OCR Maths Core 1

Mark Scheme Pack

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OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS **4721**

Core Mathematics 1

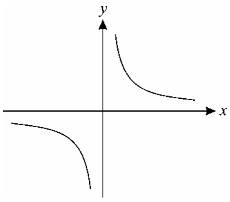
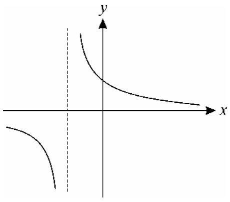
MARK SCHEME

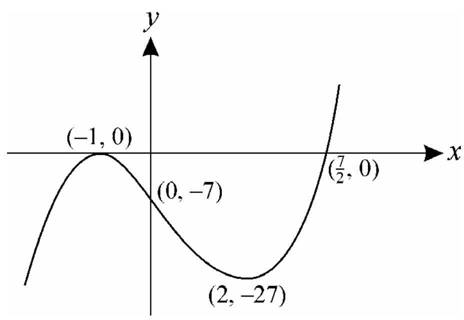
Specimen Paper

MAXIMUM MARK	72

This mark scheme consists of 4 printed pages.

1	**(i)** $\frac{1}{16}$	B1	**1**	For correct value (fraction or exact decimal)
	(ii) 8	B1	**1**	For correct value 8 only
	(iii) 6	M1 A1	**2** **[4]**	For $1^3+2^3+3^3=36$ seen or implied For correct value 6 only
2	**(i)** $x^2-8x+3=(x-4)^2-13$ i.e. $a=-4, b=-13$	B1 M1 A1	 **3**	For $(x-4)^2$ seen, or statement $a=-4$ For use of (implied) relation $a^2+b=3$ For correct value of b stated or implied
	(ii) Minimum point is $(4, -13)$	B1√ B1√	 **2** **[5]**	For x-coordinate equal to their $(-a)$ For y-coordinate equal to their b
3	**(i)** Discriminant is k^2-4k	M1 A1	 **2**	For attempted use of the discriminant For correct expression (in any form)
	(ii) For no real roots, $k^2-4k<0$ Hence $k(k-4)<0$ So $0<k<4$	M1 M1 A1 A1	 **4** **[6]**	For stating their $\Delta<0$ For factorising attempt (or other soln method) For both correct critical values 0 and 4 seen For correct pair of inequalities
4	**(i)** $\frac{dy}{dx}=12x^2$	M1 A1	 **2**	For clear attempt at nx^{n-1} For completely correct answer
	(ii) $y=x^4+2x^2$ Hence $\frac{dy}{dx}=4x^3+4x$	B1 M1 A1√	 **3**	For correct expansion For correct differentiation of at least one term For correct differentiation of their 2 terms
	(iii) $\frac{dy}{dx}=\frac{1}{2}x^{-\frac{1}{2}}$	M1 A1	 **2** **[7]**	For clear differentiation attempt of $x^{\frac{1}{2}}$ For correct answer, in any form
5	**(i)** $x^2-3x+2=3x-7 \Rightarrow x^2-6x+9=0$ Hence $(x-3)^2=0$ So $x=3$ and $y=2$	M1 A1 M1 A1 A1	 **5**	For equating two expressions for y For correct 3-term quadratic in x For factorising, or other solution method For correct value of x For correct value of y
	(ii) The line $y=3x-7$ is the tangent to the curve $y=x^2-3x+2$ at the point $(3, 2)$	B1 B1	 **2**	For stating tangency For identifying $x=3, y=2$ as coordinates
	(iii) Gradient of tangent is 3 Hence gradient of normal is $-\frac{1}{3}$ Equation of normal is $y-2=-\frac{1}{3}(x-3)$ i.e. $x+3y-9=0$	B1 B1√ M1 A1	 **4** **[11]**	For stating correct gradient of given line For stating corresponding perpendicular grad For appropriate use of straight line equation For correct equation in required form

6 **(i)**		B1 B1 **2**	For correct 1st quadrant branch For both branches correct and nothing else
(ii)	Translation of 2 units in the negative x-direction	B1 B1 B1	For translation parallel to the x-axis For correct magnitude For correct direction
		B1√ B1 **5**	For correct sketch of new curve For some indication of location, e.g. $\frac{1}{2}$ at y-intersection or -2 at asymptote
(iii)	Derivative is $-x^{-2}$	M1 A1 **2**	For correct power -2 in answer For correct coefficient -1
(iv)	Gradient of $y=\dfrac{1}{x}$ at $x=2$ is required This is -2^{-2}, which is $-\frac{1}{4}$	B1 M1 A1 **3**	For correctly using the translation For substituting $x=2$ in their **(iii)** For correct answer
		12	
7 **(i)**	$AB^2=(10-2)^2+(3-9)^2=100$ Hence the radius is 5 Mid-point of AB is $\left(\dfrac{2+10}{2},\dfrac{9+3}{2}\right)$ Hence centre is $(6, 6)$	M1 A1 M1 A1 **4**	For correct calculation method for AB^2 For correct value for radius For correct calculation method for mid-point For both coordinates correct
(ii)	Equation is $(x-6)^2+(y-6)^2=5^2$ This is $x^2-12x+36+y^2-12y+36=25$ i.e. $x^2+y^2-12x-12y+47=0$, as required	M1 A1 A1 **3**	For using correct basic form of circle equn For expanding at least one bracket correctly For showing given answer correctly
(iii)	Gradient of AB is $\dfrac{3-9}{10-2}=-\dfrac{3}{4}$ Hence perpendicular gradient is $\dfrac{4}{3}$ Equation of tangent is $y-3=\frac{4}{3}(x-10)$ Hence C is the point $(\frac{31}{4}, 0)$	M1 A1 A1√ M1 M1 A1 **6**	For finding the gradient of AB For correct value $-\frac{3}{4}$ or equivalent For relevant perpendicular gradient For using their perp grad and B correctly For substituting $y=0$ in their tangent eqn For correct value $x=\frac{31}{4}$
		13	

8	**(i)** $\frac{dy}{dx} = 6x^2 - 6x - 12$	M1		For differentiation with at least 1 term OK
		A1		For completely correct derivative
	Hence $x^2 - x - 2 = 0$	M1		For equating their derivative to zero
	$(x-2)(x+1) = 0 \Rightarrow x = 2$ or -1	M1		For factorising or other solution method
		A1		For both correct x-coordinates
	Stationary points are $(2, -27)$ and $(-1, 0)$	A1	6	For both correct y-coordinates
	(ii) $\frac{d^2y}{dx^2} = 12x - 6 = \begin{cases} +18 \text{ when } x = 2 \\ -18 \text{ when } x = -1 \end{cases}$	M1		For attempt at second derivative and at least one relevant evaluation
	Hence $(2, -27)$ is a min and $(-1, 0)$ is a max	A1		For either one correctly identified
		A1	3	For both correctly identified (Alternative methods, e.g. based on gradients either side, are equally acceptable)
	(iii) RHS $= (x^2 + 2x + 1)(2x - 7)$	M1		For squaring correctly and attempting
	$= 2x^3 - 7x^2 + 4x^2 - 14x + 2x - 7$			complete expansion process
	$= 2x^3 - 3x^2 - 12x - 7$, as required	A1	2	For obtaining given answer correctly
	(iv) 	B1		For correct cubic shape
		B1		For maximum point lying on x-axis
		B1	3	For $x = \frac{7}{2}$ and $y = -7$ at intersections
			14	

1 (i)	$11^{-2} = \frac{1}{121}$	B1 1	$\frac{1}{121}$ ($\frac{1}{11^2}$ = B0)
(ii)	$100^{\frac{3}{2}} = 1000$	M1 A1 2	Square rooting or cubing soi 1000
(iii)	$\sqrt{50} + \frac{6}{\sqrt{3}}$	B1	$5\sqrt{2}$ (allow $\pm$)
	$= 5\sqrt{2} + \frac{6\sqrt{3}}{3}$	M1	Attempt to rationalise $\frac{6}{\sqrt{3}}$
	$= 5\sqrt{2} + 2\sqrt{3}$	A1 3	cao
		<u>6</u>	
2	*q*=2	B1	(allow embedded values)
	r=3	B1	
		M1	$qr^2 + 10 = p$ or other correct method
	p=28	A1 $\sqrt{}$ 4	
		<u>4</u>	
3(i)	$y = 5\sqrt{2x}$	M1	$\sqrt{2x}$ or $\sqrt{\frac{x}{2}}$ seen
		A1 2	$y = 5\sqrt{2x}$
(ii)	Translation $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$	B1	Translation
		B1 2	$\begin{pmatrix} 0 \\ -3 \end{pmatrix}$ o.e.
		<u>4</u>	

4	Either $y = 2x + 1$ or $y = \frac{x^2 + 11}{3}$	M1	Substitute for x/y or attempt to get an equation in 1 variable only
	$x^2 - 6x + 8 = 0$	A1	Obtain correct 3 term quadratic
	$(x-2)(x-4) = 0$	M1	Correct method to solve 3 term quadratic
	$x = 2 \quad x = 4$	A1	<u>or</u> one correct pair of values B1
	$y = 5 \quad y = 9$	A1	second correct pair of values B1 c.a.o
	OR $x = \frac{y-1}{2}$ $\frac{(y-1)^2}{4} - 3y + 11 = 0$ $y^2 - 14y + 45 = 0$ $(y-5)(y-9) = 0$ $y = 5 \quad y = 9$ $x = 2 \quad x = 4$		<u>SR</u> If solution by graphical methods: setting out to draw a parabola <u>and</u> a line M1 both correct A1 reading off of coordinates at intersection point(s) M1 one correct pair A1 second correct pair A1 OR No working shown: one correct pair B1 second correct pair B1 full justification that these are the only solutions B3
		<u>5</u>	

5 (i)		B1	Correct curve in +ve quadrant
		B1 2	in –ve quadrant
(ii)		M1	Positive cubic with clearly seen max and min points
		A1	(-1,0) (0,0) (1,0) Any one point stated or marked on sketch
	(-1,0) (0,0) (1,0)	A1 3	Curve passes through all 3 points and no extras stated or marked on sketch
(iii)		B1	Graph only in bottom right hand quadrant
		B1 2	Correct graph, passing through origin
		7	

6 (i)	$49-4\times-2\times3=73$	M1	Uses b^2-4ac
	2 real roots	A1	73
		B1√ 3	2 real roots (ft from their value)
(ii)	$(p+1)^2-64=0$ or $2[(x+\frac{p+1}{4})^2-\frac{(p+1)^2}{16}+4]=0$	M1	Attempts b^2-4ac = 0 (involving p) or attempts to complete square (involving p)
		A1	$(p+1)^2-64=0$ aef
	p= -9,7	B1	*p*= -9
		B1 4	p= 7
		7	

7 (i)	$\frac{dy}{dx} = 2x^3 - 3$	B1 B1 2	1 term correct Completely correct (+c is an error, but only penalise once)
(ii)	$y = 2x^3 + 2x^2 + 3x + 3$ $\frac{dy}{dx} = 6x^2 + 4x + 3$	M1 A1 A1 A1 4	Attempt to expand brackets $2x^3 + 2x^2 + 3x + 3$ 2 terms correct Completely correct SR Recognisable attempt at product rule M1 one part correct A1 second part correct A1 final simplified answer A1
(iii)	$y = x^{\frac{1}{5}}$ $\frac{dy}{dx} = \frac{1}{5}x^{-\frac{4}{5}}$	B1 B1 B1 3 9	$x^{\frac{1}{5}}$ soi $\frac{1}{5}x^c$ $kx^{-\frac{4}{5}}$
8(i)	$2[10 + x + x] > 64$	B1 1	$20 + 4x > 64$ o.e.
(ii)	$x(x+10) < 299$ $x^2 + 10x - 299 < 0$ $(x-13)(x+23) < 0$	B1 B1 2	$x(x+10) < 299$ Correctly shows $(x-13)(x+23) < 0$ **AG** SR Complete proof worked backward B2
(iii)	$x > 11$ $(x-13)(x+23) < 0$ $-23 < x < 13$ $\therefore 11 < x < 13$	B1√ M2 A1 B1 5 8	$x > 11$ ft from their (i) Correct method to solve $(x-13)(x+23) < 0$ eg graph $-23 < x < 13$ seen in this form or as number line SR if seen with no working B1

9(i)	$\frac{dy}{dx} = 4x$	B1	$4x$
	At x=3 , $\frac{dy}{dx} = 12$	B1 2	12
(ii)	Gradient of tangent = - 8	M1	$\frac{dy}{dx} = -8$
	$4x = -8$ $x = -2$	A1	x=-2
	$y = 8$	A1 3	y=8
(iii)	Gradient = 6	B1 1	Gradient = or approaches 6
(iv)	$\frac{dy}{dx} = 2kx$	M1	$\frac{dy}{dx} = 2kx$
	$x = 1$ $\frac{dy}{dx} = 2k$	M1	$\frac{dy}{dx} = 2k$
	$k = 3$	A1 $\sqrt{}$ 3	$k = 3$ CWO
		<u>9</u>	

10(i)	Gradient DE = $-\frac{1}{2}$	B1 1	$-\frac{1}{2}$ (any working seen must be correct)
(ii)	$y-3=-\frac{1}{2}(x-2)$	M1	Correct equation for straight line, any gradient, passing through F
		A1	$y-3=-\frac{1}{2}(x-2)$ aef
	$x+2y-8=0$	A1 3	$x+2y-8=0$ (this form but can have fractional coefficients e.g. ½x + y – 4 = 0
(iii)	Gradient EF = $\frac{4}{2}$ =2	B1	Correct supporting working must be seen
	$-\frac{1}{2}\times 2=-1$	B1 2	Attempt to show that product of their gradients = - 1 o.e.
(iv)	DF= $\sqrt{4^2+3^2}=5$	M1	$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ used
		A1 2	5
(v)	DF is a diameter as angle DEF is a right angle.	B1	Justification that DF is a diameter
	Mid-point of DF <u>or</u> centre of circle is $(0,1\frac{1}{2})$	B1	Mid-point of DF <u>or</u> centre of circle is $(0,1\frac{1}{2})$
	Radius = 2.5	B1	Radius = 2.5
	$x^2+(y-\left(\frac{3}{2}\right))^2=\left(\frac{5}{2}\right)^2$	B1√	$x^2+(y-\left(\frac{3}{2}\right))^2=\left(\frac{5}{2}\right)^2$
	$x^2+y^2-3y+\frac{9}{4}=\frac{25}{4}$		
	$x^2+y^2-3y-4=0$	B1 5	$x^2+y^2-3y-4=0$ obtained correctly with at least one line of intermediate working. <u>SR</u> For working that only shows $x^2+y^2-3y-4=0$ is equation for a circle with centre $(0,1\frac{1}{2})$ B1 radius 2.5 B1
		<u>13</u>	

Mark Scheme 4721
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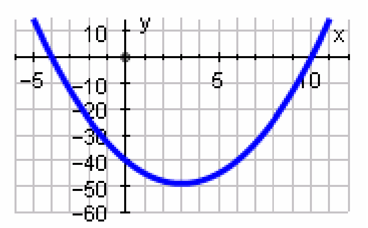
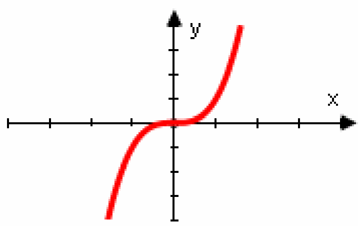
1	$x^2 - 6x - 40 \geq 0$ $(x+4)(x-10) \geq 0$  $x \leq -4, \quad x \geq 10$	M1 A1 M1 A1 4 **4**	Correct method to find roots -4, 10 Correct method to solve quadratic inequality e.g. +ve quadratic graph $x \leq -4, \quad x \geq 10$ (not wrapped, not strict inequalities, no 'and')
2(i) (ii)	EITHER $3(x^2+4x)+7$ $3(x+2)^2 - 12 + 7$ $3(x+2)^2 - 5$ OR $3(x^2+2ax+a^2)+b$ $3x^2 + 6ax + 3a^2 + b$ $6a = 12$ $a = 2$ $3a^2 + b = 7$ $b = -5$ $x = -2$	M1 A1 M1 A1 4 B1 ft 1 **5**	$a = \frac{12}{6 \text{ or } 2}$ a = 2 $7 - a^2$ or $7 - 3a^2$ or $\frac{7}{3} - a^2$ (their a) b = –5 $x = -2$
3 (i) (ii) (iii)	 Reflection in *x*-axis or reflection in *y*-axis $y = (x-p)^3$	B1 1 B1 B1 2 M1 A1 2 **5**	Correct sketch showing point of inflection at origin Reflection In *x*-axis or *y*=0 or *y*-axis or *x*=0 $y = (x \pm p)^3$ $y = (x-p)^3$

4	$k = x^3$ $k^2 + 26k - 27 = 0$ $k = -27, 1$ $x = -3, 1$	*M1 A1 A1 DM1 A1 5 **5**	Attempt a substitution to obtain a quadratic $k^2 + 26k - 27 = 0$ –27, 1 Attempt cube root x = –3, 1 (no extras) **(SR:** x = 1 seen www **B1** x = -3 seen www **B1)**
5 (a) (b) (c)	$2x^{\frac{2}{3}} \times 3x^{-1}$ $= 6x^{\frac{-1}{3}}$ $2^{40} \times 4^{30}$ $= 2^{40} \times 2^{60}$ $= 2^{100}$ $\frac{26\left(4+\sqrt{3}\right)}{\left(4-\sqrt{3}\right)\left(4+\sqrt{3}\right)}$ $= 8 + 2\sqrt{3}$	M1 A1 2 M1 A1 2 M1 A1 A1 3 **7**	Adds indices $6x^{\frac{-1}{3}}$ 2^{60} or 4^{20} 2^{100} Multiply top and bottom by $\left(4+\sqrt{3}\right)$ or $\left(-4-\sqrt{3}\right)$ $\left(4-\sqrt{3}\right)\left(4+\sqrt{3}\right) = 13$ $8 + 2\sqrt{3}$
6 (i) (ii) (iii)	$\left(x^2 + 2x + 1\right)\left(3x - 4\right)$ $= 3x^3 + 2x^2 - 5x - 4$ $9x^2 + 4x - 5$ $18x + 4$	M1 A1 A1 3 B1 ft B1 ft 2 M1 A1 ft 2 **7**	Expand 2 brackets to give an expression of the form $ax^2 + bx + c$ ($a \neq 0$, $b \neq 0$, $c \neq 0$) and attempt to multiply by third bracket $3x^3 + 2x^2 - 5x - 4$ 3 correct simplified terms Completely correct $9x^2 + 4x - 5$ 1 term correct Completely correct (3 terms) Attempt to differentiate their (ii) $18x + 4$ (2 terms) **(SR** (ii) $3ax^2 + 2bx + c$ **B1** (iii) $6ax + 2b$ **B1)**

