

Tuesday 18 June 2013 – Morning

A2 GCE MATHEMATICS

4724/01 Core Mathematics 4

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4724/01
- List of Formulae (MF1)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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1 Express $\frac{(x-7)(x-2)}{(x+2)(x-1)^2}$ in partial fractions. [5]

2 Find $\int x^8 \ln(3x) dx$. [5]

3 Determine whether the lines whose equations are

$$\mathbf{r} = (1 + 2\lambda)\mathbf{i} - \lambda\mathbf{j} + (3 + 5\lambda)\mathbf{k} \quad \text{and} \quad \mathbf{r} = (\mu - 1)\mathbf{i} + (5 - \mu)\mathbf{j} + (2 - 5\mu)\mathbf{k}$$

are parallel, intersect or are skew. [6]

4 The equation of a curve is $y = \cos 2x + 2 \sin x$. Find $\frac{dy}{dx}$ and hence find the coordinates of the stationary points on the curve for $0 < x < \pi$. [6]

5 (i) Show that $\frac{1}{1 - \tan x} - \frac{1}{1 + \tan x} \equiv \tan 2x$. [2]

(ii) Hence evaluate $\int_{\frac{1}{12}\pi}^{\frac{1}{6}\pi} \left(\frac{1}{1 - \tan x} - \frac{1}{1 + \tan x} \right) dx$, giving your answer in the form $a \ln b$. [5]

6 Use the substitution $u = 1 + \ln x$ to find $\int \frac{\ln x}{x(1 + \ln x)^2} dx$. [6]

7 Points $A(2, 2, 5)$, $B(1, -1, -4)$, $C(3, 3, 10)$ and $D(8, 6, 3)$ are the vertices of a pyramid with a triangular base.

(i) Calculate the lengths AB and AC , and the angle BAC . [4]

(ii) Show that \overrightarrow{AD} is perpendicular to both \overrightarrow{AB} and \overrightarrow{AC} . [3]

(iii) Calculate the volume of the pyramid $ABCD$. [3]

[The volume of the pyramid is $V = \frac{1}{3} \times \text{base area} \times \text{perpendicular height}$.]

8 At time t seconds, the radius of a spherical balloon is r cm. The balloon is being inflated so that the rate of increase of its radius is inversely proportional to the square root of its radius. When $t = 5$, $r = 9$ and, at this instant, the radius is increasing at 1.08 cm s^{-1} .

(i) Write down a differential equation to model this situation, and solve it to express r in terms of t . [7]

(ii) How much air is in the balloon initially? [2]

[The volume of a sphere is $V = \frac{4}{3}\pi r^3$.]

- 9 A curve has parametric equations $x = \frac{1}{t} - 1$ and $y = 2t + \frac{1}{t^2}$.
- (i) Find $\frac{dy}{dx}$ in terms of t , simplifying your answer. [3]
 - (ii) Find the coordinates of the stationary point and, by considering the gradient of the curve on either side of this point, determine its nature. [4]
 - (iii) Find a cartesian equation of the curve. [2]
- 10 (i) Show that $\frac{x}{(1-x)^3} \approx x + 3x^2 + 6x^3$ for small values of x . [2]
- (ii) Use this result, together with a suitable value of x , to obtain a decimal estimate of the value of $\frac{100}{729}$. [2]
- (iii) Show that $\frac{x}{(1-x)^3} = -\frac{1}{x^2} \left(1 - \frac{1}{x}\right)^{-3}$. Hence find the first three terms of the binomial expansion of $\frac{x}{(1-x)^3}$ in powers of $\frac{1}{x}$. [4]
- (iv) Comment on the suitability of substituting the same value of x as used in part (ii) in the expansion in part (iii) to estimate the value of $\frac{100}{729}$. [1]

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE.



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