ln sec$x$ or –ln cos$x$
			Subst $x = 0$, $y = 0$ into their equation containing f(x), g(y) and c	M1	S.R. Using def integrals: M1 $\int_0^x \quad = \int_0^y$ followed by A2 or A0
			$c = \frac{1}{2}$ **WWW** (or poss $-\frac{1}{2}$ if c on LHS)	A1	
			$y = \frac{1}{2}\ln(1-2\ln\lvert\sec x\rvert)$ or $\frac{1}{2}\ln(1+2\ln\lvert\cos x\rvert)$ oe WWW	A1	Accept omission of modulus
				[6]	

Question			Answer	Marks	Guidance
5	(i)		Use $\cos\theta = \frac{\mathbf{a}.\mathbf{b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}$	M1	
			Obtain $\left(\cos\theta = \frac{6}{12}\right)$ $\theta = 60$ or $\frac{1}{3}\pi$ or 1.05 or better	A1	Better: 1.0471976 (rot)
				[2]	
5	(ii)		Indicate **a** – **b** is vector joining ends of **a** and **b** or equiv $\lvert\mathbf{a}-\mathbf{b}\rvert = \lvert\mathbf{a}\rvert - \lvert\mathbf{b}\rvert$,or anything similar, → M0	M1	
			Use cosine rule correctly on 3, 4 and included (i) angle	M1	Or any other correct method
			Obtain $\sqrt{13}$ or 3.61 or better (No ft from wrong θ)	A1	3.6055513 (rot)
				[3]	
6			Attempt diff to connect du and dx or find $\frac{du}{dx}$ or $\frac{dx}{du}$	M1	no accuracy, not just du = dx
			Correct e.g. $\frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$ or $dx = (2u-2)du$ AEF	*A1	
			Indefinite integral in terms of u = $\int \frac{2u-2}{u}(du)$	A1dep*	
			Provided of form $\int \frac{au+b}{u}(du)$, change to $\int a + \frac{b}{u}(du)$	M1	Or by parts
			Integrate to $au + b\ln\lvert u\rvert$ or $au + b\ln u$	A1 ft	
			Use correct variable for limits after attempt at integral of f(u)	M1	i.e. use new values of u (usually) or orig values of x (if resubst)
			Show as $8 - 2\ln 4 - 6 + 2\ln 3$ (oe) = $2 + 2\ln\frac{3}{4}$ **AG** WWW	A1	Some 'numerical' working must be shown before giving final ans
				[7]	

Question			Answer	Marks	Guidance
7			Satisfactory start method eg attempt square of $(1-\sin 3x)$	M1	Not e.g. $\frac{(1-\sin 3x)^3}{3}$.
			[N.B. The squaring process might include a term $\sin^2 9x$]		
			The next **2** marks are awarded for integrating $-2\sin 3x$		
			Obtain $\int -2\sin 3x\,dx = \frac{2}{3}\cos 3x$	*A1	
			Obtain $-\frac{2}{3}$ or $(...+0...)-(...+\frac{2}{3}...)$	A1dep*	
			The next **3** marks are awarded for integrating $\sin^2 3x$		or for integrating $\sin^2 ax$ where $a = 6$ or 9 **only**
			Use $\sin^2 3x = k(+/-1+/-\cos 6x)$	M1	$\sin^2 ax = k(+/-1+/-\cos 2ax)$
			Correct version $= \frac{1}{2}(1-\cos 6x)$	A1	Correct $= \frac{1}{2}(1-\cos 2ax)$
			$\int \cos 6x\,dx = \frac{1}{6}\sin 6x$, seen anywhere, indep	B1	or $\int \cos 2ax\,dx = \frac{1}{2a}\sin 2ax$
			Final answer $= \frac{1}{4}\pi + their - \frac{2}{3}$	A1	Check that the $\frac{1}{4}\pi$ is from $\left[\frac{3}{2}x - \frac{1}{12}\sin 6x\right]_0^{\frac{1}{6}\pi}$
				[7]	

Question			Answer	Marks	Guidance
8	(a)		$\frac{d}{dx}(xy) = x\frac{dy}{dx} + y$	B1	
			$\frac{d}{dx}(y^2) = 2y\frac{dy}{dx}$	B1	
			Substitute $(-1,-1)$ for (x, y) & attempt to solve for $\frac{dy}{dx}$	M1	or solve then substitute
			Obtain $\frac{dy}{dx} = -1$ WWW	A1	
				[4]	
8	(b)	(i)	Tangent parallel y-axis $\rightarrow \frac{dx}{dt} = 0$ or $\frac{dy}{dx} \rightarrow \infty$ or $\frac{dy}{dx} = \infty$	M1	Accept clear intention
			Obtain $t = 0$	A1	
			$(-1,0)$ with no other possibilities	A1	Accept $x = -1, y = 0$
				[3]	
8	(b)	(ii)	State or imply or use $\frac{dy}{dt} = \frac{dx}{dt}$	M1	
			Produce $3t^2 + 1 = 4t$ oe	A1	
			$t = \frac{1}{3}$ or 1	A1	
				[3]	

Question			Answer	Marks	Guidance
9	(i)		$\dfrac{A}{x+1}+\dfrac{B}{x-2}+\dfrac{C}{(x-2)^2}$	B1	i.e. correct partial fractions
			$A(x-2)^2+B(x+1)(x-2)+C(x+1)=x^2-x-11$	M1	or equivalent identity or method
			$A=-1$	A1	B1 if cover up method used
			$B=2$	A1	
			$C=-3$	A1	B1 if cover up method used
				[5]	
			<u>Special Cases</u> The problems arise when we see how candidates deal with the denominator $(x-2)^2$: $\dfrac{A}{x+1}+\dfrac{Bx+C}{(x-2)^2}$; allow B1 for PF format, M1 for associated identity, B1 for $A=-1$ (max 3) $\dfrac{A}{x+1}+\dfrac{B}{x-2}+\dfrac{Cx+D}{(x-2)^2}$; allow B1 for PF format, M1 for assoc identity, B1 for $A=-1$ (max 3) $\dfrac{A}{x+1}+\dfrac{Bx}{(x-2)^2}$; allow B0 for PF format, M1 for associated identity (max 1, even if $A=-1$) $\dfrac{A}{x+1}+\dfrac{B}{(x-2)^2}$: allow B0 for PF format, M1 for associated identity (max 1, even if $A=-1$)		
9	(ii)		No marks are to be awarded for integrating a fraction with a zero numerator. Irrespective of the format used for the Partial Fractions in part (i), award marks as follow:		
			$\int\dfrac{\lambda}{x+1}\,dx=\left(\lambda \text{ or } \dfrac{1}{\lambda}\right)\ln(x+1)$ or….	B1	$\int\dfrac{\lambda}{x-2}\,dx=\left(\lambda \text{ or } \dfrac{1}{\lambda}\right)\ln(x-2)$
			$\int\dfrac{\mu}{(x-2)^2}\,dx=-\left(\mu \text{ or } \dfrac{1}{\mu}\right).\dfrac{1}{x-2}$	B1	
			$-\dfrac{3}{2}$	B1 ft	ft $\dfrac{C}{2}$
			$.....+\ln\dfrac{16}{5}$ ISW for either term	B1 ft	ft $.....+\ln\left\{\left(\dfrac{5}{4}\right)^A.2^B\right\}$
				[4]	

Question			Answer	Marks	Guidance
10			If MR, mark according to the scheme & follow-through from candidate's data. Award M, A & B marks (where possible) & apply penalty of 1 mark (by withholding one A mark in the question). E.g. in (i), product to be 'correct' & 'not perpendicular' to be stated.		
	(i)		α. Full justification that $t=-1$. May be 'by inspection'. [No equations not satisfied by $t=-1$ to be shown] ['unusual' attempts must be carefully checked; if convinced, award the B1 e.g. displacement vector between (-3**i** + 6**k**) and (-**i** + 2**j** + 7**k**) = ±(2**i** + 2**j** + **k**)]	B1	No other t = to be mentioned
			β. Consider scalar product $\begin{pmatrix}-3\\0\\6\end{pmatrix}.\begin{pmatrix}2\\2\\1\end{pmatrix}$	M1	
			Show **– 6 + (0) + 6** = 0 and somewhere state perpendicularity oe	A1	
			[If $\cos\theta=\frac{\mathbf{a.b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}$ quoted, ignore accuracy of work involving $\lvert\mathbf{a}\rvert$ and $\lvert\mathbf{b}\rvert$]	**[3]**	
10	(ii)		Use **r** = v (-3**i** +6**k)** and ℓ_2	*M1	or $(-3\mathbf{i}+6\mathbf{k})+v(-3\mathbf{i}+6\mathbf{k})$
			Attempt to produce at least two relevant equations	M1dep*	
			Solve two equations & produce $(v,s)=\left(\frac{1}{3},-3\right)$ soi	A1	$(v,s)=\left(-\frac{2}{3},-3\right)$
			Demonstrate clearly that these satisfy third equation	B1 **[4]**	Numerical proof required
10	(iii)		Method for finding $\lvert\overrightarrow{OB}\rvert$ or $\lvert\overrightarrow{OA}\rvert$ or $\lvert\overrightarrow{AB}\rvert$	M1	Method for finding $\overrightarrow{OB}$ or $\overrightarrow{BO}$ or $\overrightarrow{AB}$ or $\overrightarrow{BA}$
			$\lvert\overrightarrow{OB}\rvert=\sqrt{5}$ or $\lvert\overrightarrow{OA}\rvert=\sqrt{45}$ oe or $\lvert\overrightarrow{BA}\rvert=\sqrt{20}$ oe	A1	$\overrightarrow{OB}=\begin{pmatrix}-1\\0\\2\end{pmatrix}$ or $\overrightarrow{BA}=\begin{pmatrix}-2\\0\\4\end{pmatrix}$
			Obtain 3:2 oe	A1 **[3]**	Answer 3:2 WW $\rightarrow$ B3

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