7 (i)	$b^2 - 4ac$ (a) 36 – 9 x 4 = 0 (b) 100 – 48 = 52 (c) 4 – 20 = -16	M1 A1 A1 3	Uses $b^2 - 4ac$ 1 correct 3 correct **SR** All 3 values correct but √ used **B1**
(ii)	(a) Fig 3 (b) Fig 2 (c) Fig 5 (a) 1 root, touches *x*-axis once, line of symmetry *x*= -3 or root *x* =-3 (b) 2 roots, meets *x*-axis twice, line of symmetry *x*=5 (c) No real roots, does not meet *x*-axis	B1 B1 B1 B1 4 **7**	1 correct matching 3 correct matchings 1 correct comment relating roots to touching/crossing *x*-axis or about line of symmetry or vertex o.e. for one graph 2 further correct comments about roots, line of symmetry o.e. for the other 2 graphs
8 (i)	Circle, centre (0, 0), radius 5	B1 B1 2	Circle centre (0, 0) Radius 5
(ii)	$y = 5 - 2x$ $x^2 + (5-2x)^2 = 25$ $5x^2 - 20x = 0$ OR $x = \frac{5-y}{2}$ $\frac{(5-y)^2}{4} + y^2 = 25$ $y^2 - 2y - 15 = 0$ $x = 0, 4$ $y = 5, -3$	M1 *M1 DM1 M1 A1 A1 6 **8**	Attempt to solve equations simultaneously Substitute for *x*/*y* or correct attempt at elimination of one variable (NOT for 2 linear equations) Obtain quadratic ax^2 + bx + c = 0 (a ≠ 0, b ≠ 0) Correct method to solve quadratic $x = 0, 4$ or $y = 5, -3$ $y = 5, -3$ or $x = 0, 4$ **SR** one correct pair **www** **B1** <u>SR</u> If solution by graphical methods: Drawing circle, centre (0,0) radius 5 B1 Drawing line B1 Looking for intersection M1 (0,5) correct A1 (4, –3) correct A2

9 (i)	$y=\frac{4}{3}x+\frac{5}{3}$ gradient $=\frac{4}{3}$	B1 1	$\frac{4}{3}$ or 1.33 or better
(ii)	gradient of $\perp^r=-\frac{3}{4}$	B1 ft	$-\frac{3}{4}$ seen or implied
	$y-2=-\frac{3}{4}(x-1)$	M1	Attempts equation of straight line through (1, 2) with any gradient
	$4y+3x=11$	A1	$y-2=-\frac{3}{4}(x-1)$
		A1 4	$3x+4y-11=0$ (not aef)
(iii)	$P\left(-\frac{5}{4},0\right)$	B1	$\left(-\frac{5}{4},0\right)$ seen or implied
	$Q\left(0,\frac{11}{4}\right)$	B1 ft	$\left(0,\frac{11}{4}\right)$ seen or implied (from a straight line equation in (ii))
	$\left(-\frac{5}{8},\frac{11}{8}\right)$	B1 ft 3	$\left(-\frac{5}{8},\frac{11}{8}\right)$ aef
(iv)	$\sqrt{\left(\frac{5}{4}\right)^2+\left(\frac{11}{4}\right)^2}$	M1	Correct method to find line length using Pythagoras' theorem
	$\frac{\sqrt{146}}{4}$	A1	$\sqrt{\left(\frac{5}{4}\right)^2+\left(\frac{11}{4}\right)^2}$
		A1 3	$\frac{\sqrt{146}}{4}$
		11	

10 (i)	$\frac{dy}{dx} = x^2 - 9$	B1 B1 2	$x^2 - 9$ 1 term correct Both terms correct
(ii)	$x^2 - 9 = 0$ $x = 3, -3$ $y = -18, 18$	*M1 A1 A1 3	uses $\frac{dy}{dx} = 0$ $x = 3, -3$ $y = -18, 18$ (1 correct pair A1 A0)
(iii)	$\frac{d^2y}{dx^2} = 2x$ $x = 3 \quad \frac{d^2y}{dx^2} = 6$ $x = -3 \quad \frac{d^2y}{dx^2} = -6$	DM1 A1 A1 3	Looks at sign of $\frac{d^2y}{dx^2}$ or other correct method $x = 3$ minimum $x =$ -3 maximum (N.B. If no method shown but min and max correctly stated, award all 3 marks unless earlier incorrect working)
(iv)	gradient of $24x + 3y + 2 = 0$ is -8 $x^2 - 9 = -8$ $x = \pm 1$ For line $x = 1,\ y = -8\frac{2}{3}$ $x = -1,\ y = 7\frac{1}{3}$ For curve $x = 1,\ y = -8\frac{2}{3}$ $x = -1,\ y = 8\frac{2}{3}$ $\therefore\ p = 1,\ q = -8\frac{2}{3}$	B1 M1 M1 M1 A1 5 **13**	Gradient = – 8 $x^2 - 9 = -8$ one of their *x* values substituted in both line <u>and</u> curve second *x* value substituted in both line and curve **<u>or</u>** justification that first point is the correct one $p = 1,\ q = -8\frac{2}{3}$ seen <u>Alternative methods:</u> <u>Either:</u> Solve equations for curve and line simultaneously to get one solution (either *x* = 1 or *x* = -2) M1 Gradient of line = -8 B1 Substitution of one *x* value into their gradient formula and check for -8 M1 Substitution of other *x* value into gradient formula and check for -8 or justification as above M1 Correct *q* value A1 <u>Or:</u> Solve equations for curve and line simultaneously to get one solution M1 Factorise to $(x-1)^2(x+2)$ B1 State that a double root implies a tangent at x = 1 M2 Correct value for y A1

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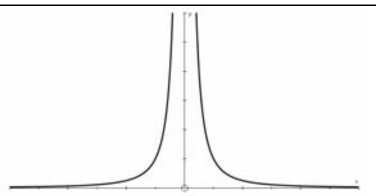
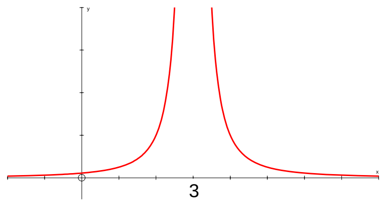
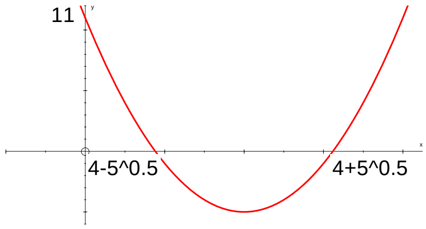
1	(i)	$x^{\frac{1}{3}} = 2$ $x = 8$	B1	**1**	(allow embedded values throughout question 1) 8
	(ii)	$10^t = 1$ $t = 0$	B1	**1**	0
	(iii)	$\left(y^{-2}\right)^2 = \frac{1}{81}$ $y^{-4} = \frac{1}{81}$ $y = \pm 3$	B1 B1	**2**	$y = 3$ $y = -3$
2	(i)	$(3x+1)^2 - 2(2x-3)^2$ $= (9x^2+6x+1) - 2(4x^2-12x+9)$ $= x^2+30x-17$	M1 A1 A1	**3**	Square to get at least one 3 or 4 term quadratic $9x^2+6x+1$ or $4x^2-12x+9$ soi $x^2+30x-17$
	(ii)	$2x^3+6x^3+4x^3 = 12x^3$ 12	B1 B1	**2**	2 of $2x^3$, $6x^3$, $4x^3$ soi **N.B. www for these terms,** must be positive 12 or $12x^3$
3	(i)	$\frac{dy}{dx} = 15x^4 - \frac{1}{2}x^{-\frac{1}{2}}$	B1 B1 B1	**3**	$15x^4$ $kx^{-\frac{1}{2}}$ $cx^4 - \frac{1}{2}x^{-\frac{1}{2}}$ only
	(ii)	$\frac{d^2y}{dx^2} = 60x^3 + \frac{1}{4}x^{-\frac{3}{2}}$	M1 A1	**2**	Attempt to differentiate their 2 term $\frac{dy}{dx}$ and get one correctly differentiated term $60x^3 + \frac{1}{4}x^{-\frac{3}{2}}$
4	(i)		B1 B1	**2**	Correct curve in one quadrant Completely correct
	(ii)		M1 A1√	**2**	Translate (i) horizontally Translates all of their (i) $\begin{pmatrix}3\\0\end{pmatrix}$ 3 must be labelled or stated
	(iii)	(One-way) stretch, sf 2, parallel to the *y*-axis	B1 B1 B1	**3**	Stretch (Scale) factor 2 Parallel to *y*-axis o.e. **SR** Stretch B1 Sf $\sqrt{2}$ parallel to *x*-axis B2

5	(i)	$x^2+3x=\left(x+\frac{3}{2}\right)^2-\frac{9}{4}$	B1 B1	2	$a=\frac{3}{2}$ $b=-\frac{9}{4}$ o.e.
	(ii)	$y^2-4y-\frac{11}{4}=(y-2)^2-\frac{27}{4}$	B1 B1	2	$p=-2$ $q=-\frac{27}{4}$ o.e.
	(iii)	Centre $\left(-\frac{3}{2},2\right)$	B1√	1	$\left(-\frac{3}{2},2\right)$ N.B. If question is restarted in this part, ft from part (iii) working only
	(iv)	Radius $=\sqrt{\frac{27}{4}+\frac{9}{4}}$ $=\sqrt{9}$ $=3$	M1 A1	2	$\sqrt{-their'b'-their'q'}$ or use $\sqrt{(f^2+g^2-c)}$ 3 (±3 scores A0)
6	(i)	$y=x^3-3x^2+4$ $\frac{dy}{dx}=3x^2-6x$ $3x^2-6x=0$ $3x(x-2)=0$ $x=0$ $x=2$ $y=4$ $y=0$	B1 B1 M1 M1 A1 A1√	6	$3x^2-6x$ 1 term correct Completely correct $\frac{dy}{dx}=0$ Correct method to solve quadratic x = 0, 2 y = 4, 0 **SR** one correct (x,y) pair **www** **B1**
	(ii)	$\frac{d^2y}{dx^2}=6x-6$ $x=0$ $y''=-6$ $-$ve max $x=2$ $y''=6$ $+$ve min	M1 B1 B1	3	Correct method to find nature of stationary points (can be a sketch) x = 0 max x = 2 min (N.B. If no method shown but both min and max correctly stated, award all 3 marks)
	(iii)	Increasing $x<0$ $x>2$	M1 A1	2	Any inequality (or inequalities) involving both their x values from part (i) Allow $x\leq 0$ $x\geq 2$

7	(i)	$x = \frac{8 \pm \sqrt{64-44}}{2}$	M1		Correct use of formula
		$= \frac{8 \pm \sqrt{20}}{2}$	A1		$\frac{8 \pm \sqrt{20}}{2}$ aef
			B1		$\sqrt{20} = 2\sqrt{5}$ soi
		$= 4 \pm \sqrt{5}$	A1	**4**	$4 \pm \sqrt{5}$
					Alternative method $(x-4)^2 - 16 + 11 = 0$ M1 $(x-4)^2 = 5$ A1 $x = 4 + \sqrt{5}$ A1 or $4 - \sqrt{5}$ A1
	(ii)		B1		+ve parabola
			B1√		Root(s) in correct places
			B1	**3**	Completely correct curve with roots and (0, 11) labelled or referenced
	(iii)	$y = x^2 = \left(4 \pm \sqrt{5}\right)^2$	M1 M1		$y = x^2$ soi Attempt to square at least one answer from part (i)
		$= 16 + 5 \pm 8\sqrt{5}$	A1√		Correct evaluation of $(a + b\sqrt{c})^2$ $(a,b,c \neq 0)$
		$= 21 \pm 8\sqrt{5}$	A1	**4**	$21 \pm 8\sqrt{5}$

8	(i)	$y = x^2 - 5x + 15$ $y = 5x - 10$ $x^2 - 5x + 15 = 5x - 10$ $x^2 - 10x + 25 = 0$	M1 A1	 **2**	Attempt to eliminate y $x^2 - 10x + 25 = 0$ **AG** Obtained with no wrong working seen
	(ii)	$b^2 - 4ac = 100 - 100$ $= 0$	B1	**1**	0 Do not allow $\sqrt{(b^2 - 4ac)}$
	(iii)	Line is a tangent to the curve	B1√	**1**	Tangent or 'touches' N.B. Strict ft from their discriminant
	(iv)	$x^2 - 10x + 25 = 0$ $(x-5)^2 = 0$ $x = 5$ $y = 15$	M1 A1 A1	 **3**	Correct method to solve 3 term quadratic $x = 5$ $y = 15$
	(v)	Gradient of tangent = 5 Gradient of normal = $-\frac{1}{5}$ $y - 15 = -\frac{1}{5}(x - 5)$ $x + 5y = 80$	B1 B1√ M1 A1	 **4**	Gradient of tangent = 5 Gradient of normal = $-\frac{1}{5}$ Correct equation of straight line, any gradient, passing through (5, 15) $x + 5y = 80$

9	(i)	Length AC = $\sqrt{(8-5)^2+(2-1)^2}$ $=\sqrt{3^2+1^2}$ $=\sqrt{10}$	M1 A1		Uses $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $\sqrt{10}$ (± $\sqrt{10}$ scores A0)
		Length AB = $\sqrt{(p-5)^2+(7-1)^2}$ $=\sqrt{(p-5)^2+36}$	A1		$\sqrt{(p-5)^2+(7-1)^2}$
		$\sqrt{(p-5)^2+36}=2\sqrt{10}$ $p^2-10p+25+36=40$ $p^2-10p+21=0$ $(p-7)(p-3)=0$ $p=7,3$	M1 M1 A1 A1	7	AB = 2AC (with algebraic expression) used Obtains 3 term quadratic = 0 suitable for solving <u>or</u> $(p-5)^2=4$ $p = 7$ $p = 3$ **SR <u>If no working seen</u>, and one correct value found, award B2 in place of the final 4 marks in part (i)**
	(ii)	$7 = 3x - 14$ $x = 7$ (5, 1) (7, 7) Mid-point (6, 4)	M1 A1 M1 A1√	4	Correct method to find x $x = 7$ Use $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$ (6, 4) or correct midpoint for their AB <u>Alternative method</u> y coordinate of midpoint = 4 M1 A1 sub 4 into equation of line M1 obtains $x = 6$ A1

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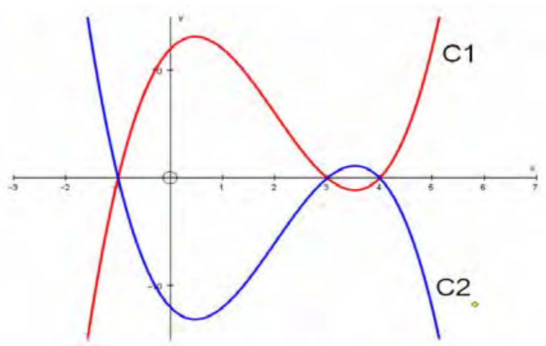
1	(i)	$\frac{21-3}{4-1}=\frac{18}{3}=6$	M1 A1	 2	Uses $\frac{y_2-y_1}{x_2-x_1}$ 6 (not left as $\frac{18}{3}$)
	(ii)	$\frac{dy}{dx}=2x+1$ $2\times3+1=7$	B1 B1	 2	
2	(i)	$27^{-\frac{2}{3}}=\frac{1}{27^{\frac{2}{3}}}=\frac{1}{9}$	M1 A1	 2	$\frac{1}{27^{\frac{2}{3}}}$ or $27^{\frac{2}{3}}=9$ or 3^{-2} soi $\frac{1}{9}$
	(ii)	$5\sqrt{5}=5^{\frac{3}{2}}$	B1	1	
	(iii)	$\frac{1-\sqrt{5}}{3+\sqrt{5}}=\frac{(1-\sqrt{5})(3-\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})}$ $=\frac{8-4\sqrt{5}}{4}$ $=2-\sqrt{5}$	M1 B1 A1	 3	Multiply numerator and denominator by conjugate $(\sqrt{5})^2=5$ soi $2-\sqrt{5}$
3	(i)	$2x^2+12x+13=2(x^2+6x)+13$ $=2[(x+3)^2-9]+13$ $=2(x+3)^2-5$	B1 B1 M1 A1	 4	*a* = 2 *b* = 3 $13-2b^2$ or 13 – b^2 or $\frac{13}{2}$ - b^2 (their *b*) *c*= –5
	(ii)	$2(x+3)^2-5=0$ $(x+3)^2=\frac{5}{2}$ $x=-3\pm\sqrt{\frac{5}{2}}$	M1 A1 A1	 3	Uses correct quadratic formula or completing square method $x=\frac{-12\pm\sqrt{40}}{4}$ or $(x+3)^2=\frac{5}{2}$ $x=-3\pm\sqrt{\frac{5}{2}}$ or $-3\pm\frac{1}{2}\sqrt{10}$

4	(i)	$(x-4)(x-3)(x+1)$ $\equiv (x^2-7x+12)(x+1)$ $\equiv x^3+x^2-7x^2-7x+12x+12$ $\equiv x^3-6x^2+5x+12$	B1 M1 A1	 3	$x^2-7x+12$ or x^2-2x-3 or x^2-3x-4 seen Attempt to multiply a quadratic by a linear factor or attempt to list an 8 term expansion of all 3 brackets $x^3-6x^2+5x+12$ **(AG)** obtained (no wrong working seen)
	(ii) (iii)		B1 B1 B1 M1 A1√	 3 2	+ve cubic with 3 roots (not 3 line segments) (0, 12) labelled or indicated on *y*-axis (– 1, 0), (3,0), (4, 0) labelled or indicated on *x*-axis Reflect *their* (ii) in either *x*- or *y*-axis Reflect *their* (ii) in *x*-axis
5	(i)	$1 < 4x-9 < 5$ $10 < 4x < 14$ $2.5 < x < 3.5$	M1 A1 A1	 3	2 equations or inequalities both dealing with all 3 terms 2.5 and 3.5 seen oe 2.5 < *x* < 3.5 (or '*x* > 2.5 <u>and</u> *x* < 3.5')
	(ii)	$y^2 \geq 4y+5$ $y^2-4y-5 \geq 0$ $(y-5)(y+1) \geq 0$ $y \leq -1, y \geq 5$	B1 M1 A1 M1 A1	 5	$y^2-4y-5=0$ soi Correct method to solve quadratic -1, 5 (**SR** If **both** values obtained from trial and improvement, award **B3**) Correct method to solve inequality $y \leq -1, y \geq 5$

6	(i)	$x^4 - 10x^2 + 25 = 0$ Let $y = x^2$ $y^2 - 10y + 25 = 0$ $(y-5)^2 = 0$ $y = 5$ $x^2 = 5$ $x = \pm\sqrt{5}$	*M1 dep*M1 A1 A1	 4	Use a substitution to obtain a quadratic or $(x^2 - 5)(x^2 - 5) = 0$ Correct method to solve a quadratic 5 (not $x = 5$ with no subsequent working) $x = \pm\sqrt{5}$
	(ii)	$y = \frac{2x^5}{5} - \frac{20x^3}{3} + 50x + 3$ $\frac{dy}{dx} = 2x^4 - 20x^2 + 50$	B1 B1	 2	$2x^4$ or $- 20x^2$ oe seen $2x^4 - 20x^2 + 50$ (integers required)
	(iii)	$2x^4 - 20x^2 + 50 = 0$ $x^4 - 10x^2 + 25 = 0$ which has 2 roots	M1 A1	 2	*their* $\frac{dy}{dx} = 0$ seen (or implied by correct answer) 2 stationary points **www in any part**
7	(i)	$y = x^2 - 5x + 4$ $y = x - 1$ $x^2 - 5x + 4 = x - 1$ $x^2 - 6x + 5 = 0$ $(x-1)(x-5) = 0$ $x = 1 \quad x = 5$ $y = 0 \quad y = 4$	M1 M1 A1 A1	 4	Substitute to find an equation in *x* (or *y*) Correct method to solve quadratic *x* = 1, 5 *y* = 0, 4 (**N.B.** This final A1 may be awarded in part (ii) if y coordinates only seen in part (ii)) **SR** one correct (*x*,y) pair **www** **B1**
	(ii)	2 points of intersection	B1	1	
	(iii)	EITHER $x^2 - 5x + 4 = x + c$ has 1 solution $x^2 - 6x + (4 - c) = 0$ $b^2 - 4ac = 0$ $36 - 4(4 - c) = 0$ $c = -5$ OR $\frac{dy}{dx} = 1 = 2x - 5$ $x = 3 \quad y = -2$ $-2 = 3 + c$ $c = -5$	M1 M1 A1 A1 M1 A1 A1 A1	 4 4	$x^2 - 5x + 4 = x + c$ has 1 soln seen or implied Discriminant = 0 or $(x - a)^2 = 0$ soi 36 – 4(4 – *c*) = 0 or 9 = 4 – *c* *c* = – 5 Algebraic expression for gradient of curve = non-zero gradient of line used 2*x* – 5 = 1 *x* = 3 *c* = –5 **SR** *c* = -5 without any working **B1**

8	(i)	Height of box = $\frac{8}{x^2}$	*B1		Area of 1 vertical face $= \frac{8}{x^2} \times x$ $= \frac{8}{x}$
		4 vertical faces $= 4 \times \frac{8}{x}$ $= \frac{32}{x}$	*B1		
		Total surface area = $x^2 + x^2 + \frac{32}{x}$	B1 dep on both **		Correct final expression
		$A = 2x^2 + \frac{32}{x}$		3	
	(ii)	$\frac{dA}{dx} = 4x - \frac{32}{x^2}$	B1 B1 B1	3	$4x$ kx^{-2} $-32x^{-2}$
	(iii)	$4x - \frac{32}{x^2} = 0$	M1		$\frac{dA}{dx} = 0$ soi
		$4x^3 = 32$ $x = 2$	A1		$x = 2$
			M1 A1	4	Check for minimum Correctly justified **SR** If $x = 2$ stated **www** but with no evidence of differentiated expression(s) having been used in part (iii) **B1**

9	(i)	$\left(\frac{4+10}{2}, \frac{-2+6}{2}\right)$	M1		Uses $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$
		(7, 2)	A1	2	(7, 2) (integers required)
	(ii)	$\sqrt{(7-4)^2+(2--2)^2}$	M1		Uses $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
		$=\sqrt{3^2+4^2}$ $=5$	A1	2	5
	(iii)	$(x-7)^2+(y-2)^2=25$	B1√		$(x-7)^2$ and $(y-2)^2$ used (*their* centre)
			B1√		$r^2 = 25$ used (*their* r^2)
			B1	3	$(x-7)^2+(y-2)^2=25$ cao <u>Expanded form:</u> -14x and -4y used B1√ $r = \sqrt{g^2+f^2-c}$ used B1√ $x^2+y^2-14x-4y+28=0$ B1 cao <u>By using ends of diameter:</u> (x - 4)(x - 10) + (y + 2)(y – 6) = 0 Both x brackets correct B1 Both y brackets correct B1 Final equation fully correct B1
	(iv)	Gradient of *AB* = $\frac{6--2}{10-4}=\frac{4}{3}$	B1		oe
		Gradient of tangent = $-\frac{3}{4}$	B1√		
			M1		Correct equation of straight line through *A*, any non-zero gradient
		$y--2=-\frac{3}{4}(x-4)$	A1		
		$3x+4y=4$	A1	5	*a* ,*b*, *c* need not be integers

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1	$\frac{5}{2-\sqrt{3}}$ x $\frac{2+\sqrt{3}}{2+\sqrt{3}}$ $= \frac{5(2+\sqrt{3})}{4-3}$ $= 10+5\sqrt{3}$	M1 A1 A1 3 **3**	Multiply top and bottom by $\pm(2+\sqrt{3})$ $(2+\sqrt{3})(2-\sqrt{3})=1$ (may be implied) $10+5\sqrt{3}$
2(i) (ii)	1 $\frac{1}{2}\times 2^4$ $= 8$	B1 1 M1 M1 A1 3 **4**	 $2^{-1}=\frac{1}{2}$ **<u>or</u>** $32^{\frac{1}{5}}=2$ **<u>or</u>** $2^5=32$ soi $32^{\frac{4}{5}}=2^4$ or 16 seen or implied 8
3(i) (ii)	$3x-15\le 24$ $3x\le 39$ $x\le 13$ **or** $x-5\le 8$ M1 $x\le 13$ A1 $5x^2>80$ $x^2>16$ $x>4$ or $x<-4$	M1 A1 2 M1 B1 A1 3 **5**	Attempt to simplify expression by multiplying out brackets $x\le 13$ Attempt to simplify expression by dividing through by 3 Attempt to rearrange inequality or equation to combine the constant terms $x>4$ fully correct, not wrapped, not 'and' **SR** B1 for $x\ge 4$, $x\le -4$

4	Let $y = x^{\frac{1}{3}}$ $y^2 + 3y - 10 = 0$ $(y-2)(y+5) = 0$ $y = 2, y = -5$ $x = 2^3, x = (-5)^3$ $x = 8, x = -125$	*M1 DM1 A1 DM1 A1 ft 5 **5**	Attempt a substitution to obtain a quadratic or factorise with $\sqrt[3]{x}$ in each bracket Correct attempt to solve quadratic Both values correct Attempt cube Both answers correctly followed through **SR** B2 $x = 8$ from T & I
5 (i)		M1 A1 2	Reflection in either axis Correct reflection in x axis
(ii)	(1, 3)	B1 B1 2	Correct x coordinate Correct y coordinate **SR** B1 for (3, 1)
(iii)	Translation 2 units in negative x direction	B1 B1 2 **6**	
6 (i)	$2(x^2 - 12x + 40)$ $= 2\left[(x-6)^2 - 36 + 40\right]$ $= 2\left[(x-6)^2 + 4\right]$ $= 2(x-6)^2 + 8$	B1 B1 M1 A1 4	$a = 2$ $b = 6$ $80 - 2b^2$ or $40 - b^2$ or $80 - b^2$ or $40 - 2b^2$ (their b) $c = 8$
(ii)	$x = 6$	B1 ft 1	
(iii)	$y = 8$	B1 ft 1 **6**	

7(i)	$\frac{dy}{dx}=5$	B1 1	
(ii)	$y=2x^{-2}$	B1	x^{-2} soi
	$\frac{dy}{dx}=-4x^{-3}$	B1	$-4x^{c}$
		B1 3	kx^{-3}
(iii)	$y=10x^2-14x+5x-7$	M1	Expand the brackets to give an expression of form ax^2+bx+c $(a\neq 0, b\neq 0, c\neq 0)$
	$y=10x^2-9x-7$	A1	Completely correct (allow 2 x-terms)
	$\frac{dy}{dx}=20x-9$	B1 ft B1 ft 4	1 term correctly differentiated Completely correct (2 terms)
		8	
8 (i)	$\frac{dy}{dx}=9-6x-3x^2$	*M1	Attempt to differentiate y or $-y$ (at least one correct term)
		A1	3 correct terms
	At stationary points, $9-6x-3x^2=0$	M1	Use of $\frac{dy}{dx}=0$ (for y or $-y$)
	$3(3+x)(1-x)=0$	DM1	Correct method to solve 3 term quadratic
	$x=-3$ or $x=1$	A1	$x=-3,\ 1$
	$y=0,\ 32$	A1ft 6	y = 0, 32 (1 correct pair www A1 A0)
(ii)	$\frac{d^2y}{dx^2}=-6x-6$	M1	Looks at sign of $\frac{d^2y}{dx^2}$, derived correctly from $k\frac{dy}{dx}$, or other correct method
	When $x=-3, \frac{d^2y}{dx^2}>0$	A1	$x=-3$ minimum
	When $x=1, \frac{d^2y}{dx^2}<0$	A1 3	$x=1$ maximum
(iii)	$-3<x<1$	M1	Uses the x values of both turning points in inequality/inequalities
		A1 2	Correct inequality or inequalities. Allow $\leq$
		11	

9 (i)	Gradient = 4 $y - 7 = 4(x - 2)$ $y = 4x - 1$	B1 M1 A1 3	Gradient of 4 soi Attempts equation of straight line through (2, 7) with any gradient
(ii)	$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ $= \sqrt{(2 - {}^-1)^2 + (7 - {}^-2)^2}$ $= \sqrt{3^2 + 9^2}$ $= \sqrt{90}$ $= 3\sqrt{10}$	M1 A1 A1 3	Use of correct formula for d or d^2 (3 values correctly substituted) $\sqrt{3^2 + 9^2}$ Correct simplified surd
(iii)	Gradient of AB = 3 Gradient of perpendicular line = $-\frac{1}{3}$ Midpoint of AB = $\left(\frac{1}{2}, \frac{5}{2}\right)$ $y - \frac{5}{2} = -\frac{1}{3}\left(x - \frac{1}{2}\right)$ $x + 3y - 8 = 0$	B1 B1 ft B1 M1 A1 A1 6 **12**	 **SR** Allow B1 for $-\frac{1}{4}$ Attempts equation of straight line through their midpoint with any non-zero gradient y - $\frac{5}{2}$ = $\frac{-1}{3}$ (x - $\frac{1}{2}$) x + 3y – 8 = 0

10 (i)	Centre (-1, 2) $(x+1)^2 - 1 + (y-2)^2 - 4 - 8 = 0$ $(x+1)^2 + (y-2)^2 = 13$ Radius $\sqrt{13}$	B1 M1 A1 3	Correct centre Attempt at completing the square Correct radius <u>Alternative method:</u> Centre ($-g$, $-f$) is (-1, 2) B1 $g^2 + f^2 - c$ M1 Radius = $\sqrt{13}$ A1
(ii)	$(2)^2 + (k-2)^2 = 13$ $(k-2)^2 = 9$ $k - 2 = \pm 3$ $k = -1$	M1 M1 A1 3	Attempt to substitute x = -3 into circle equation Correct method to solve quadratic $k = -1$ (negative value chosen)
(iii)	EITHER $y = 6 - x$ $(x+1)^2 + (6 - x - 2)^2 = 13$ $(x+1)^2 + (4 - x)^2 = 13$ $x^2 + 2x + 1 + 16 - 8x + x^2 = 13$ $2x^2 - 6x + 4 = 0$ $2(x-1)(x-2) = 0$ $x = 1 , 2$ $\therefore y = 5 , 4$ OR $x = 6 - y$ $(6 - y + 1)^2 + (y-2)^2 = 13$ $(7 - y)^2 + (y-2)^2 = 13$ $49 - 14y + y^2 + y^2 - 4y + 4 = 13$ $2y^2 - 18y + 40 = 0$ $2(y-4)(y-5) = 0$ $y = 4 , 5$ $\therefore x = 2 , 1$	M1 M1 A1 M1 A1 A1 6 **12**	Attempt to solve equations simultaneously Substitute into their circle equation for x/y or attempt to get an equation in 1 variable only Obtain correct 3 term quadratic Correct method to solve quadratic of form $ax^2 + bx + c = 0$ ($b \neq 0$) Both x values correct Both y values correct <u>or</u> one correct pair of values **www** B1 second correct pair of values B1 **SR** **<u>T & I</u>** M1 A1 One correct x (or y) value A1 Correct associated coordinate

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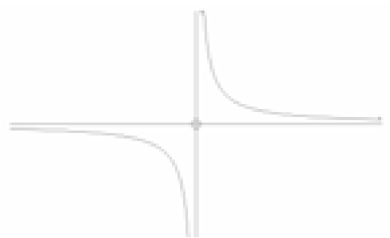
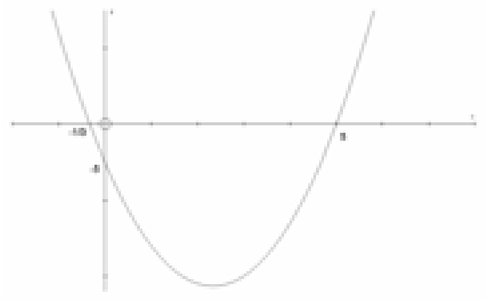
1	$(4x^2 + 20x + 25) - (x^2 - 6x + 9)$ $= 3x^2 + 26x + 16$	M1 A1 A1 3	Square one bracket to give an expression of the form $ax^2 + bx + c$ $(a \neq 0, b \neq 0, c \neq 0)$ One squared bracket fully correct All 3 terms of final answer correct
	Alternative method using difference of two squares: $(2x + 5 + (x - 3))(2x + 5 - (x - 3))$ $= (3x + 2)(x + 8)$ $= 3x^2 + 26x + 16$	**3**	M1 2 brackets with same terms but different signs A1 One bracket correctly simplified A1 All 3 terms of final answer correct
2 (a)(i)		B1 B1 2	Excellent curve for $\frac{1}{x}$ in either quadrant Excellent curve for $\frac{1}{x}$ in other quadrant **SR B1** Reasonably correct curves in 1^{st} and 3^{rd} quadrants
(ii)		B1 1	Correct graph, minimum point at origin, symmetrical
(b)	Stretch Scale factor 8 in y direction **or** scale factor ½ in x direction	B1 B1 2 **5**	
3 (i)	$3\sqrt{20}$ or $3\sqrt{2}\ \sqrt{5} \times \sqrt{2}$ or $\sqrt{180}$ or $\sqrt{90} \times \sqrt{2}$ $= 6\sqrt{5}$	M1 A1 2	 Correctly simplified answer
(ii)	$10\sqrt{5} + 5\sqrt{5}$ $= 15\sqrt{5}$	M1 B1 A1 3 **5**	Attempt to change both surds to $\sqrt{5}$ One part correct and fully simplified cao

4 (i)	$(-4)^2 - 4 \times k \times k$ $= 16 - 4k^2$	M1 A1 2	Uses $b^2 - 4ac$ (involving k) $16 - 4k^2$
(ii)	$16 - 4k^2 = 0$ $k^2 = 4$ $k = 2$ or $k = -2$	M1 B1 B1 3 **5**	Attempts $b^2 - 4ac = 0$ (involving k) or attempts to complete square (involving k)
5 (i)	Length = 20 – 2x Area = x(20 – 2x) $= 20x - 2x^2$	M1 A1 2	Expression for length of enclosure in terms of x Correctly shows that area = $20x - 2x^2$ **AG**
(ii)	$\frac{dA}{dx} = 20 - 4x$ For max, 20 – 4x = 0 x = 5 only Area = 50	M1 M1 A1 A1 4 **6**	Differentiates area expression Uses $\frac{dy}{dx} = 0$
6	Let $y = (x + 2)^2$ $y^2 + 5y - 6 = 0$ (y + 6)(y - 1) = 0 y = -6 or y = 1 $(x + 2)^2 = 1$ x = -1 or x = -3	B1 M1 A1 M1 A1 A1 6 **6**	Substitute for $(x + 2)^2$ to get $y^2 + 5y - 6$ (= 0) Correct method to find roots Both values for y correct Attempt to work out x One correct value Second correct value and no extra real values
7 (a)	$f(x) = x + 3x^{-1}$ $f'(x) = 1 - 3x^{-2}$	M1 A1 A1 A1 4	Attempt to differentiate First term correct x^{-2} soi **www** Fully correct answer
(b)	$\frac{dy}{dx} = \frac{5}{2}x^{\frac{3}{2}}$ When x = 4, $\frac{dy}{dx} = \frac{5}{2}\sqrt{4^3}$ = 20	M1 B1 B1 M1 A1 5 **9**	Use of differentiation to find gradient $\frac{5}{2}x^c$ $kx^{\frac{3}{2}}$ $\sqrt{4^3}$ soi **SR** If 0 scored for first 3 marks, award B1 if $\sqrt{4^n}$ correctly evaluated.

8 (i)	$(x + 4)^2 - 16 + 15$ $= (x + 4)^2 - 1$	B1 M1 A1 3	a = 4 15 – their a^2 cao in required form
(ii)	(-4, -1)	B1 ft B1 ft 2	Correct x coordinate Correct y coordinate
		M1 A1	Correct method to find roots -5, -3
(iii)	$x^2 + 8x + 15 > 0$ $(x + 5)(x + 3) > 0$ $x < -5, x > -3$	M1 A1 4 **9**	Correct method to solve quadratic inequality eg +ve quadratic graph $x < -5, x > -3$ (not wrapped, strict inequalities, no 'and')
9 (i)	$(x - 3)^2 - 9 + y^2 - k = 0$ $(x - 3)^2 + y^2 = 9 + k$ Centre (3, 0) $9 + k = 4^2$ $k = 7$	B1 B1 M1 A1 4	$(x-3)^2$ soi Correct centre Correct value for *k* (may be embedded) <u>Alternative method using expanded form:</u> Centre $(-g, -f)$ M1 Centre (3, 0) A1 $4 = \sqrt{f^2 + g^2 - (-k)}$ M1 $k = 7$ A1
(ii)	$(3 - 3)^2 + y^2 = 16$ $y^2 = 16$ $y = 4$	M1 A1	Attempt to substitute x = 3 into original equation or their equation y = 4 (do not allow ± 4)
	Length of AB = $\sqrt{(-1-3)^2 + (0-4)^2}$ $= \sqrt{32}$ $= 4\sqrt{2}$	M1 A1 ft A1 5	Correct method to find line length using Pythagoras' theorem $\sqrt{32}$ or $\sqrt{16 + a^2}$ cao
(iii)	Gradient of AB = 1 or $\frac{a}{4}$ $y - 0 = m(x + 1)$ or $y - 4 = m(x - 3)$ $y = x + 1$	B1 ft M1 A1 3 **12**	 Attempts equation of straight line through their A or B with their gradient Correct equation in any form with simplified constants

10 (i)	(3x + 1)(x – 5) = 0 $x = \frac{-1}{3}$ or x = 5	M1 A1 A1 3	Correct method to find roots Correct brackets or formula Both values correct **SR B1** for x = 5 spotted **www**
(ii)		B1 B1 B1 ft 3	Positive quadratic (must be reasonably symmetrical) y intercept correct both x intercepts correct
(iii)	$\frac{dy}{dx} = 6x - 14$ 6x – 14 = 4 x = 3 On curve, when x = 3, y = -20 -20 = (4 x 3) + c c = -32 Alternative method: $3x^2 - 14x - 5 = 4x + c$ $3x^2 - 18x - 5 - c = 0$ has one solution $b^2 - 4ac = 0$ $(-18)^2 - (4 \times 3 \times (-5 - c)) = 0$ c = -32	M1* M1* A1 A1 ft M1dep A1 6 M1 B1 M1 M1 A1 A1 **12**	Use of differentiation to find gradient of curve Equating their gradient expression to 4 Finding y co ordinate for their x value N.B. dependent on both previous M marks Equate curve and line (or substitute for x) Statement that only one solution for a tangent (may be implied by next line) Use of discriminant = 0 Attempt to use a, b, c from their equation Correct equation c = -32

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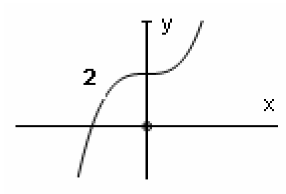
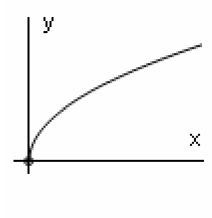
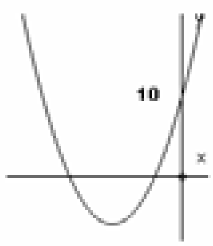
1	$\frac{4(3+\sqrt{7})}{(3-\sqrt{7})(3+\sqrt{7})}$ $= \frac{12+4\sqrt{7}}{9-7}$ $= 6 + 2\sqrt{7}$	M1 B1 A1 3 **3**	Multiply top and bottom by conjugate 9 ± 7 soi in denominator $6 + 2\sqrt{7}$
2(i) (ii)	$x^2 + y^2 = 49$ $x^2 + y^2 - 6x - 10y - 30 = 0$ $(x-3)^2 - 9 + (y-5)^2 - 25 - 30 = 0$ $(x-3)^2 + (y-5)^2 = 64$ $r^2 = 64$ $r = 8$	B1 1 M1 A1 2 **3**	$x^2 + y^2 = 49$ 3^2 5^2 30 with consistent signs soi 8 cao
3	$a(x+3)^2 + c = 3x^2 + bx + 10$ $3(x^2 + 6x + 9) + c = 3x^2 + bx + 10$ $3x^2 + 18x + 27 + c = 3x^2 + bx + 10$ $c = -17$	B1 B1 M1 A1 4 **4**	$a = 3$ soi $b = 18$ soi $c = 10 - 9a$ or $c = 10 - \frac{b^2}{12}$ $c = -17$
4(i) (ii) (iii)	p = -1 $\sqrt{25k^2} = 15$ $25k^2 = 225$ $k^2 = 9$ $k = \pm 3$ $\sqrt[3]{t} = 2$ $t = 8$	B1 1 M1 A1 A1 3 M1 A1 2 **6**	p = -1 Attempt to square 15 or attempt to square root $25k^2$ k = 3 k = -3 $\frac{1}{t^{\frac{1}{3}}} = \frac{1}{2}$ or $t^{\frac{1}{3}} = 2$ soi $t = 8$

5(i)		B1 B1 2	+ve cubic +ve or -ve cubic with point of inflection at (0, 2) and no max/min points
(ii)		B1 B1 2	curve with correct curvature in +ve quadrant only completely correct curve
(iii)	Stretch scale factor 1.5 parallel to y-axis	B1 B1 B1 3 **7**	stretch factor 1.5 parallel to y-axis or in y-direction
6(i)	EITHER $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \dfrac{-8 \pm \sqrt{64-40}}{2}$ $x = \dfrac{-8 \pm \sqrt{24}}{2}$ $x = \dfrac{-8 \pm 2\sqrt{6}}{2}$ $x = -4 \pm \sqrt{6}$ OR $(x+4)^2 - 16 + 10 = 0$ $(x+4)^2 = 6$ $x + 4 = \pm\sqrt{6}$ M1 A1 $x = \pm\sqrt{6} - 4$ A1	M1 A1 A1 3	Correct method to solve quadratic $x = \dfrac{-8 \pm \sqrt{24}}{2}$ $x = -4 \pm \sqrt{6}$
(ii)		B1 B1 B1 3	+ve parabola parabola cutting *y*-axis at (0, 10) where (0, 10) is not min/max point parabola with 2 negative roots
(iii)	$x \le -\sqrt{6} - 4, x \ge \sqrt{6} - 4$	M1 A1 ft 2 **8**	$x \le$ lower root $x \ge$ higher root (allow < , >) Fully correct answer, ft from roots found in (i)

7(i)	Gradient = $-\frac{1}{2}$	B1 1	$-\frac{1}{2}$
(ii)	$y-5=-\frac{1}{2}(x-6)$	M1	Equation of straight line through (6, 5) with any non-zero numerical gradient
		B1 ft	Uses gradient found in (i) in their equation of line
	$2y-10=-x+6$ $x+2y-16=0$	A1 3	Correct answer in correct form (integer coefficients)
(iii)	EITHER $\frac{4-x}{2}=x^2+x+1$	*M1	Substitute to find an equation in *x* (or *y*)
	$4-x=2x^2+2x+2$ $2x^2+3x-2=0$ $(2x-1)(x+2)=0$	DM1	Correct method to solve quadratic
	$x=\frac{1}{2}, x=-2$	A1	$x=\frac{1}{2}, x=-2$
	$y=\frac{7}{4}, y=3$	A1 4	$y=\frac{7}{4}, y=3$ **SR** one correct (*x*,y) pair **www** **B1**
	OR $y=(4-2y)^2+(4-2y)+1$ *M $y=16-16y+4y^2+4-2y+1$ $0=21-19y+4y^2$ $0=(4y-7)(y-3)$ DM1 $y=\frac{7}{4}, y=3$ A1 $x=\frac{1}{2}, x=-2$ A1	**8**	

8(i)	$\frac{dy}{dx} = 3x^2 + 2x - 1$	*M1 A1	Attempt to differentiate (at least one correct term) 3 correct terms
	At stationary points, $3x^2 + 2x - 1 = 0$	M1	Use of $\frac{dy}{dx} = 0$
	$(3x-1)(x+1) = 0$	DM1	Correct method to solve 3 term quadratic
	$x = \frac{1}{3},\ x = -1$	A1	$x = \frac{1}{3},\ x = -1$
	$y = \frac{76}{27},\ y = 4$	A1 6	$y = \frac{76}{27},\ 4$
			SR one correct (x,y) pair **www** **B1**
(ii)	$\frac{d^2y}{dx^2} = 6x + 2$	M1	Looks at sign of $\frac{d^2y}{dx^2}$ for at least one of their x-values or other correct method
	$x = \frac{1}{3},\ \frac{d^2y}{dx^2} > 0$	A1	$x = \frac{1}{3}$, minimum point CWO
	$x = -1,\ \frac{d^2y}{dx^2} < 0$	A1 3	$x = -1$, maximum point CWO
(iii)	$-1 < x < \frac{1}{3}$	M1	Any inequality (or inequalities) involving both their x values from part (i)
		A1 2	Correct inequality (allow < or ≤)
		11	

9(i)	Gradient of AB = $\frac{-2-1}{-5-3}$ $=\frac{3}{8}$	B1	$\frac{3}{8}$ oe
	$y-1=\frac{3}{8}(x-3)$ $8y-8=3x-9$	M1	Equation of line through either A or B, any non-zero numerical gradient
	$3x-8y-1=0$	A1 3	Correct equation in correct form
(ii)	$\left(\frac{-5+3}{2},\frac{-2+1}{2}\right)$	M1	Uses $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$
	$=(-1,\ -\frac{1}{2})$	A1 2	$(-1,\ -\frac{1}{2})$
(iii)	$AC=\sqrt{(-5+3)^2+(-2-4)^2}$	M1	Uses $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
	$=\sqrt{2^2+6^2}$ $=\sqrt{40}$	A1	$\sqrt{40}$
	$=2\sqrt{10}$	A1 3	Correctly simplified surd
(iv)	Gradient of AC $=\frac{-2-4}{-5+3}=3$	B1	3 oe
	Gradient of BC $=\frac{4-1}{-3-3}=-\frac{1}{2}$	B1	$-\frac{1}{2}$ oe
	$3\times-\frac{1}{2}\neq-1$ so lines are not	M1	Attempts to check $m_1\times m_2$
	perpendicular	A1 4	Correct conclusion **www**
		12	

10(i)	$24x^2 - 3x^{-4}$	B1 B1 B1	$24x^2$ kx^{-4} $-3x^{-4}$
	$48x + 12x^{-5}$	M1 A1 5	Attempt to differentiate their (i) Fully correct
(ii)	$8x^3 + \frac{1}{x^3} = -9$ $8x^6 + 1 = -9x^3$ $8x^6 + 9x^3 + 1 = 0$	*M1	Use a substitution to obtain a 3-term quadratic
	Let $y = x^3$ $8y^2 + 9y + 1 = 0$ $(8y+1)(y+1) = 0$ $y = -\frac{1}{8}, y = -1$ $x = -\frac{1}{2}, x = -1$	DM1 A1 M1 A1 5	Correct method to solve quadratic $-\frac{1}{8}$, -1 Attempt to cube root at least one of their y-values $-\frac{1}{2}$, -1 **SR** one correct x value **www** **B1** **SR for trial and improvement:** x = -1 B1 $x = -\frac{1}{2}$ B2
		10	Justification that there are no further solutions B2

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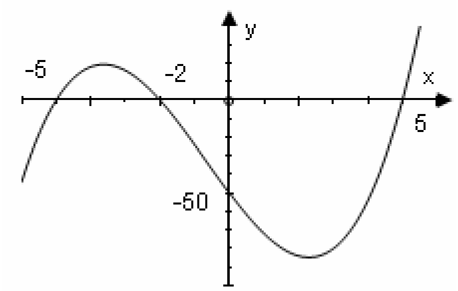
Q	Part	Answer	Mark	Guidance
1	(i)	$n = -2$	B1 [1]	
	(ii)	$n = 3$	B1 [1]	
	(iii)		M1	$\sqrt{4^3}$ or $64^{\frac{1}{2}}$ or $\left(4^{\frac{1}{2}}\right)^3$ or $\left(4^3\right)^{\frac{1}{2}}$ or $4\times\sqrt{4}$ with brackets correct if used
		$n = \frac{3}{2}$	A1 [2]	
2	(i)		M1	$y = (x \pm 2)^2$
		$y = (x-2)^2$	A1 [2]	
	(ii)	$y = -(x^3 - 4)$	B1 [1]	oe
3	(i)	$\sqrt{2\times 100} = 10\sqrt{2}$	B1 [1]	
	(ii)	$\frac{12}{\sqrt{2}} = \frac{12\sqrt{2}}{2} = 6\sqrt{2}$	B1 [1]	
	(iii)		M1	Attempt to express $5\sqrt{8}$ in terms of $\sqrt{2}$
		$10\sqrt{2} - 3\sqrt{2} = 7\sqrt{2}$	A1 [2]	
4		$y = x^{\frac{1}{2}}$		
		$2y^2 - 7y + 3 = 0$	M1*	Use a substitution to obtain a quadratic or factorise into 2 brackets each containing $x^{\frac{1}{2}}$
		$(2y-1)(y-3) = 0$	M1dep	Correct method to solve a quadratic
		$y = \frac{1}{2}, y = 3$	A1	
			M1	Attempt to square to obtain x
		$x = \frac{1}{4}, x = 9$	A1	
			SR If first M1 not gained and 3 and ½ given as final answers, award B1 [5]	

5			**M1**	Attempt to differentiate
			A1	$kx^{-\frac{1}{2}}$
		$\frac{dy}{dx} = 4x^{-\frac{1}{2}} + 1$	**A1**	
		$= 4\left(\frac{1}{\sqrt{9}}\right) + 1$	**M1**	Correct substitution of $x = 9$ into their
	$\frac{dy}{dx}$	$= \frac{7}{3}$	**A1**	$\frac{7}{3}$ only
			[5]	
6	**(i)**	$(x-5)(x+2)(x+5)$	**B1**	$x^2 - 3x - 10$ or $x^2 + 7x + 10$ or $x^2 - 25$ seen
		$= (x^2 - 3x - 10)(x+5)$	**M1**	Attempt to multiply a quadratic by a linear factor
		$= x^3 + 2x^2 - 25x - 50$	**A1**	
			[3]	
	(ii)		**B1**	+ve cubic with 3 roots (not 3 line segments)
			B1√	(0, -50) labelled or indicated on *y*-axis
			B1	(–5, 0), (–2, 0), (5, 0) labelled or indicated on *x*-axis and no other *x*- intercepts
			[3]	
7	**(i)**	$8 < 3x - 2 < 11$	**M1**	2 equations or inequalities both dealing with all 3 terms resulting in $a < kx < b$
		$10 < 3x < 13$	**A1**	10 and 13 seen
		$\frac{10}{3} < x < \frac{13}{3}$	**A1**	
			[3]	
	(ii)	$x(x+2) \geq 0$	**M1**	Correct method to solve a quadratic
			A1	0, –2
			M1	Correct method to solve inequality
		$x \geq 0,\ x \leq -2$	**A1**	
			[4]	

8	(i)	$\frac{dy}{dx} = 3x^2 - 2kx + 1$	B1	One term correct
			B1	Fully correct
			2	
	(ii)	$3x^2 - 2kx + 1 = 0$ when $x = 1$	M1	their $\frac{dy}{dx} = 0$ soi
		$3 - 2k + 1 = 0$	M1	$x = 1$ substituted into their $\frac{dy}{dx} = 0$
		$k = 2$	A1√	
			3	
	(iii)	$\frac{d^2y}{dx^2} = 6x - 4$	M1	Substitutes $x = 1$ into their $\frac{d^2y}{dx^2}$ and looks at sign
		When $x = 1$, $\frac{d^2y}{dx^2} > 0 \therefore$ min pt	A1	States minimum **CWO**
			2	
	(iv)	$3x^2 - 4x + 1 = 0$	M1	their $\frac{dy}{dx} = 0$
		$(3x - 1)(x - 1) = 0$	M1	correct method to solve 3-term quadratic
		$x = \frac{1}{3}, x = 1$		
		$x = \frac{1}{3}$	A1	**WWW** at any stage
			3	

9 (i)		**B1**	$(x-2)^2$ and $(y-1)^2$ seen
	$(x-2)^2+(y-1)^2=100$	**B1**	$(x\pm 2)^2+(y\pm 1)^2=100$
	$x^2+y^2-4x-2y-95=0$	**B1**	correct form
		3	
(ii)	$(5-2)^2+(k-1)^2=100$	**M1**	x = 5 substituted into their equation
	$(k-1)^2=91$ or $k^2-2k-90=0$	**A1**	correct, simplified quadratic in k (or y) obtained
	$k=1+\sqrt{91}$	**A1**	cao
		3	
(iii)	distance from (–3, 9) to (2, 1)		
	$=\sqrt{(2--3)^2+(1-9)^2}$	**M1**	Uses $(x_2-x_1)^2+(y_2-y_1)^2$
	$=\sqrt{25+64}$	**A1**	
	$=\sqrt{89}$		
	$\sqrt{89}<10$ so point is inside	**B1**	compares their distance with 10 and makes consistent conclusion
		3	
(iv)	gradient of radius $=\frac{9-1}{8-2}$	**M1**	uses $\frac{y_2-y_1}{x_2-x_1}$
	$=\frac{4}{3}$	**A1**	oe
	gradient of tangent $=-\frac{3}{4}$	**B1√**	oe
	$y-9=-\frac{3}{4}(x-8)$	**M1**	correct equation of straight line through (8, 9), any non-zero gradient
	$y-9=-\frac{3}{4}x+6$		
	$y=-\frac{3}{4}x+15$	**A1**	oe 3 term equation
		5	

10	**(i)**	$2(x^2-3x)+11$	**B1**	$p = 2$
		$=2\left[\left(x-\frac{3}{2}\right)^2-\frac{9}{4}\right]+11$	**B1**	$q = -\frac{3}{2}$
		$=2\left(x-\frac{3}{2}\right)^2+\frac{13}{2}$	**M1**	$r=11-2q^2$ or $\frac{11}{2}-q^2$
			A1	$r=\frac{13}{2}$
			4	
	(ii)	$\left(\frac{3}{2},\frac{13}{2}\right)$	**B1**√	
			B1√	
			2	
	(iii)	$36-4\times2\times11$	**M1**	uses b^2-4ac
		$=-52$	**A1**	
			2	
	(iv)	0 real roots	**B1**	cao
			1	
	(v)	$2x^2-6x+11=14-7x$	**M1***	substitute for *x*/*y* or attempt to get an equation in 1 variable only
		$2x^2+x-3=0$	**A1**	obtain correct 3 term quadratic
		$(2x+3)(x-1)=0$	**M1dep**	correct method to solve 3 term quadratic
		$x=-\frac{3}{2},\ x=1$	**A1**	
		$y=\frac{49}{2},\ y=7$	**A1**	
				SR If A0 A0, one correct pair of values, spotted or from correct factorisation **www** **B1**
			5	

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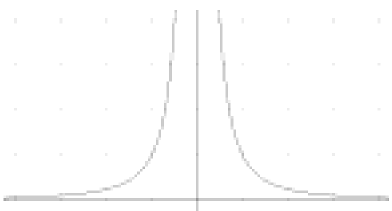
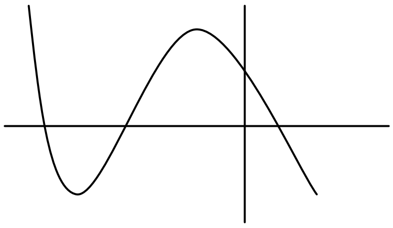
1	$3\sqrt{5}+\frac{20\sqrt{5}}{5}$ $=7\sqrt{5}$	B1 M1 A1 3 [3]	$3\sqrt{5}$ soi Attempt to rationalise $\frac{20}{\sqrt{5}}$ cao
2 (i)	x^2	B1 1	cao
(ii)	$\frac{3y^4 \times 1000y^3}{2y^5}$ $=1500y^2$	B1 B1 B1 3 [4]	$1000y^3$ soi 1500 y^2
3	*Let* $y = x^{\frac{1}{3}}$ $3y^2+y-2=0$ $(3y-2)(y+1)=0$ $y=\frac{2}{3}, y=-1$ $x=\left(\frac{2}{3}\right)^3, x=(-1)^3$ $x=\frac{8}{27}, x=-1$	*M1 DM1 A1 DM1 A1 ft 5 [5]	Attempt a substitution to obtain a quadratic or factorise with $\sqrt[3]{x}$ in each bracket Correct method to find roots Both values correct Attempt cube of at least one value Both answers correctly followed through **SR** If M1* not awarded, **B1** x = -1 from T & I
4 (i)		B1 B1 2	Excellent curve in one quadrant or roughly correct curves in correct 2 quadrants Completely correct
(ii)	$y=\frac{1}{(x+3)^2}$	M1 A1 2	$\frac{1}{(x\pm 3)^2}$ $y=\frac{1}{(x+3)^2}$
(iii)	*(1, 4)*	B1 B1 2 [6]	Correct x coordinate Correct y coordinate

