OCR
RECOGNISING ACHIEVEMENT

Tuesday 18 June 2013 – Morning

A2 GCE MATHEMATICS

4724/01   Core Mathematics 4

QUESTION PAPER

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

*   The Question Paper will be found in the centre of the Printed Answer Book.
*   Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
*   **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
*   Use black ink. HB pencil may be used for graphs and diagrams only.
*   Answer **all** the questions.
*   Read each question carefully. Make sure you know what you have to do before starting your answer.
*   Do **not** write in the bar codes.
*   You are permitted to use a scientific or graphical calculator in this paper.
*   Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

*   The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
*   **You are reminded of the need for clear presentation in your answers.**
*   The total number of marks for this paper is **72**.
*   The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

*   Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Turn over

1 Express $\dfrac{(x-7)(x-2)}{(x+2)(x-1)^2}$ in partial fractions. **[5]**

2 Find $\displaystyle\int x^8 \ln(3x)\,dx$. **[5]**

3 Determine whether the lines whose equations are

$$\mathbf{r} = (1+2\lambda)\mathbf{i} - \lambda\mathbf{j} + (3+5\lambda)\mathbf{k} \ \ \text{and} \ \ \mathbf{r} = (\mu-1)\mathbf{i} + (5-\mu)\mathbf{j} + (2-5\mu)\mathbf{k}$$

are parallel, intersect or are skew. **[6]**

4 The equation of a curve is $y = \cos 2x + 2\sin x$. Find $\dfrac{dy}{dx}$ and hence find the coordinates of the stationary points on the curve for $0 < x < \pi$. **[6]**

5 **(i)** Show that $\dfrac{1}{1-\tan x} - \dfrac{1}{1+\tan x} \equiv \tan 2x$. **[2]**

(ii) Hence evaluate $\displaystyle\int_{\frac{1}{12}\pi}^{\frac{1}{6}\pi}\left(\dfrac{1}{1-\tan x} - \dfrac{1}{1+\tan x}\right)dx$, giving your answer in the form $a\ln b$. **[5]**

6 Use the substitution $u = 1 + \ln x$ to find $\displaystyle\int \dfrac{\ln x}{x(1+\ln x)^2}\,dx$. **[6]**

7 Points $A\,(2, 2, 5)$, $B\,(1, -1, -4)$, $C\,(3, 3, 10)$ and $D\,(8, 6, 3)$ are the vertices of a pyramid with a triangular base.

(i) Calculate the lengths AB and AC, and the angle BAC. **[4]**

(ii) Show that $\overrightarrow{AD}$ is perpendicular to both $\overrightarrow{AB}$ and $\overrightarrow{AC}$. **[3]**

(iii) Calculate the volume of the pyramid $ABCD$. **[3]**

[The volume of the pyramid is $V = \dfrac{1}{3} \times$ base area $\times$ perpendicular height.]

8 At time t seconds, the radius of a spherical balloon is $r\,\text{cm}$. The balloon is being inflated so that the rate of increase of its radius is inversely proportional to the square root of its radius. When $t = 5$, $r = 9$ and, at this instant, the radius is increasing at $1.08\,\text{cm}\,\text{s}^{-1}$.

(i) Write down a differential equation to model this situation, and solve it to express r in terms of t. **[7]**

(ii) How much air is in the balloon initially? **[2]**

[The volume of a sphere is $V = \frac{4}{3}\pi r^3$.]

9 A curve has parametric equations $x = \dfrac{1}{t} - 1$ and $y = 2t + \dfrac{1}{t^2}$.

   (i) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t, simplifying your answer. **[3]**

   (ii) Find the coordinates of the stationary point and, by considering the gradient of the curve on either side of this point, determine its nature. **[4]**

   (iii) Find a cartesian equation of the curve. **[2]**

10   **(i)** Show that $\dfrac{x}{(1-x)^3} \approx x + 3x^2 + 6x^3$ for small values of x. **[2]**

   (ii) Use this result, together with a suitable value of x, to obtain a decimal estimate of the value of $\dfrac{100}{729}$. **[2]**

   (iii) Show that $\dfrac{x}{(1-x)^3} = -\dfrac{1}{x^2}\left(1 - \dfrac{1}{x}\right)^{-3}$. Hence find the first three terms of the binomial expansion of $\dfrac{x}{(1-x)^3}$ in powers of $\dfrac{1}{x}$. **[4]**

   (iv) Comment on the suitability of substituting the same value of x as used in part **(ii)** in the expansion in part **(iii)** to estimate the value of $\dfrac{100}{729}$. **[1]**

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE.