GCE

Mathematics

Unit **4724**: Core Mathematics 4

Advanced GCE

Mark Scheme for June 2014

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

1. **Annotations and abbreviations**

Annotation in scoris	**Meaning**
BP	Blank Page – this annotation **must** be used on all blank pages within an answer booklet (structured or unstructured) and on each page of an additional object where there is no candidate response.
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

2. Subject-specific Marking Instructions for GCE Mathematics (OCR) Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1			$x(1-x^2)+(1+x)+2(1-x)$ oe	M1	condone one sign error	if M0B0, SC1 for any pair of terms correctly combined into a single fraction, may be unsimplified
			$1-x^2$ oe	B1	any correct denominator common to all three fractions	
			$\dfrac{3-x^3}{1-x^2}$ oe cao	A1	must be fully simplified; mark the final answer	eg $\dfrac{x(3-x^3)}{x(1-x^2)}$ oe may score a maximum of M1B1A0
				[3]		
2			**±** ((3 – 2)**i** + (− 3 − 8)**j** + (6 – 2)**k**) soi	B1	NB **i** – 11**j** + 4**k**	or
			their **±** (**i** – 11**j** + 4**k**). ±(5**i** + 5**j** + 8**k**) both diagonals used ; evaluation not essential	M1	if M0 SC2 for 84° (or 84.5°), or 52(.3°) or 39° or (38.5° or 43(.2°) or 46(.0°) found from scalar product or SC1 for the equivalent obtuse angle	B3 for correct use of Cosine Rule (using the midpoint of the diagonals of the parallelogram) $[\cos\theta]=\dfrac{34.5+28.5-72}{2\sqrt{34.5}\sqrt{28.5}}$ oe
			± (1 × 5 + (− 11) × 5 + 4 × 8) $=\sqrt{1^2+11^2+4^2}\times\sqrt{5^2+5^2+8^2}\cos\theta$ oe	A1	must be fully correct	
			$\theta=\cos^{-1}\dfrac{\pm18}{\sqrt{138}\times\sqrt{114}}$	A1		
			81.7 to 82°	A1 **[5]**	1.4 to 1.43 rad	B2 for 81.7 to 82° unsupported or B3 + B2 possible for Cosine Rule

Question			Answer	Marks	Guidance	
3	(i)		$1+(-\frac{1}{2})(-2x)+(-\frac{1}{2})(\frac{-3}{2})\frac{(\pm 2x)^2}{2!}[+...]$	B1 B1	first two terms third term	allow recovery from omission of brackets do not allow $2x^2$ unless fully recovered in answer
			$1+x+\frac{3}{2}x^2$ oe	B1 **[3]**		
	(ii)		use of $(x+3)\times$ their$(1+x+\frac{3}{2}x^2)$	M1		
			coefficient is 5.5 oe	A1 **[2]**	or **B2** www in either part	may be embedded (eg $5.5x^2$ alone or in expansion)
4			$\int \frac{\cos 2x}{1+\sin 2x}\,(dx)$	B1* B1*	$\cos 2x = 1 - 2\sin^2 x$ or $(1 +)\sin 2x = (1 +)\ 2\sin x\cos x$ seen numerator and denominator both correct in the integral soi	if B0B0M0A0, SC4 for $F[x] = ½\ln(1 + 2\sin x\cos x)$ or $½\ln(1 + \sin 2x)$ final mark may still be awarded
			$F[x] = k\ln(1 + \sin 2x)$ soi	M1dep*	or $k\ln(1 + u)$ or $k\ln(u)$ following their substitution www	
			$k = ½$	A1	correct k for their substitution	
			$½\ln(1 + \sin(^{\pi}/_{2})) - ½\ln(1 + 0)$ $= ½\ln 2$	A1 AG **[5]**	correct use of limits www	minimum working: $½\ln 2 - ½\ln 1$ or $½\ln(1 + 1)$ oe