5 (i)	$\frac{dy}{dx} = -50x^{-6}$	M1 A1 2	kx^{-6} Fully correct answer
(ii)	$y = x^{\frac{1}{4}}$ $\frac{dy}{dx} = \frac{1}{4}x^{-\frac{3}{4}}$	B1 B1 B1 3	$\sqrt[4]{x} = x^{\frac{1}{4}}$ soi $\frac{1}{4}x^{c}$ $kx^{-\frac{3}{4}}$
(iii)	$y = (x^2 + 3x)(1 - 5x)$ $= 3x - 14x^2 - 5x^3$ $\frac{\mathrm{d}y}{\mathrm{d}x} = 3 - 28x - 15x^2$	M1 A1 M1 A1 4 **9**	Attempt to multiply out fully Correct expression (may have 4 terms) Two terms correctly differentiated from their expanded expression Completely correct (3 terms)
6(i)	$5(x^2 + 4x) - 8$ $= 5\left[(x+2)^2 - 4\right] - 8$ $= 5(x+2)^2 - 20 - 8$ $= 5(x+2)^2 - 28$	B1 B1 M1 A1 4	$p = 5$ $(x+2)^2$ seen or $q = 2$ $-8 - 5q^2$ or $-\frac{8}{5} - q^2$ $r = -28$
(ii)	$x = -2$	B1 ft 1	
(iii)	$20^2 - 4 \times 5 \times -8$ $= 560$	M1 A1 2	Uses $b^2 - 4ac$ 560
(iv)	2 real roots	B1 1 **8**	2 real roots
7(i)	$30 + 4k - 10 = 0$ $\therefore k = -5$	M1 A1 2	Attempt to substitute x = 10 into equation of line
(ii)	$\sqrt{(10-2)^2 + (-5-1)^2}$ $= \sqrt{64 + 36}$ $= 10$	M1 A1 2	Correct method to find line length using Pythagoras' theorem cao, dependent on correct value of k in (i)
(iii)	Centre (6, -2) Radius 5	B1 B1 2	
(iv)	Midpoint of AB = (6, -2) Length of AB = 2 x radius Both A and B lie on circumference Centre lies on line 3x + 4y – 10 = 0	B1 B1 2 **8**	One correct statement of verification Complete verification

8 (i)	$x = \dfrac{8 \pm \sqrt{(-8)^2 - (4\times -1 \times 5)}}{-2}$	M1	Correct method to solve quadratic
	$= \dfrac{8 \pm \sqrt{84}}{-2}$	A1	$x = \dfrac{8 \pm \sqrt{84}}{-2}$
	$= -4 - \sqrt{21}$ or $= -4 + \sqrt{21}$	A1 3	Both roots correct and simplified
(ii)	$x \le -4 - \sqrt{21}$, $x \ge -4 + \sqrt{21}$	M1	Identifying $x \le$ their lower root, $x \ge$ their higher root
		A1 2	$x \le -4 - \sqrt{21}$, $x \ge -4 + \sqrt{21}$ (not wrapped, no 'and')
(iii)		B1	Roughly correct negative cubic with max and min
		B1	(-4, 0)
		B1	(0, 20)
		B1	Cubic with 3 distinct real roots
		B1 5	Completely correct graph
		10	
9	$\dfrac{dy}{dx} = 3x^2 + 2px$	M1 A1	Attempt to differentiate Correct expression cao
	When $x = 4$, $\dfrac{dy}{dx} = 0$	M1	Setting their $\dfrac{dy}{dx} = 0$
	$\therefore 3\times 4^2 + 8p = 0$	M1	Substitution of $x = 4$ into their $\frac{dy}{dx} = 0$ to evaluate p
	$8p = -48$		
	$p = -6$	A1	
	$\dfrac{d^2y}{dx^2} = 6x - 12$	M1	Looks at sign of $\dfrac{d^2y}{dx^2}$, derived correctly from their $\dfrac{dy}{dx}$, or other correct method
	When $x = 4$, $6x - 12 > 0$		
	Minimum point	A1 7	Minimum point CWO
		7	

10(i)	$\frac{dy}{dx} = 2x + 1$ $= 5$	M1 A1 2	Attempt to differentiate *y* cao
(ii)	Gradient of normal = $-\frac{1}{5}$ When $x = 2$, $y = 6$ $y - 6 = -\frac{1}{5}(x - 2)$ $x + 5y - 32 = 0$	B1 ft B1 M1 A1 4	ft from a non-zero numerical value in (i) May be embedded in equation of line Equation of line, any non-zero gradient, their *y* coordinate Correct equation in correct form
(iii)	$x^2 + x = kx - 4$ $x^2 + (1-k)x + 4 = 0$ One solution => $b^2 - 4ac = 0$ $(1-k)^2 - 4 \times 1 \times 4 = 0$ $(1-k)^2 = 16$ $1 - k = \pm 4$ $k = -3$ or 5	*M1 DM1 DM1 A1 DM1 A1 6 **12**	Equating $y_1 = y_2$ Statement that discriminant = 0 Attempt (involving *k*) to use a, b, c from their equation Correct equation (may be unsimplified) Correct method to find *k*, dep on 1st 3Ms Both values correct

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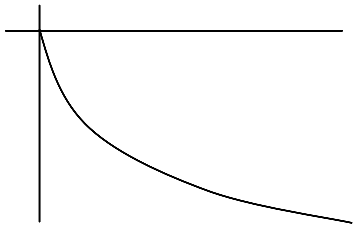
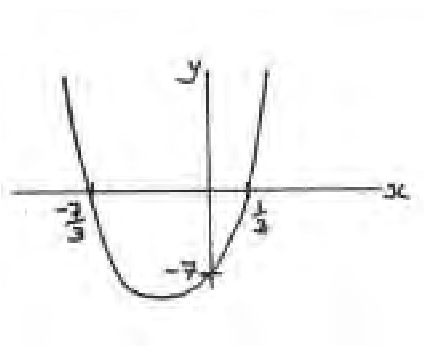
1 (i)	$\frac{dy}{dx} = 5x^4 - 2x^{-3}$	**B1** **M1** **A1 3**	$5x^4$ x^{-2} before differentiation or kx^{-3} in $\frac{dy}{dx}$ soi $-2x^{-3}$
(ii)	$\frac{d^2y}{dx^2} = 20x^3 + 6x^{-4}$	**M1** **A1 2** **5**	Attempt to differentiate their (i) – at least one term correct cao
2	$\frac{(8+\sqrt{7})(2-\sqrt{7})}{(2+\sqrt{7})(2-\sqrt{7})}$ $= \frac{9-6\sqrt{7}}{4-7}$ $= -3+2\sqrt{7}$	**M1** **A1** **A1** **A1 4** **4**	Multiply numerator and denominator by conjugate Numerator correct and simplified Denominator correct and simplified cao
3 (i)	3^{-2}	**B1 1**	
(ii)	$3^{\frac{1}{3}}$	**B1 1**	
(iii)	$3^{10} \times 3^{30}$ $= 3^{40}$	**M1** **A1 2** **4**	3^{30} or 9^{20} soi
4	$y = 2x - 4$ $4x^2 + (2x-4)^2 = 10$ $8x^2 - 16x + 16 = 10$ $8x^2 - 16x + 6 = 0$ $4x^2 - 8x + 3 = 0$ $(2x-1)(2x-3) = 0$ $x = \frac{1}{2}$, $x = \frac{3}{2}$ $y = -3$, $y = -1$	**M1*** **A1** **M1dep*** **A1** **A1** **A1 6** **6**	Attempt to get an equation in 1 variable only Obtain correct 3 term quadratic (aef) Correct method to solve quadratic of form $ax^2 + bx + c = 0$ $(b \neq 0)$ Correct factorisation oe Both x values correct Both y values correct <u>or</u> one correct pair of values www B1 second correct pair of values B1

5 **(i)**	$(2x^2-5x-3)(x+4)$ $=2x^3+8x^2-5x^2-20x-3x-12$ $=2x^3+3x^2-23x-12$	**M1** **A1** **A1 3**	Attempt to multiply a quadratic by a linear factor or to expand all 3 brackets with an appropriate number of terms (including an x^3 term) Expansion with no more than one incorrect term
(ii)	$2x^4+7x^4$ $=9x^4$ 9	**B1** **B1 2** **5**	$2x^4$ or $7x^4$ soi www $9x^4$ or 9
6 **(i)**		**B1** **B1 2**	One to one graph <u>only</u> in bottom right hand quadrant Correct graph, passing through origin
(ii)	Translation Parallel to *y*-axis, 5 units	**B1** **B1 2**	
(iii)	$y=-\sqrt{\frac{x}{2}}$	**M1** **A1 2** **6**	$\sqrt{2x}$ or $\sqrt{\frac{x}{2}}$ seen cao
7 **(i)**	$\left(x-\frac{5}{2}\right)^2-\left(\frac{5}{2}\right)^2+\frac{1}{4}$ $=\left(x-\frac{5}{2}\right)^2-6$	**B1** **M1** **A1 3**	$a=\frac{5}{2}$ $\frac{1}{4}-a^2$ cao
(ii)	$\left(x-\frac{5}{2}\right)^2-6+y^2=0$ Centre $\left(\frac{5}{2},0\right)$ Radius = $\sqrt{6}$	**B1** **B1** **B1 3** **6**	Correct *x* coordinate Correct *y* coordinate

8 (i)	$-42 < 6x < -6$	**M1**	2 equations or inequalities both dealing with all 3 terms
	$-7 < x < -1$	**A1**	-7 and -1 seen oe
		A1 3	$-7 < x < -1$ (or $x > -7$ <u>and</u> $x < -1$)
(ii)	$x^2 > 16$	**B1**	±4 oe seen
	$x > 4$	**B1**	$x > 4$
	or $x < -4$	**B1 3**	$x < -4$ not wrapped, not 'and'
		6	
9 (i)	$\sqrt{(^-1-4)^2+(9-{^-3})^2}$	**M1**	Correct method to find line length using Pythagoras' theorem
	$=13$	**A1 2**	cao
(ii)	$\left(\frac{4+{^-1}}{2}, \frac{^-3+9}{2}\right)$	**M1**	Correct method to find midpoint
	$\left(\frac{3}{2}, 3\right)$	**A1 2**	
(iii)	Gradient of $AB = -\frac{12}{5}$	**B1**	
	$y-3=-\frac{12}{5}(x-1)$	**M1**	Correct equation for line, any gradient, through (1, 3)
	$12x+5y-27=0$	**A1**	Correct equation in any form with gradient simplified
		A1 4	$12x+5y-27=0$
		8	
10 (i)	$(3x+7)(3x-1)=0$	**M1**	Correct method to find roots
	$x=-\frac{7}{3}, x=\frac{1}{3}$	**A1**	Correct factorisation oe
		A1 3	Correct roots
(ii)	$\frac{dy}{dx}=18x+18$	**M1**	Attempt to differentiate y
	$18x+18=0$	**M1**	Uses $\frac{dy}{dx}=0$
	$x=-1$	**A1**	
	$y=-16$	**A1 ft 4**	
(iii)		**B1**	Positive quadratic curve
		B1	y intercept (0, -7)
		B1 3	Good graph, with correct roots indicated and minimum point in correct quadrant
(iv)	$x > -1$	**B1 1**	
		11	

11 **(i)**	Gradient of normal = $-\frac{2}{3}$	**B1**	
	$\frac{dy}{dx} = \frac{1}{2}kx^{-\frac{1}{2}}$	**M1*** **A1**	Attempt to differentiate equation of curve $\frac{1}{2}kx^{-\frac{1}{2}}$
	When $x = 4$, $\frac{dy}{dx} = \frac{k}{4}$	**M1dep***	Attempt to substitute $x = 4$ into their $\frac{dy}{dx}$ soi
	$\therefore \frac{k}{4} = \frac{3}{2}$ $k = 6$	**M1dep*** **A1 6**	Equate their gradient expression to negative reciprocal of their gradient of normal cao
(ii)	*P* is point (4, 12)	**B1 ft**	
	Q is point (22, 0)	**M1** **A1**	Correct method to find coordinates of *Q* Correct *x* coordinate
	Area of triangle = $\frac{1}{2} \times 12 \times 22$ = 132 sq. units	**M1** **A1 5** **11**	Must use *y* coordinate of P and *x* coordinate of Q

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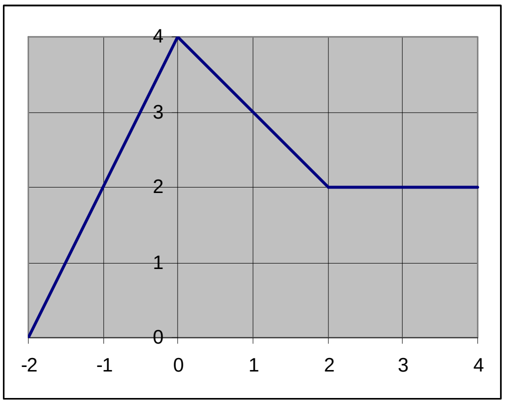
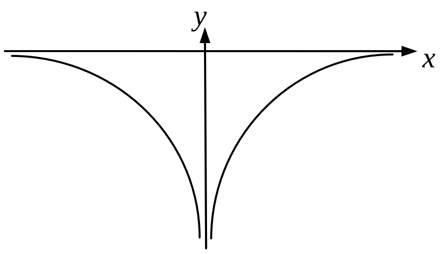
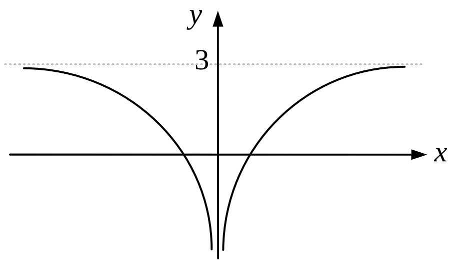
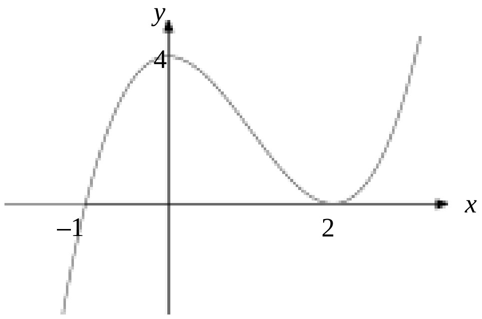
1		$[(x-6)^2-36]+1$	B1		$(x-6)^2$
		$=(x-6)^2-35$	M1		$q = 1 - (\text{their } p)^2$
			A1	3	$q = -35$
				[3]	
2	(i)		B1		For $x < 0$, straight line joining (–2, 0) and (0, 4)
			B1	2	For $x > 0$, line joining (0,4) to (2, 2) and horizontal line joining (2,2) and (4,2)
	(ii)	Translation	B1		
		1 unit right parallel to x axis	B1	2	Allow: 1 unit right, 1 along the x axis, 1 in x **direction**, allow vector notation e.g. $\begin{pmatrix}1\\0\end{pmatrix}$, 1 unit horizontally
				[4]	
3		$\frac{dy}{dx}=3x^2-8x$	M1		Attempt to differentiate (one of $3x^2$, $-8x$)
			A1		Correct derivative
		When $x = 2$, $\frac{dy}{dx}=-4$	M1		Substitutes $x = 2$ into their $\frac{dy}{dx}$
			A1		
		$\therefore$ Gradient of normal to curve = $\frac{1}{4}$	B1 ft		Must be numerical $= -1 \div$ their m
		$y+1=\frac{1}{4}(x-2)$	M1		Correct equation of straight line through (2, –1), any non-zero numerical gradient
		$x-4y-6=0$	A1	7	Correct equation in required form
				[7]	

4	(i)	$m = 4$	B1	1	May be embedded
	(ii)	$6p^2 = 24$ $p^2 = 4$ $p = 2$ or $p = -2$	M1 A1 A1	 3	$(\pm)6p^2 = 24$ or $36p^4 = 576$
	(iii)	$5^{2n+4} = 25$	M1		Addition of indices as powers of 5
		$\therefore 2n+4=2$	M1	3	Equate powers of 5 or 25
		$n = -1$	A1	7	
5		$k = \sqrt{x}$			
		$k^2 - 8k + 13 = 0$	M1*		Use a substitution to obtain a quadratic (may be implied by squaring or rooting later) or factorise into 2 brackets each containing $\sqrt{x}$
		$k - 4 = \pm\sqrt{3}$ or $k = \dfrac{8 \pm \sqrt{(-8)^2 - 4\times1\times13}}{2}$	M1 dep A1		Correct method to solve resulting quadratic
		$k = 4 \pm \sqrt{3}$	A1		$k = 4 \pm \sqrt{3}$ or $k = \dfrac{8 \pm \sqrt{12}}{2}$ or $k = 4 \pm \dfrac{\sqrt{12}}{2}$
		$\therefore x = (4+\sqrt{3})^2$ or $x = (4-\sqrt{3})^2$	M1 M1		Recognise the need to square to obtain x Correct method for squaring $a + \sqrt{b}$ (3 or 4 term expansion)
		$x = 19 \pm 8\sqrt{3}$ or $19 \pm 4\sqrt{12}$	A1	7 7	
6	(i)	$\dfrac{dy}{dx} = 2x$	B1*		
		When $x = 1$, $\dfrac{dy}{dx} = 2$	B1 dep	2	
	(ii)	$\dfrac{a^2+5-6}{a-1} = 2.3$	M1 A1		uses $\dfrac{y_2 - y_1}{x_2 - x_1}$ correct expression
		$a^2 - 2.3a + 1.3 = 0$ $(a-1.3)(a-1) = 0$	M1		correct method to solve a quadratic or correct factorisation and cancelling to get $a + 1 = 2.3$
		$a = 1.3$	A1	4	1.3 only

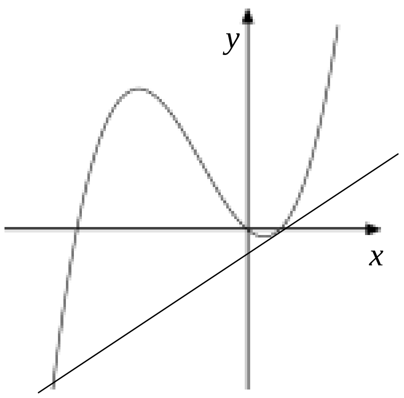
		Alternative method: Equation of straight line through (1,6) with m = 2.3 found then $a^2 + 5 = 2.3a + \text{“}c\text{”}$ seen **M1** with $c = 3.7$ **A1** then as main scheme			
	(iii)	A value between 2 and 2.3	**B1**	1 7	2 < value < 2.3 (strict inequality signs)
7	**(i)**	(a) Fig 3 (b) Fig 1 (c) Fig 4	**B1** **B1** **B1**	3	
	(ii)	$-(x-3)^2$	**M1**		Quadratic expression with correct x^2 term and correct y-intercept and/or roots for **their unmatched diagram** (e.g. negative quadratic with y-intercept of –9 or root of 3 for Fig 2)
		$y = -(x-3)^2$	**A1**	2 5	Completely correct equation for **Fig 2**
8	**(i)**	Centre (–3, 2)	**B1**		
		$(x+3)^2 - 9 + (y-2)^2 - 4 - 4 = 0$ $r^2 = 17$	**M1**		Correct method to find r^2
		$r = \sqrt{17}$	**A1**	3	Correct radius
	(ii)	$x^2 + (3x+4)^2 + 6x - 4(3x+4) - 4 = 0$	**M1***		substitute for x/y or attempt to get an equation in 1 variable only
			A1		correct unsimplified expression
		$10x^2 + 18x - 4 = 0$	**A1**		obtain correct 3 term quadratic
		$(5x-1)(x+2) = 0$	**M1** **dep**		correct method to solve their quadratic
		$x = \frac{1}{5}$ or $x = -2$	**A1**		
		$y = \frac{23}{5}$ or $y = -2$	**A1**	6 9	**SR** If A0 A0, one correct pair of values, spotted or from correct factorisation **www** **B1**
9	**(i)**	$f'(x) = -x^{-2} - \frac{1}{2}x^{-\frac{1}{2}}$	**M1**		Attempt to differentiate
			A1		$-x^{-2}$ or $-\frac{1}{2}kx^{-\frac{1}{2}}$ **www**
			A1	3	Fully correct expression

	(ii)	$f''(x) = 2x^{-3} + \frac{1}{4}x^{-\frac{3}{2}}$	M1		Attempt to differentiate their $f'(x)$
			A1 ft		One correctly differentiated term
			A1		Fully correct expression **www** in either part of the question
		$f''(4) = \frac{2}{4^3} + \frac{1}{4}.\frac{1}{8}$	M1		Substitution of $x = 4$ into their $f''(x)$
		$= \frac{1}{16}$	A1	5 [8]	oe single fraction **www** in either part of the question
10		$(-30)^2 - 4 \times k \times 25k = 0$	M1		Attempts $b^2 - 4ac$ involving k
		$900 - 100k^2 = 0$	M1		States their discriminant = 0
		$k = 3$	B1		
		or $k = -3$	B1	4 [4]	
11	(i)	$P = 2 + x + 3x + 2 + 5x + 5x$ $= 14x + 4$	M1		Adds lengths of all 4 edges with attempt to use Pythagoras to find the missing length
			A1	2	May be left unsimplified
	(ii)	Area of rectangle = $3x(2 + x) = 6x + 3x^2$	M1		Correct method – splitting or formula for area of trapezium
		Area of triangle $= \frac{1}{2}(3x)(4x) = 6x^2$			
		Total area = $9x^2 + 6x$	A1	2	Convincing working leading to given expression **AG**
	(iii)	$14x + 4 \geq 39$	B1 ft		ft on their expression for P from **(i)** unless restarted in **(iii)**. (Allow >)
		$\frac{5}{2}$	B1		o.e. (e.g. $\frac{35}{14}$) soi by subsequent working
		$9x^2 + 6x < 99$	B1		
		$3x^2 + 2x - 33 < 0$			Allow ≤
		$(3x + 11)(x - 3) < 0$	M1		Correct method to find critical values
		$\left(-\frac{11}{3} <\right) x < 3$	B1		$x < 3$ identified
			M1		root from linear < x < upper root from quadratic
		$\therefore \frac{5}{2} \leq x < 3$	A1	7 [11]	Fully correct including inequality signs or exact equivalent in words **cwo**
		Total		**72**	

Question	Answer	Mark	Total	Guidance
1 (i)	1	**B1**	**1**	
(ii)	$\frac{1}{3}$	**M1**		$\frac{1}{9^{\frac{1}{2}}}$ or $\frac{1}{\sqrt{9}}$ soi
		A1	**2**	cao
			3	
2 (i)		**B1***		Reasonably correct curve for $y=-\frac{1}{x^2}$ in 3rd and 4th quadrants only
		B1 dep*	**2**	Very good curves in curve for $y=-\frac{1}{x^2}$ in 3rd and 4th quadrants **SC** If 0, very good single curve in either 3rd or 4th quadrant and nothing in other three quadrants. **B1**
(ii)		**M1**		Translation of their $y=-\frac{1}{x^2}$ vertically
		A1	**2**	Reasonably correct curve, horizontal asymptote soi at $y = 3$
(iii)	$y=-\frac{2}{x^2}$	**B1**	**1**	
			5	
3 (i)	$\frac{12(3-\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})}$	**M1**		Multiply numerator and denom by $3-\sqrt{5}$
	$=\frac{12(3-\sqrt{5})}{9-5}$	**A1**		$(3+\sqrt{5})(3-\sqrt{5})=9-5$
	$=9-3\sqrt{5}$	**A1**	**3**	
(ii)	$3\sqrt{2}-\sqrt{2}$	**M1**		Attempt to express $\sqrt{18}$ as $k\sqrt{2}$
	$=2\sqrt{2}$	**A1**	**2**	
			5	

4 (i)	$(x^2 - 4x + 4)(x + 1)$	**M1**		Attempt to multiply a 3 term quadratic by a linear factor or to expand all 3 brackets with an appropriate number of terms (including an x^3 term)
		A1		Expansion with at most 1 incorrect term
	$= x^3 - 3x^2 + 4$	**A1**	3	Correct, simplified answer
(ii)		**B1**		+ve cubic with 2 or 3 roots
		B1		Intercept of curve labelled (0, 4) or indicated on *y*-axis
		B1	3	(–1, 0) and turning point at (2, 0) labelled or indicated on *x*-axis and no other *x* intercepts
			[6]	
5	$k = x^2$ $4k^2 + 3k - 1 = 0$	**M1***		Use a substitution to obtain a quadratic or factorise into 2 brackets each containing x^2
	$(4k - 1)(k + 1) = 0$	**M1 dep**		Correct method to solve a quadratic
	$k = \frac{1}{4}$ (or k = -1)	**A1**		
	$x = \pm\frac{1}{2}$	**M1** **A1**	5	Attempt to square root to obtain *x* $\pm\frac{1}{2}$ and no other values
			[5]	
6	$y = 2x + 6x^{-\frac{1}{2}}$	**M1**		Attempt to differentiate
	$\frac{dy}{dx} = 2 - 3x^{-\frac{3}{2}}$	**A1**		$kx^{-\frac{3}{2}}$
		A1		Completely correct expression (no +c)
	When *x* = 4, gradient = $2 - \frac{3}{\sqrt{4^3}}$	**M1**		Correct evaluation of either $4^{-\frac{3}{2}}$ or $4^{-\frac{1}{2}}$
	$= \frac{13}{8}$	**A1**	5	
			[5]	
7	$2(6 - 2y)^2 + y^2 = 57$	**M1***		substitute for *x*/*y* or attempt to get an equation in 1 variable only
		A1		correct unsimplified expression
	$2(36 - 24y + 4y^2) + y^2 = 57$ $9y^2 - 48y + 15 = 0$	**A1**		obtain correct 3 term quadratic
	$3y^2 - 16y + 5 = 0$ $(3y - 1)(y - 5) = 0$	**M1 dep**		correct method to solve 3 term quadratic
	$y = \frac{1}{3}$ or *y* = 5	**A1**		
	$x = \frac{16}{3}$ or *x* = -4	**A1**	6 **[6]**	**SC** If A0 A0, one correct pair of values, spotted or from correct factorisation **www B1**

8 (i)	$2(x^2+\frac{5}{2}x)$	B1		$\left(x+\frac{5}{4}\right)^2$
	$=2\left[\left(x+\frac{5}{4}\right)^2-\frac{25}{16}\right]$	M1		$q=-2p^2$
	$=2\left(x+\frac{5}{4}\right)^2-\frac{25}{8}$	A1	3	$q=-\frac{25}{8}$ c.w.o.
(ii)	$\left(-\frac{5}{4},-\frac{25}{8}\right)$	B1√ B1√	2	
(iii)	$x=-\frac{5}{4}$	B1	1	
(iv)	$x(2x+5)>0$	M1		Correct method to find roots
		A1		0, $-\frac{5}{2}$ seen
	$x<-\frac{5}{2},\ x>0$	M1		Correct method to solve quadratic inequality.
		A1	4 **10**	(not wrapped, strict inequalities, no 'and')
9 (i)	$\frac{4+p}{2}=-1,\quad \frac{5+q}{2}=3$	M1		Correct method (may be implied by one correct coordinate)
	p = -6 q = 1	A1 A1	3	
(ii)	$r^2=(4-{}^-1)^2+(5-3)^2$	M1		Use of $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ for either radius or diameter
	$r=\sqrt{29}$	A1	2	
(iii)	$(x+1)^2+(y-3)^2=29$	M1		$(x+1)^2$ and $(y-3)^2$ seen
		M1		$(x\pm1)^2+(y\pm3)^2=$ their r^2
	$x^2+y^2+2x-6y-19=0$	A1	3	Correct equation in correct form
(iv)	gradient of radius = $\frac{3-5}{-1-4}$	M1		uses $\frac{y_2-y_1}{x_2-x_1}$
	$=\frac{2}{5}$	A1		oe
	gradient of tangent = $-\frac{5}{2}$	B1√		oe
	$y-5=-\frac{5}{2}(x-4)$	M1		correct equation of straight line through (4, 5), any non-zero gradient
	$y=-\frac{5}{2}x+15$	A1	5 **13**	oe 3 term equation e.g. $5x+2y=30$

10(i)	$\frac{dy}{dx} = 6x^2 + 10x - 4$	**B1** **B1**		1 term correct Completely correct (no +c)
	$6x^2 + 10x - 4 = 0$	**M1***		Sets their $\frac{dy}{dx} = 0$
	$2(3x^2 + 5x - 2) = 0$ $(3x - 1)(x + 2) = 0$	**M1 dep***		Correct method to solve quadratic
	$x = \frac{1}{3}$ or $x = -2$	**A1**		**SC** If A0 A0, one correct pair of values, spotted or from correct factorisation **www B1**
	$y = -\frac{19}{27}$ or $y = 12$	**A1**	6	
(ii)	$-2 < x < \frac{1}{3}$	**M1** **A1**	2	Any inequality (or inequalities) involving both their *x* values from part (i) Allow ≤ and ≥
(iii)	When $x = \frac{1}{2}$, $6x^2 + 10x - 4 = \frac{5}{2}$	**M1**		Substitute $x = \frac{1}{2}$ into their $\frac{dy}{dx}$
	and $2x^3 + 5x^2 - 4x = -\frac{1}{2}$	**B1**		Correct y coordinate
	$y + \frac{1}{2} = \frac{5}{2}\left(x - \frac{1}{2}\right)$	**M1**		Correct equation of straight line using their values. Must use their $\frac{dy}{dx}$ value not e.g. the negative reciprocal
	$10x - 4y - 7 = 0$	**A1**	4	Shows rearrangement to given equation **CWO** throughout for A1
(iv)		**B1**		Sketch of a cubic with a tangent which meets it at 2 points only
		B1	2 **14**	+ve cubic with max/min points and line with +ve gradient as tangent to the curve to the right of the min **SC1** **B1** Convincing algebra to show that the cubic $8x^3 + 20x^2 – 26x + 7 = 0$ factorises into $(2x – 1)(2x – 1)(x + 7)$ **B1** Correct argument to say there are 2 distinct roots **SC2 B1** Recognising y = 2.5x -7/4 is tangent from part (iii) **B1** As second B1 on main scheme

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Unit **4721:** Core Mathematics 1

Mark Scheme for January 2011

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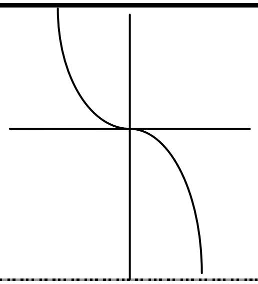
1 (i)	$\sqrt{(-2-6)^2+(7-1)^2}$	**M1**		Use of $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$	3 out of 4 substitutions correct
	$= 10$	**A1**	**2**		Look out for no square root, $(x_2+x_1)^2$ etc. **M0**
(ii)	$\dfrac{7-1}{-2-6}$	**M1**		uses $\dfrac{y_2-y_1}{x_2-x_1}$	3 out of 4 substitutions correct
	$= -\dfrac{3}{4}$	**A1**	**2**	o.e. **ISW**	Allow –0.75 $\dfrac{3}{-4}$ etc.
(iii)	Gradient of given line = $\dfrac{4}{3}$	**M1**		Attempt to rearrange equation to make *y* the subject **OR** attempt to find the gradient using points on the line	Must at least isolate *y*
	$-\dfrac{3}{4}\times\dfrac{4}{3}=-1$	**B1ft**		Correct conclusion for their gradients	
	So lines are perpendicular	**B1**	**3** **7**	States $-\dfrac{3}{4}\times\dfrac{4}{3}=-1$ or "negative reciprocal" relating to the correct values **www**	
2	$2x^3+9x^2-2px^2-9px+10x-10p$ $= 2x^3+qx^2-8x-4q$	**M1***		Attempt to expand both sides **OR** to substitute 2 values of *x* into both expressions **OR** to express at least one side as a product of three factors	If expanding, minimum of 5 terms on LHS and 3terms on RHS
		DM1		Valid method to obtain either p or q	If comparing coefficients, must be of corresponding terms
	p = 2 and *q* = 5	**A1**	**3** **3**	Both values correct	**SR** Spotted solutions **B1** one correct **B2** other correct
3 (i)	$8^{\frac{1}{2}}$	**B1**	**1**		Allow $8^{0.5}$ Condone p= $\frac{1}{2}$, just "$\frac{1}{2}$" seen as answer **www**
(ii)	8^{-2}	**B1**	**1**		Condone p = -2, just "-2" seen as answer **www** $\dfrac{1}{8^2}$ only not enough
(iii)	$2^8 = \left(8^{\frac{1}{3}}\right)^8$	**M1**		2^8 or $2^6 = 8^2$ soi	Condone p= $\dfrac{8}{3}$, just "$\dfrac{8}{3}$" seen as answer **www**
	$= 8^{\frac{8}{3}}$	**M1**		$2 = 8^{\frac{1}{3}}$ soi	2^3= 8 not enough for second **M** mark
		A1	**3** **5**	o.e.	

4	$u^2 - 5u + 4 = 0$ $(u-1)(u-4) = 0$ $u = 1$ or $u = 4$ $3x - 2 = \pm 1$ or $3x - 2 = \pm 2$ $x = 1$ or $\frac{1}{3}$ or $\frac{4}{3}$ or 0	**M1*** **DM1** **A1** **M1** **A1** **A1**	 **6** **6**	Use the given substitution to obtain a quadratic or factorise into 2 brackets each containing $(3x-2)^2$ Correct method to solve a quadratic Correct values for u Attempt to square root and rearrange to obtain x **OR** to expand, rearrange and solve quadratic (at least one) 2 correct values All 4 correct values ($\frac{0}{3}$ = **A0**)	**No marks** if evidence of "square rooting" e.g. "$(3x-2)^2$ - 5$(3x-2)$ + 2 (or 4) = 0" **No marks** if straight to quadratic formula to get x = "1" x = "4" and no further working **SR 1)** If **M0** Spotted solutions **www B1** each Justifies 4 solutions exactly **B2** **SR 2)** If first 3 marks awarded, spotted solutions 2 correct **B1** Other 2 correct **B1** Justifies 4 solutions exactly **B1** <u>Alternative scheme for candidates who multiply out:</u> Attempt to expand $(3x-2)^4$ and $(3x-2)^2$ **M1** $81x^4 - 216x^3 + 171x^2 - 36x = 0$ **A1** $x = 0$ a solution or x a factor of the quartic **A1** Attempt to use factor theorem to factorise their cubic **M1*** Correct method to solve quadratic **DM1** All 4 solutions correct **A1**
5 (i)		**M1** **A1**	 **2**	Negative cubic through (0, 0) (may have max and min) Must have reasonable rotational symmetry. Cannot be a finite "plot". Allow negative gradient at origin. Correct curvature at both ends.	Must be continuous. Allow slight curve towards or away from y-axis at one end, but not both.
(ii)	$y = -(x-3)^3$	**M1** **A1**	 **2**	$\pm(x-3)^3$ seen or $y = (3-x)^3$	Must have "$y =$ " for A mark **SR** $y = -(x-3)^2$ **B1**
(iii)	Stretch scale factor 5 parallel to y-axis	**B1** **B1**	 **2** **6**	o.e. e.g. scale factor $\frac{1}{\sqrt[3]{5}}$ parallel to the x axis.	Allow "factor" for "scale factor" For "parallel to the y axis" allow "vertically", "in the y direction". **Do not accept** "in/on/across/up/along the y axis"

6 (i)	$y = 5x^{-2} - \frac{1}{4}x^{-1} + x$ $\frac{dy}{dx} = -10x^{-3} + \frac{1}{4}x^{-2} + 1$	M1 A1 A1 A1	 4	x^{-2} used for $\frac{1}{x^2}$ **OR** x^{-1} used for $\frac{1}{x}$ soi, **OR** x correctly differentiated kx^{-3} or kx^{-2} from differentiating Two fully correct terms Completely correct	Look out for: $y = 5x^{-2} - 4x^{-1} + x$ followed by $\frac{dy}{dx} = -10x^{-3} + 4x^{-2} + 1$ and then the correct answer. This is **M1 A1 A1 A0** $4x^{-1}$ is **NOT** a misread
(ii)	$\frac{d^2y}{dx^2} = 30x^{-4} - \frac{1}{2}x^{-3}$	M1 A1	 2 **6**	Attempt to differentiate their $\frac{dy}{dx}$ (one term correctly differentiated) Completely correct	Allow a sign slip in coefficient for M mark **NB** Only penalise “+ c” first time seen in the question