Question			Answer	Marks	Guidance	
5	(i)		$1 - s = 2 + t$ $4 + 2s = 8 + 3t$ $1 + 2s = 2 + 5t$	B1	for all three equations NB third equation may appear later, or with values already substituted	or M1 for one value (of s or t) found from one pair of equations
			value of either s or t obtained from valid method	M1	eqns (i) and (ii): $s = 0.2$, $t = -1.2$	A1 for substitution of this value (of s or t) in third equation and obtaining the other parameter (ie of t or s);
			correct pair of values	A1	eqns (i) and (iii): $s = {}^{-4}/_{7}$, $t = {}^{-3}/_{7}$ eqns (ii) and (iii) $s = 4.25$, $t = 1.5$	NB (0.2, − 0.12) or $({}^{-4}/_{7}, {}^{-12}/_{7})$ or (4.25, − 5.25) if s found first and (– 2.5, –1.2) or $({}^{19}/_{14}, {}^{-3}/_{7})$ or (– 2.5, 1.5) if t found first
			eg $1 + 2\times0.2 \neq 2 + 5\times-1.2$ oe isw NB A0 for $1 + 2\times0.2 = 2 + 5\times-1.2$ unless clarified by suitable comment	A1 **[4]**	correct substitution of correct values in correct equation	**or** find same parameter from second pair of equations A1 for correct demonstration of inconsistency NB clear statement needed if two different values of same parameter found
5	(ii)		$2\mathbf{i} - 4\mathbf{j} - 4\mathbf{k} = -2(-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$ oe	B1	allow equivalent in words, but scale factors must be correct	eg direction of A is −½×direction of C
			eg line A goes through (1, 4, 1) but line C goes through (1, 15, 11), so they do not coincide so the lines are parallel eg demonstration of different y or z values on each line for (say) $x = 1$, so lines are parallel	B1 **[2]**		

Question			Answer	Marks	Guidance	
6			$3y^2\dfrac{dy}{dx}$	B1	or $2x\dfrac{dx}{dy}$	if B0B0 M0
			$2x-12\dfrac{dy}{dx}-8$	B1	$3y^2-8\dfrac{dx}{dy}-12$	SC2 for $\dfrac{dy}{dx}=$
			their $3y^2\dfrac{dy}{dx}-12\dfrac{dy}{dx}=8-2x$ soi must be two terms on each side and must follow from RHS = 0	M1	their $2x\dfrac{dx}{dy}-8\dfrac{dx}{dy}=-3y^2+12$ must be two terms on each side must follow from RHS = 0	$\dfrac{1}{3}(-x^2+8x+12y+4)^{\frac{-2}{3}}\times(-2x+8+12\dfrac{dy}{dx})$ M1 may be earned for setting correct denominator equal to 0
			$\dfrac{dy}{dx}=\dfrac{8-2x}{3y^2-12}$ oe	A1	This mark may be implied if $\dfrac{dx}{dy}=0$ is substituted and there is no evidence for an incorrect expression for $\dfrac{dx}{dy}$	
			their $3y^2-12=0$	M1*		$x\neq 4$ not required
			$y=(\pm)\ 2$	A1	A0 if $\dfrac{dy}{dx}$ incorrect	
			substitution of their positive y value in original equation	M1dep*		ignore substitution of -2
			$x=10$, $x=-2$ and no others cao	A1	A0 if $\dfrac{dy}{dx}$ incorrect	condone omission of formal statement of coordinates (10, 2) and (–2, 2)
				[8]		