7 (i)	$4(x^2+3x)-3$ $=4\left[\left(x+\frac{3}{2}\right)^2-\frac{9}{4}\right]-3$ $=4\left(x+\frac{3}{2}\right)^2-12$	B1 B1 M1 A1 **4**	$p = 4$ $q=\frac{3}{2}$ $r=-3-4q^2$ or $r=-\frac{3}{4}-q^2$ $r = -12$ (from $q=\pm1.5$)	If *p*, *q*, *r* found correctly, then **ISW** slips in format. $4(x + 1.5)^2 + 12$ **B1 B1 M0 A0** $4(x + 1.5) - 12$ **B1 B1 M1 A1 (BOD)** $4(x + 1.5x)^2 - 12$ **B1 B0 M1 A0** $4(x^2 + 1.5)^2 - 12$ **B1 B0 M1 A0** $4(x - 1.5)^2 - 12$ **B1 B0 M1 A1** $4x(x + 1.5)^2 - 12$ **B0 B1M1A1**
(ii)	$\frac{-12\pm\sqrt{12^2-4\times4\times-3}}{2\times4}$ $=\frac{-12\pm\sqrt{192}}{8}$ $=\frac{-12\pm8\sqrt{3}}{8}$ $=-\frac{3}{2}\pm\sqrt{3}$ OR: $4\left(x+\frac{3}{2}\right)^2-12=0$ $x+\frac{3}{2}=\pm\sqrt{3}$ $x=-\frac{3}{2}\pm\sqrt{3}$	M1 A1 B1 A1 M1 A1ft A1 A1 **4**	Correct method to solve quadratic $\frac{-12\pm\sqrt{192}}{8}$ or $\frac{-3\pm\sqrt{12}}{2}$ $\sqrt{192}=8\sqrt{3}$ or $\sqrt{12}=2\sqrt{3}$ from correct b^2-4ac $\frac{-3\pm2\sqrt{3}}{2}$ or $-\frac{12}{8}\pm\sqrt{3}$, $-\frac{6}{4}\pm\sqrt{3}$ Must have ± for method mark $x + 1.5$ **ft** $x + q$ from part(i) www in LHS in part (ii) $\pm\sqrt{3}$ Do not **ISW**	Not for $2(x + q) = ...$ **SR** One correct root **www B1**
(iii)	$12^2-4\times4\times(-k)=0$ $144+16k=0$ $k=-9$ **OR (see next page)**	M1 A1 A1	Attempts $b^2-4ac=0$ or $\sqrt{b^2-4ac}=0$ involving k. If b^2-4ac not quoted then expression must be correct. Correct, unsimplified expression	<u>Other alternative methods</u> a) Attempt to factorise into two equal brackets, (may divide by 4 first – must be correct) **M1** Equate coefficient of *x* to 12 (or 3) **A1** $k=-9$ **A1** b) Uses differentiation to find x ordinate of turning point and uses this to form equation in *k* **M1** Correct equation in *k* **A1** $k=-9$ **A1**

7(iii) cont.	$4x^2+12x=k$ $4(x+\frac{3}{2})^2-9=k$	M1		Attempts completing the square in given equation or factorises to $(2x+3)^2-9=k$	Must involve k in their working to gain the method marks in this scheme
	Equal roots when $x=-\frac{3}{2}$	M1		Substitutes $x=-\frac{3}{2}$	
	$k=-9$	A1	3 **11**		
8 (i)	$\frac{dy}{dx}=6-2x$	M1 A1		Attempt to differentiate $\pm y$ Correct expression **cao**	One correct non-zero term
	When $x=5$, $6-2x=-4$	M1		Substitute $x=5$ into their $\frac{dy}{dx}$	
	When $x = 5$, $y = 12$	B1		Correct y coordinate	
	$y-12=-4(x-5)$	M1		Correct equation of straight line through (5, their y), their non-zero, numerical gradient	Allow $\frac{y-12}{x-5}$ = their gradient If using $y = mx + c$ must attempt at evaluating c
	$4x+y-32=0$	A1	6	Shows rearrangement to correct form	Allow any correct form e.g. $0=2y+8x-64$ etc.
(ii)	Q is point (8, 0)	B1ft		ft from line in (i)	.
	Midpoint of $PQ=\left(\frac{5+8}{2},\frac{12+0}{2}\right)$	M1		Uses $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$ o.e. for their P,Q	
	$=\left(\frac{13}{2},6\right)$	A1	3		Do not accept $\left(\frac{13}{2},\frac{12}{2}\right)$
(iii)	$6 - 2x = 0$	M1		Solution of their $\frac{dy}{dx}=0$	Alternatives for Method Mark a) attempts completion of square with $\pm(x-3)^2$ b) attempts to solve quadratic (usual scheme) and to find the mid-point of the two roots c) attempts to use $x=-\frac{b}{2a}$ (allow one sign slip on substitution)
	(Line of symmetry is) $x = 3$	A1	2	Allow from $\pm[16-(x-3)^2]$, $\pm[6-2x=0]$	
(iv)	$x < 3$	M1		$x <$ their3 or $x >$ their3 **OR** attempt to solve their $\frac{dy}{dx}>0$	May solve $\frac{dy}{dx}=0$ then use $\frac{d^2y}{dx^2}<0$ implies maximum point for the method mark, or sketch of curve
		A1	2 **13**	Allow from $\pm[16-(x-3)^2]$**,** $\pm[6-2x=0]$ in (iii)	Allow $x\le 3$

9 (i)	Centre (4, 1) $(x-4)^2+(y-1)^2-16-1-3=0$ $(x-4)^2+(y-1)^2=20$ Radius $=\sqrt{20}$	**B1** **M1** **A1**	 **3**	Correct centre Correct method to find r^2 Correct radius	 $r^2=(\pm \text{ their } 4)^2+(\pm\text{their } 1)^2+3$ soi $\pm\sqrt{20}$ is **A0** Ignore incorrect simplification of $\sqrt{20}$
(ii)	$k=1\pm\sqrt{20}$ $k=1\pm 2\sqrt{5}$	**M1** **A1ft** **A1**	 **3**	y ordinate of their centre ± their radius or Both correct, unsimplified values **cao**	Alternatives for method mark : a) Substitutes k for y and uses $b^2-4ac=0$ to obtain quadratic in k b) Recognises x = 4 is equation of normal, substitutes into circle equation and solves for k. **SR** $k = 1+\sqrt{20}$ or $k = 1-\sqrt{20}$ or better **www B1**
(iii)	$MT^2=r^2-2^2$ $MT = 4$ $ST = 8$	**M1** **A1ft** **A1**	 **3**	Correct use of Pythagoras' theorem involving MT (or SM) Correct value of MT for their r **cao**	**SR** ST=8 from particular S and T co-ordinates [e.g. horizontal chord calculated as (0,3) and (8,3)] **B1** Justifies solution the same for all possible chords **B2**
(iv)	$x=2y+12$ $(2y+8)^2+(y-1)^2=20$ $4y^2+32y+64+y^2-2y+1=20$ $5y^2+30y+45=0$ $y^2+6y+9=0$ $(y+3)^2=0$ $y=-3$ $x=6$ **OR** $y-1=-2(x-4)$ Solve simultaneously with $y=\frac{1}{2}x-6$ $x=6$ $y=-3$ States line is tangent as meets at one point or verifies (6, -3) lies on circle	**M1*** **A1** **A1** **DM1** **A1** **A1** **M1** **A1** **M1** **A1** **A1** **B1**	 **6** **15**	Attempt to solve equations simultaneously Correct unsimplified expression, may be $(12+2y)^2+y^2-8(12+2y)-2y-3=0$ Obtain correct 3 term quadratic Correct method to solve quadratic of form $ax^2+bx+c=0$ $(b\neq 0)$ y value correct, no extra solutions x value correct **ISW** Attempt to find equation of radius/normal Correct equation Allow showing distance between (6,-3) and (4,1) = $\sqrt{20}$	Must be a clear attempt to reduce to one variable using equation of line and either form of equation of circle. Condone poor algebra for first mark. If y eliminated: $(x-4)^2+\left(\frac{1}{2}x-7\right)^2=20$ Or $x^2+\left(\frac{1}{2}x-6\right)^2-8x-2\left(\frac{1}{2}x-6\right)-3=0$ Leading to $x^2-12x+36=0$ **SR** Correct coordinates spotted or from trial and improvement **www B2**

Allocation of method mark for solving a quadratic

e.g. $4x^2 + 12x - 3 = 0$

By factorisation
– when expanded, quadratic term and one other term must be correct (with correct sign):

$(2x+1)(2x-3)=0$	M1	$4x^2$ and -3 obtained from expansion
$(4x+4)(x+2)=0$	M1	$4x^2$ and $+12x$ obtained from expansion
$(4x-1)(x-3)=0$	M0	only x^2 term correct

By formula
- if the formula is quoted correctly first, allow one sign slip in substituting values into it:
$a = 4$, $b = 12$, $c = -3$

$$\frac{12 \pm \sqrt{(12)^2 - 4\times 4\times -3}}{8}$$ gains M1 (minus sign incorrect at start of formula)

$$\frac{-12 \pm \sqrt{(12)^2 - 4\times 4\times 3}}{8}$$ gains M1 (3 for c instead of −3)

$$\frac{12 \pm \sqrt{(12)^2 - 4\times 4\times 3}}{8}$$ M0 (2 sign errors: initial sign and c incorrect)

- if the formula is not quoted, then no errors at all are allowed in substitution.

By completing the square

$$4x^2 + 12x - 3 = 0$$
$$4\left[\left(x+\frac{3}{2}\right)^2 - \frac{9}{4}\right] - 3 = 0$$
$$\left(x+\frac{3}{2}\right)^2 = 3$$
$$x + \frac{3}{2} = \pm\sqrt{3}$$

The method mark is awarded only at the last line of working
i.e. when ±√ combined constants is seen.

N.B. The value of the combined constants does not have to be correct for the M1 mark

Condone "invisible brackets" if justified by correct later working

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Unit **4721:** Core Mathematics 1

Mark Scheme for June 2011

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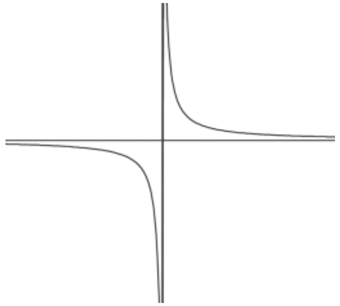
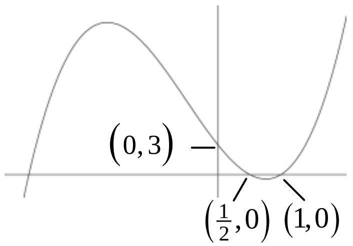
1	$3(x^2-6x)+4$ $=3\left[(x-3)^2-9\right]+4$ $=3(x-3)^2-23$	B1 B1 M1 A1	 4 **4**	$p=3$ $(x-3)^2$ seen or $q=-3$ $4-3q^2$ or $\frac{4}{3}-q^2$ (their q) $r=-23$	If p, q, r found correctly, then **ISW** slips in format. $3(x-3)^2+23$ **B1 B1 M0 A0** $3(x-3)-23$ **B1 B1 M1 A1 (BOD)** $3(x-3x)^2-23$ **B1 B0 M1 A0** $3(x^2-3)^2-23$ **B1 B0 M1 A0** $3(x+3)^2-23$ **B1 B0 M1 A1 (BOD)** $3x(x-3)^2-23$ **B0 B1M1A1**
2 (i)		B1 B1	 2	Reasonably correct curve for $y=\frac{1}{x}$ in 1st and 3rd quadrants only Very good curves for $y=\frac{1}{x}$ in 1st and 3rd quadrants **SC** If 0, very good single curve in either 1st or 3rd quadrant and nothing in other three quadrants. **B1**	N.B. Ignore 'feathering' now that answers are scanned. Reasonably correct shape, not touching axes more than twice. Correct shape, not touching axes, asymptotes clearly the axes. Allow slight movement away from asymptote at one end but not more. Not finite.
(ii)	Translation 4 units parallel to y axis	B1 B1	 2 **4**	**Must** be translation/translated – not shift, move etc. Or $\begin{pmatrix}0\\4\end{pmatrix}$	For "parallel to the y axis" allow "vertically", "up", "in the (positive) y direction". **Do not accept** "in/on/across/up/along the y axis"
3 (i)	$\frac{16x^2\times 2x^3}{x}$ $=32x^4$	B1 B1	 2	32 x^4	
(ii)	$\frac{1}{6}x$	M1 A1 B1	 3 **5**	6 or $\frac{1}{36^{\frac{1}{2}}}$ or $\frac{1}{\sqrt{36}}$ seen $\frac{1}{6}$ in final answer x (Allow x^1) in final answer	$\dfrac{1}{\frac{1}{\sqrt{36}}}$ is M0 $\pm\frac{1}{6}$ is **A0**

4	$2x^2 - 8x + 8 = 26 - 3x$	**M1**		Attempt to eliminate *x* or *y*	Must be a clear attempt to reduce to one variable. Condone poor algebra for first mark.
	$2x^2 - 5x - 18 (= 0)$	**A1**		Correct 3 term quadratic (not necessarily all in one side)	If *x* eliminated:
	$(2x - 9)(x + 2) (= 0)$	**M1**		Correct method to solve quadratic	$y = 2(\frac{26 - y}{3} - 2)^2$
	$x = \frac{9}{2}, x = -2$	**A1**		*x* values correct	Leading to $2y^2 - 89y + 800 = 0$
	$y = \frac{25}{2}, y = 32$	**A1**	5	*y* values correct	(2y – 25)(y – 32) =0 etc.
			5	**SR** If A0 A0, one correct pair of values, spotted or from correct factorisation **www B1**	
5 (i)	$10\sqrt{3} - 4\sqrt{3}$	**M1**		Attempt to express both surds in terms of $\sqrt{3}$	e.g. $\sqrt{3x100} - \sqrt{3x16}$
		B1		One term correct	
	$= 6\sqrt{3}$	**A1**	3	Fully correct (not $\pm 6\sqrt{3}$)	
(ii)	$\frac{\sqrt{5}(15 + \sqrt{40})}{5}$	**M1**		Multiply numerator and denominator by $\sqrt{5}$ or - $\sqrt{5}$ **or** attempt to express both terms of numerator in terms of $\sqrt{5}$ (e.g. dividing both terms by $\sqrt{5}$)	Check both numerator and denominator have been multiplied
	$= \frac{15\sqrt{5} + 10\sqrt{2}}{5}$	**B1**		One of a, b correctly obtained	
	$= 3\sqrt{5} + 2\sqrt{2}$	**A1**	3 **6**	Both a = 3 and b=2 correctly obtained	

6	$k = x^{\frac{1}{4}}$	M1*		Use a substitution to obtain a quadratic or factorise into 2 brackets each containing $x^{\frac{1}{4}}$	**No marks** unless evidence of substitution (quadratic seen or square rooting or squaring of roots found). = 0 may be implied.
	$3k^2 - 8k + 4 = 0$ $(3k-2)(k-2) = 0$	DM1		Correct method to solve a quadratic	Allow $x = x^{\frac{1}{4}}$ as a substitution.
	$k = \frac{2}{3}$ or $k = 2$	A1			**No marks** if straight to quadratic formula to get x = "$\frac{2}{3}$" x = "2" and no further working
	$x = \left(\frac{2}{3}\right)^4$ or $x = 2^4$	M1		Attempt to calculate k^4	**No marks** if $k = x^{\frac{1}{4}}$ then $3k - 8k^2 + 4 = 0$
	$x = \frac{16}{81}$ or $x = 16$	A1	5 **5**		**SC** If **M0** Spotted solutions **www B1** each Justifies 2 solutions exactly **B3**
	<u>If candidates use $k = x^{\frac{1}{2}}$ and rearrange:</u> $3k - 8\sqrt{k} + 4 = 0$ $8\sqrt{k} = 3k + 4$ $64k = 9k^2 + 24k + 16$ $9k^2 - 40k + 16 = 0$	M1*		Substitute, rearrange and square both sides	
	$(9k-4)(k-4)=0$	DM1		Correct method to solve quadratic	
	$k = \frac{4}{9}$ or $k = 4$	A1			
	$x = \left(\frac{4}{9}\right)^2$ or $x = 4^2$	M1		Attempt to calculate k^2	
	$x = \frac{16}{81}$ or $x = 16$	A1			
7 (i)	$-14 \le 6x \le -5$	M1		2 equations or inequalities both dealing with all 3 terms resulting in $a \le 6x \le b$, $a \ne -9$, $b \ne 0$	**Do not ISW** after correct answer if contradictory inequality seen.
	$-\frac{7}{3} \le x \le -\frac{5}{6}$	A1		-14 and -5 seen **www**	
		A1	3	Accept as two separate inequalities provided not linked by "or" (must be ≤)	Allow $-\frac{14}{6} \le x \le -\frac{5}{6}$
(ii)	$0 < x^2 - 4x - 12$	M1		Rearrange to collect all terms on one side	**Do not ISW** after correct answer if contradictory inequality seen.
	$(x-6)(x+2)$	M1		Correct method to find roots	
		A1		6, -2 seen	
		M1		Correct method to solve quadratic inequality i.e. $x >$ their higher root, $x <$ their lower root	
	$x > 6$, $x < -2$		5		
		A1	**8**	(not wrapped, strict inequalities, no 'and')	e.g. for last two marks, $-2 > x > 6$ scores **M1 A0**

8 (i)	$\frac{dy}{dx} = 6x + 6x^{-2}$ $6x + \frac{6}{x^2} = 0$ $x = -1$ $y = 7$	**M1** **A1** **M1** **A1** **A1 ft**	5	Attempt to differentiate (one non-zero term correct) Completely correct Sets their $\frac{dy}{dx} = 0$ Correct value for *x* - **www** Correct value of *y* for *their* value of *x*	**NB** – $x = -1$ (and therefore possibly $y = 7$) can be found from equating the incorrect differential $\frac{dy}{dx} = 6x + 6$ to 0. This could score **M1A0 M1A0A1 ft** If more than one value of x found, allow **A1 ft** for one correct value of y
(ii)	$\frac{d^2y}{dx^2} = 6 - 12x^{-3}$ When $x = -1$, $\frac{d^2y}{dx^2} > 0$ so minimum pt	**M1** **A1 ft**	2 **7**	Correct method e.g. substitutes their x from (i) into their $\frac{d^2y}{dx^2}$ (must involve *x*) and considers sign. **ft** from their $\frac{dy}{dx}$ differentiated correctly and correct substitution of *their* value of x and consistent final conclusion **NB** If second derivate evaluated, it must be correct (18 for $x = -1$). If more than one value of x used, max **M1 A0**	Allow comparing signs of their $\frac{dy}{dx}$ either side of their "– 1", comparing values of y to their "7" **SC** $\frac{d^2y}{dx^2}$ = a constant correctly obtained from their $\frac{dy}{dx}$ and correct conclusion (ft) **B1**