Question			Answer	Marks	Guidance	
7	(i)		$\frac{dy}{dt} = -2\sin 2t + 2\cos t$ soi	B1	NB $\frac{dx}{dt} = 2\cos t$	if B0M0A0 SC3 for $\frac{dy}{dx} = 1 - x$ from correct Cartesian equation seen in part (i) or part (ii) B1 for substitution of x = 2sin$t$
			$\frac{dy}{dx} = \text{their} \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ oe	M1		
			$\frac{-2\sin 2t + 2\cos t}{2\cos t}$ soi	A1		
			$\frac{-4\sin t\cos t + 2\cos t}{2\cos t}$ or $\frac{2\cos t(-2\sin t + 1)}{2\cos t}$ and completion to 1 – 2sin$t$ www	A1	or equivalent intermediate step	
			(1, 1½)	B1 **[5]**	NB $t = \frac{\pi}{6}$	from 1 - 2sin$t$ = 0
7	(ii)		(y =) 1 – 2sin^2t + 2sin$t$	B1	may be awarded after correct substitution for x eg (y =) 1 – $^{x^2}/_4$ – sin^2t + 2sin$t$	or (y =) x + cos2t
			substitution of sin$t$ = ½x to eliminate t	M1		substitution of t = sin^{-1}($^x/_2$) to eliminate t
			y = 1 + x – ½x^2 oe isw	A1 **[3]**	or B3 www	y = x + cos2(sin^{-1}($^x/_2$)) oe isw

Question			Answer	Marks	Guidance	
7	(iii)		$-2 \le x \le 2$ **or** $x \ge -2$ (and) $x \le 2$ or $\lvert x \rvert \le 2$	B1	cao	
			sketch of negative quadratic with endpoints in 1st and 3rd quadrants	M1	RH point must be to the right of the maximum	
			positive y-intercept and one distinguishing feature isw	A1		one from: endpoints (–2, –3) and (2, 1), vertex at (1, 1½), y – intercept is (0, 1), x-intercept is $(1 -\sqrt{3}, 0)$
				[3]		
8	(i)		t^2 in quotient and $t^3 + 2t^2$ seen	B1	or $\dfrac{t(t^2-4)+4t}{(t+2)}$	or $\dfrac{(t+2)^3-6t^2-12t-8}{(t+2)}$
			$-2t$ in quotient and $-2t^2-(-2t^2-4t) = 4t$ seen	B1	$\dfrac{t(t+2)(t-2)}{(t+2)} + \dfrac{4t}{t+2}$	$\dfrac{(t+2)^3}{(t+2)} - \dfrac{6((t+2)^2-4t-4)+12t+8}{(t+2)}$ oe
			completion to obtain correct quotient and remainder identified www	B1	$t(t-2)+\dfrac{4(t+2)-8}{t+2}$	$(t+2)^2-6(t+2)+\dfrac{12t+16}{t+2}$ oe $= t^2+4t+4-6t-12+\dfrac{12(t+2)-8}{t+2}$ oe both steps needed for final B1
				[3]		
8	(i)		alternatively $\dfrac{t^3}{t+2} \equiv At^2+Bt+C+\dfrac{D}{(t+2)}$	B1	or $t^3 \equiv (At^2+Bt+C)(t+2)+D$	or B1 for $\dfrac{t^2(t+2)-2t^2}{(t+2)}$
			equate coefficients to obtain correctly $A = 1$, $0 = 2A + B$ and $B = -2$ www	B1		B1 for $t^2+\dfrac{-2t(t+2)+4t}{(t+2)}$
			$0 = 2B + C$ and $0 = 2C + D$ obtained and solved correctly www	B1		B1 for $t^2-2t+\dfrac{4(t+2)-8}{(t+2)}$
				[3]		

Question			Answer	Marks	Guidance	
8	(ii)		integration by parts with $u = \ln(t + 2)$ and $dv = 6t^2$ to obtain $f(t) \pm \int g(t)(dt\ )$	M1*	$f(t)$ must include t^3 and $g(t)$ must **not** include a logarithm	ignore spurious dx etc
			$2t^3\ln(t+2) - \int \frac{2t^3}{t+2}(dt)$ cao	A1		alternatively, following $u = t + 2$
			result from part (i) seen in integrand; must follow award of at least first M1	M1*	no integration required for this mark	$\int 2(u^2 - 6u + 12 - \frac{8}{u})du$ oe
			$F[t] = 2t^3\ln(t+2) \pm \frac{2t^3}{3} \pm 2t^2 \pm 8t \pm 16\ln(t+2)$	A1	$2t^3\ln(t+2) - \frac{2t^3}{3} + 2t^2 - 8t + 16\ln(t+2)$	$\frac{2u^3}{3} - 6u^2 + 24u - 16\ln u$ and $2t^3\ln(t+2)$
			their F[2] – F[1]	M1dep*	at least one of their terms correctly integrated	NB limits following substitution are $u = 4$ and $u = 3$
			$-6\frac{2}{3} - 18\ln3 + 32\ln4$ oe cao	A1 **[6]**		
9			$\frac{A}{1+2x} + \frac{B}{1-x} + \frac{C}{(1-x)^2}$ may be seen in later work	B1	or $\frac{A}{1+2x} + \frac{Bx+C}{(1-x)^2}$ may be seen later in later work	if B0M0, SC1 for $\frac{1}{1+2x}$ seen
			$2 + x^2 \equiv A(1-x)^2 + B(1+2x)(1-x) + C(1+2x)$	M1	or $A(1 - x)^2 + (Bx + C)(1 + 2x)$	allow only sign errors , not algebraic errors
			$A = 1$, $B = 0$ and $C = 1$ www	A1A1A1		
			$\int \left(\frac{1}{1+2x} + \frac{1}{(1-x)^2}\right) dx =$ $a\ln(1 + 2x) + b(1 - x)^{-1}$	M1*	a and b are non-zero constants	ignore extra terms
			$F(x) = ½\ln(1 + 2x) + (1 - x)^{-1}$	A1		
			their $\frac{1}{2}\ln(\frac{3}{2}) + \frac{4}{3} - (\frac{1}{2}\ln1 + 1)$	M1dep*		

Question		Answer	Marks	Guidance	
		$\frac{1}{2}\ln(\frac{3}{2})+\frac{4}{3}-0-1$	A1 **[9]**	and completion to given result www	NB $\frac{1}{2}\ln(\frac{3}{2})+\frac{1}{3}$
10	**(i)**	$\frac{dV}{dt}=\pm 0.01$	B1		
		by similar triangles, $\frac{h}{4.5}=\frac{r}{3}$	B1	may be implied by $r=\frac{2h}{3}$ oe	
		$\frac{dV}{dh}=\frac{4}{9}\pi h^2$ oe	B1		
		$\frac{dh}{dt}=\pm 0.01\times$ their $\frac{dh}{dV}$ oe	M1	use of Chain rule	may follow from incorrect differentiation: expressions must be a function of either r or h or both
		$-0.01=(\frac{4}{9}\pi h^2)\times\frac{dh}{dt}$	A1 **[5]**	completion to given result www	$h^2\frac{dh}{dt}=\frac{-0.09}{4\pi}=\frac{-9}{400\pi}$
10	**(ii)**	$\int h^2 dh=\int\frac{-9}{400\pi}dt$ oe soi	M1	separation of variables	if no subsequent work, integral signs needed, but allow omission of dh or dt, but must be correctly placed if present;
		$\frac{h^3}{3}=\frac{-9}{400\pi}t(+c)$	A1		
		substitution of t = 0 and h = 4.5 in their expression following integration	M1	expression must include c and powers must be correct on each side	
		$h=\sqrt[3]{\frac{729}{8}-\frac{27t}{400\pi}}$ oe isw	A1 **[4]**	allow – 0.0215 or – 0.02148591…r.o.t to 4 sf or more and similarly 91.125	91.125 = $^{729}/_{8}$
10	**(iii)**	set h = 0 and solve to obtain positive t	**M1**	or $(t=)\frac{1}{3}\pi\times 3^2\times 4.5\div 0.01$ (= 1350π)	NB 1350π = 4241.150082…
		71 minutes cao	**A1** **[2]**		

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

Education and Learning
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

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