9 (i)	Gradient of $AB = \frac{1-3}{7-1} = -\frac{1}{3}$	**M1***		Uses $\frac{y_2 - y_1}{x_2 - x_1}$ for any 2 points	
	Gradient of $AC = \frac{-9-3}{-3-1} = 3$	**A1**		One correct gradient (may be for gradient of *BC* =1)	
		A1		Gradients for both *AB* and *AC* found correctly	
		M1		Attempts to show that $m_1 \times m_2 = -1$ oe, accept "negative reciprocal"	Do not allow final mark if vertex A found from wrong working. (Dependent on 1st M 1 A1 A1)
	Vertex *A*	**DB1**			Accept BÂC etc for vertex A or "between AB and AC" Allow if marked on diagram.
	OR:				
	Length of $AB = \sqrt{(7-1)^2 + (1-3)^2} = \sqrt{40}$	**M1***		Correct use of Pythagoras, square rooting not needed	
	$AC = \sqrt{(-3-1)^2 + (-9-3)^2} = \sqrt{160}$	**A1**		Any length or length squared correct	
	$BC = \sqrt{(-3-7)^2 + (-9-1)^2} = \sqrt{200}$	**A1**		All three correct	
	Shows that $AB^2 + AC^2 = BC^2$	**M1**	5	Correct use of Pythagoras to show $AB^2 + AC^2 = BC^2$	i.e must add squares of shorter two lengths
	Vertex *A*	**DB1**			
9 (ii)	Midpoint of *BC* is $\left(\frac{7+-3}{2}, \frac{1+-9}{2}\right)$ = (2, -4)	**M1***		Uses $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ o.e. for BC, AB or AC (3 out of 4 subs correct)	<u>Substitution method 1</u> (into $x^2 + y^2 + ax + by + c = 0$) Substitutes all 3 points to get 3 equations in *a*,*b*,*c* **M1** At least 2 equations correct **A1** Correct method to find one variable **M1** One of a, b, c correct **A1** Correct method to find other values **M1** All values correct **A1** Correct equation in required form **A1**
		A1		Correct centre (**cao**)	
	Length of *BC* = $\sqrt{(-3-7)^2 + (-9-1)^2} = \sqrt{200} = 10\sqrt{2}$	**M1****		Correct method to find *d* or *r* or d^2 or r^2 o.e. for BC, AB or AC (must be consistent with their midpoint if found)	<u>Alternative markscheme for last 4 marks with *f,g, c* method</u>: $x^2 - 4x + y^2 + 8y$ for their centre **DM1*** $c = (\pm 2)^2 + 4^2 - 50$ **DM1**** c = – 30 **A1** Correct equation in required form **A1**
	Radius = $5\sqrt{2}$ $(x-2)^2 + (y+4)^2 = (5\sqrt{2})^2$	**DM1***	7	$(x-a)^2 + (y-b)^2$ seen for their centre	<u>Ends of diameter method (*p, q*) to (*c, d*)</u>: Attempts to use $(x-p)(x-c) + (y-q)(y-d) = 0$ for BC,AC or AB **M2** $(x-7)(x+3) + (y-1)(y+9) = 0$ **A2** for both *x* brackets correct, **A2** for both *y* brackets correct $x^2 + y^2 - 4x + 8y - 30 = 0$ **A1** **SC** If **M2 A0 A0** then **B1** if both x brackets correct and **B1** if both y brackets correct for **AC** or **AB**
	$(x-2)^2 + (y+4)^2 = 50$	**DM1****	**12**	$(x-a)^2 + (y-b)^2 =$ their r^2	
	$x^2 + y^2 - 4x + 8y - 30 = 0$	**A1**		Correct equation	
		A1		Correct equation in required form	

					<u>Substitution method 2</u>into $(x-p)^2+(y-q)^2$ = their r^2 Correct method to find d or r or d^2 or r^2 ***M1** Substitutes all 3 points to get 3 equations in p,q **DM1** At least 2 equations correct **A1** Correct method to find one variable **M1** One of p, q correct **A1** Correct equation [$(x-2)^2+(y+4)^2=50$]**A1** Correct equation in required form [$x^2+y^2-4x+8y-30=0$]**A1**
10(i)		**B1** **B1** **B1**	 3	+ve cubic with 3 distinct roots (0, 3) labelled or indicated on y-axis (-3, 0), ($\frac{1}{2}$, 0) and (1, 0) labelled or indicated on x-axis and no other x- intercepts	For first **B1**, left end of curve must finish below x axis and right end must end above x axis. Allow slight wrong curvature at one end but not both ends. No cusp at either turning point. No straight lines drawn with a ruler. Condone (0, 3) as maximum point. To gain second and third **B** marks, there must be an attempt at a curve, not just points on axes. Final **B1** can be awarded for a negative cubic.
(ii)	$2x^2+5x-3$, x^2+2x-3, $2x^2-3x+1$ $(2x^2+5x-3)(x-1)$ $2x^3+3x^2-8x+3$ $\frac{dy}{dx}=6x^2+6x-8$ When $x = 1$, gradient = 4	**B1** **M1** **A1** **M1** **A1** **A1**	 6	Obtain one quadratic factor (can be unsimplified) Attempt to multiply a quadratic by a linear factor Attempt to differentiate (one non-zero term correct) Fully correct expression **www** Confirms gradient = 4 at x = 1 ****AG**	<u>Alternative for first 3 marks:</u> Attempt to expand all 3 brackets with an appropriate number of terms (including an x^3 term) **M1** Expansion with at most 1 incorrect term **A1** Correct, answer (can be unsimplified) **A1** Allow if done in part(i) please check.
(iii)	Gradient of l = 4 On curve, when x = -2, y = 15 $y - 15 = 4(x + 2)$ $y = 4x + 23$	**B1** **B1** **M1** **A1**	 4	May be embedded in equation of line Correct y coordinate Correct equation of line using their values Correct answer **in correct form**	 **M** mark is for any equation of line with any non-zero numerical gradient through (-2, their evaluated y)
(iv)	Attempt to find gradient of curve when x = -2 $6(-2)^2+6(-2)-8=4$ So line is a tangent	**M1** **A1** **A1**	 3 **16**	Substitute x = -2 into their $\frac{dy}{dx}$ Obtain gradient of 4 **CWO** Correct conclusion	<u>Alternatives</u> **1)** Equates equation of l to equation of curve and attempts to divide resulting cubic by $(x + 2)$ **M1** Obtains $(x+2)^2(2x-5)$ (=0) **A1** Concludes repeated root implies tangent at x = -2 **A1** **2)** Equates their gradient function to 4 and uses correct method to solve the resulting quadratic **M1** Obtains $(x + 2)(x - 1) = 0$ oe **A1** Correctly concludes gradient = 4 when x = -2 **A1**

Allocation of method mark for solving a quadratic

e.g. $2x^2 - 5x - 18 = 0$

1) If the candidate attempts to solve by factorisation, their attempt when expanded must produce the **correct quadratic term** and **one other correct term** (with correct sign):

$(2x+2)(x-9)=0$	**M1**	$2x^2$ and -18 obtained from expansion
$(2x+3)(x-4)=0$	**M1**	$2x^2$ and $-5x$ obtained from expansion
$(2x-9)(x-2)=0$	**M0**	only $2x^2$ term correct

2) If the candidate attempts to solve by using the formula

a) If the formula is quoted incorrectly then **M0.**

b) If the formula is quoted correctly then one **sign** slip is permitted. Substituting the wrong numerical value for a or b or c scores **M0**

$\dfrac{-5 \pm \sqrt{(-5)^2 - 4 \times 2 \times -18}}{2 \times 2}$ earns **M1** (minus sign incorrect at start of formula)

$\dfrac{5 \pm \sqrt{(-5)^2 - 4 \times 2 \times 18}}{2 \times 2}$ earns **M1** (18 for *c* instead of −18)

$\dfrac{-5 \pm \sqrt{(-5)^2 - 4 \times 2 \times 18}}{2 \times 2}$ **M0** (2 sign errors: initial sign and *c* incorrect)

$\dfrac{5 \pm \sqrt{(-5)^2 - 4 \times 2 \times -18}}{2 \times -5}$ **M0** (2*b* on the denominator)

Notes – for equations such as $2x^2 - 5x - 18 = 0$, then $b^2 = 5^2$ would be condoned in the discriminant and would not be counted as a sign error. Repeating the sign error for *a* in both occurrences in the formula would be two sign errors and score **M0**.

c) If the formula is not quoted at all, substitution must be completely correct to earn the **M1**

3) If the candidate attempts to complete the square, they must get to the "square root stage" involving ±; we are looking for evidence that the candidate knows a quadratic has two solutions!

$$2x^2 - 5x - 18 = 0$$

$$2\left(x^2 - \frac{5}{2}x\right) - 18 = 0$$

$$2\left[\left(x - \frac{5}{4}\right)^2 - \frac{25}{16}\right] - 18 = 0$$

$$\left(x - \frac{5}{4}\right)^2 = \frac{169}{16}$$

$$x - \frac{5}{4} = \pm\sqrt{\frac{169}{16}}$$

← This is where the **M1** is awarded – arithmetical errors may be condoned provided $x - \frac{5}{4}$ seen or implied

If a candidate makes repeated attempts (e.g. fails to factorise and then tries the formula), mark only what you consider to be their last full attempt.

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RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced Subsidiary GCE

Unit **4721:** Core Mathematics 1

Mark Scheme for January 2012

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Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

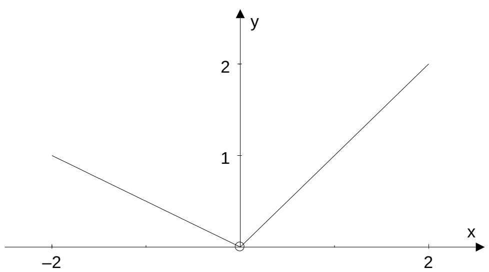
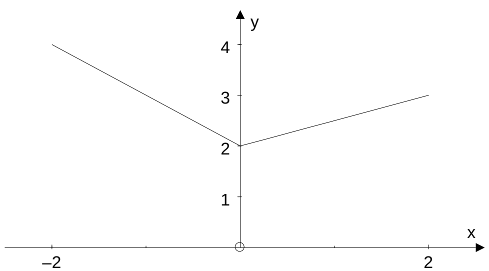
If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

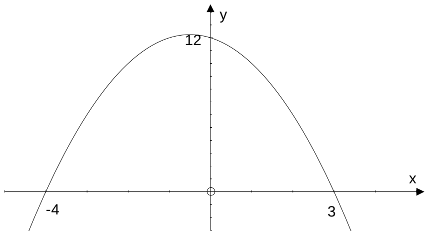
Question			Answer	Marks	Guidance	
1			$\frac{15+\sqrt{3}}{3-\sqrt{3}} \times \frac{3+\sqrt{3}}{3+\sqrt{3}}$	M1	Multiply top and bottom by $\pm(3+\sqrt{3})$	**SC** If A0A0A0 scored, both parts correct but unsimplified **B1** i.e. $\frac{45+15\sqrt{3}+3\sqrt{3}+3}{9+3\sqrt{3}-3\sqrt{3}-3}$ o.e.
			$=\frac{48+18\sqrt{3}}{9-3}$	A1	Numerator correct and simplified	<u>Alternative method:</u> Equates expression to $a+b\sqrt{3}$ and forms simultaneous equations in a and b **M1**
				A1	Denominator correct and simplified to 6	Correct method to solve simultaneous equations **M1**
			$=8+3\sqrt{3}$	A1	cao	a = 8 found **A1**
				[4]		b = 3 found **A1**
2	(i)			M1 A1 **[2]**	Reflection of given graph in either axis Correct reflection in y-axis	Clear intention to show (-2, 1), (0,0), (2,2) by numbers, dashes or co-ordinates **A0** If significantly short or long
2	(ii)			M1 A1 **[2]**	Translation of **given** graph vertically (up or down) Correct translation of two units vertically	Clear intention to show (-2, 4), (0,2), (2,3) by numbers, dashes or co-ordinates **A0** If significantly short or long

Question			Answer	Marks	Guidance	
3			$5x^2 + px - 8 = 5(x-1)^2 + r$ $= 5(x^2 - 2x + 1) + r$ $= 5x^2 - 10x + 5 + r$ $p = -10$ $r = -13$	B1 B1 M1 A1 **[4]**	$q = 5$ (may be embedded on RHS) $p = -10$ $-8 = \pm q + r$ or $\frac{-p^2}{20} - 8 = r$ $r = -13$	Allow from $p = 10$
4	(i)		$\frac{1}{9}$	B1 **[1]**		
4	(ii)		$\left(\sqrt[4]{16}\right)^3$ $= 8$	M1 A1 **[2]**	Interprets the power $\frac{3}{4}$ correctly ± 8 is **A0**	$(\sqrt[4]{16})^3$ or $(\sqrt[4]{16^3})$ or $\left(16^{\frac{1}{4}}\right)^3$ or $\left(16^3\right)^{\frac{1}{4}}$
4	(iii)		$5\sqrt{8} \div \sqrt{8}$ $= 5$	M1 A1 **[2]**	$\sqrt{100}\ \sqrt{2} \div \sqrt{4}\ \sqrt{2}$ or $\sqrt{\frac{200}{8}}$ or $\sqrt{25}\ \sqrt{8} \div \sqrt{8}$ or $\sqrt{1600} \div 8$ soi Condone ± 5	

Question			Answer	Marks	Guidance	
5			$k = \frac{1}{y^2}$	M1*	Use a correct substitution or pair of substitutions to obtain a quadratic or factorise into 2 brackets each containing $\frac{1}{y^2}$	**No marks** if straight to quadratic formula to get y = "$-\frac{2}{3}$", y = "4" unless correct substitution applied later i.e. reciprocal and square root
			$3k^2 - 10k - 8 = 0$ $(3k+2)(k-4) = 0$	M1dep	Correct method to solve a quadratic	
			$k = -\frac{2}{3}$ or $k = 4$	A1	k = 4 from correct method. If other root stated it must be correct.	**No marks** if quadratic found from incorrect substitution
			$y^2 = -\frac{3}{2}$ or $y^2 = \frac{1}{4}$	M1	Attempt to reciprocal and square root to obtain y (either term)	**SC** If **M0** Spotted solutions **www B1** each Justifies 2 solutions exactly **B3**
			$y = \pm\frac{1}{2}$	A1	No other roots given. Must be from k = 4 from correct method.	
				[5]		
			Alternative method below: $3 - 10y^2 - 8y^4 = 0$ $k = y^2$ $8k^2 + 10k - 3 = 0$ M1* $(4k-1)(2k+3) = 0$ M1 dep			
			$k = \frac{1}{4}$ or $k = -\frac{3}{2}$ A1		$k = \frac{1}{4}$ from correct method. If other root stated it must be correct.	
			$y = \pm\frac{1}{2}$ M1 A1			

Question			Answer	Marks	Guidance	
6	(i)		$f'(x) = -4x^{-2} - 3$	M1 A1 A1 **[3]**	Attempt to differentiate $-4x^{-2}$ Fully correct derivative (no "+ *c*")	kx^{-2} or -3 correctly obtained
6	(ii)		$f''(x) = 8x^{-3}$ $f''\left(\frac{1}{2}\right) = \frac{8}{\left(\frac{1}{2}\right)^3}$ = 64	M1* A1 M1dep A1 **[4]**	Attempts to differentiate their **(i)** Correct derivative Substitutes $x = \frac{1}{2}$ correctly into their $f''(x)$ e.g. $8\left(\frac{1}{2}\right)^{-3}$ (allow "invisible brackets") **www**	Must involve reducing power of an *x* term by 1 $f''(x)$ must involve *x*.
7	(i)		$x^3 - 3x^2 + 5x + 2x^2 - 6x + 10$ $= x^3 - x^2 - x + 10$ $\frac{dy}{dx} = 3x^2 - 2x - 1$ $(3x + 1)(x - 1) = 0$ $x = -\frac{1}{3}$ or $x = 1$ $\frac{d^2y}{dx^2} = 6x - 2$, $x = 1$ gives +ve (4) Min point at $x = 1$ $y = 9$ found	M1 M1 M1* M1 A1 M1dep A1 A1 **[8]**	Attempt to multiply out brackets Attempt to differentiate their cubic Sets their $\frac{dy}{dx} = 0$ Correct method to solve quadratic Correct *x* values of turning points found **www** Valid method to establish which is min point with a conclusion Correct conclusion for $x = 1$ found from correct factorisation (even if other root incorrect) **www** for (1, 9) given as minimum point (ignore other point here)	<u>Alternative for product rule</u> Attempt to use product rule **M1** Expand brackets of both parts **M1** Then as main scheme Any extra values for turning points loses all three **A** marks (eg by sketching positive cubic, second diff method for either of their *x* values, *y* co-ords etc.) If constant incorrect in initial expansion, max **5/8**

Question			Answer	Marks	Guidance	
7	(ii)		$(-3)^2 - 4 \times 1 \times 5$ $= -11$	M1 A1 **[2]**	Uses $b^2 - 4ac$	$\sqrt{b^2 - 4ac}$ is **M0**
7	(iii)			B2 **[2]**	Fully correct argument - no extra incorrect statements e.g. 1) Justifying the quadratic factor having no roots so only intersection with x-axis is at $x = -2$ and stating it's a positive cubic 2) Sketch of positive cubic with one root at (–2, 0) and a min point at (1, 9) (f/t positive $y(1)$ from (i))	Award **B1** for either of: 1) Justifying the quadratic factor having no roots so only intersection with x-axis is at $x = -2$ 2) Sketch of positive cubic with one root at (–2, 0) and a min point with y coordinate positive or 0
8			B lies on l so has coordinates $(x, 11 - 2x)$ $(x-3)^2 + (11-2x-5)^2 = \left(6\sqrt{5}\right)^2$ $5x^2 - 30x - 135 = 0$ $5(x+3)(x-9) = 0$ $x = -3,\ x = 9$ $y = 17,\ y = -7$	M1 M1 M1* M1dep A1 A1 **[6]**	Attempt to find equation of l with gradient -2 $(x-3)^2 + (y - 5)^2 = (6\sqrt{5})^2$ o.e. seen Attempts to solve the equations simultaneously to get a quadratic Correct method to solve their quadratic Both x values Both y values	e.g. by substitution as shown **SC** If A0 A0, one correct pair of values from correct factorisation **www B1**
			Alternative method: Use of (1, 2, $\sqrt{5}$) triangle with –ve gradient M1 Scaling to $6\sqrt{5}$ M1 (3, 5) + (6, -12) M1 (9, -7) A1 (3, 5) - (6, -12) M1 (-3, 17) A1		**SC** Spotted solutions Each correct pair **www B1** (May also earn first two Ms as in main scheme)* **-1** for one or two extra incorrect solutions **-2** for three or more extra incorrect solutions Checks solutions and justifies only two solutions **B2** *** NB –** First M1 may also be awarded for establishing gradient between (3,5) and their solution(s) is **–** 2	

Question			Answer	Marks	Guidance	
9	(i)		$(x-3)(x+4)=0$ $x=3$ or $x=-4$ 	M1 A1 B1 B1 B1 **[5]**	Correct method to find roots Correct roots Negative quadratic curve y intercept (0, 12) Good curve, with correct roots 3 and –4 indicated and max point in 2nd quadrant	 i.e. max at (0, 12) **B0** Curve must go below x-axis for final mark
9	(ii)		$-4 < x < 3$	M1 A1 **[2]**	Correct method to solve quadratic inequality Allow $\le$ for the method mark but not the accuracy mark	their lower root $< x <$ their higher root Allow "$x > -4, x < 3$" Allow "$x > -4$ and $x < 3$" Do not allow "$x > -4$ **or** $x < 3$"
9	(iii)		$y = 4-3x$ $12 - x - x^2 = 4 - 3x$ $x^2 - 2x - 8 = 0$ $(x-4)(x+2)=0$ $x=4$ or $x=-2$ $y=-8$ or $y=10$	 M1 A1 M1 A1 A1 **[5]**	 substitute for x/y or attempt to get an equation in 1 variable only obtain correct 3 term quadratic correct method to solve 3 term quadratic	 e.g. for first mark $3x + 12 - x - x^2 = 4$, or $y = 12 - \left(\frac{4-y}{3}\right) - \left(\frac{4-y}{3}\right)^2$ (this leads to $y^2 - 2y - 80 = 0$). Condone poor algebra for this mark. **SC** If **A0 A0**, give **B1** for one correct pair of values spotted or from correct factorisation **www**

Question			Answer	Marks	Guidance	
10	(i)		$(x+2)^2+(y-4)^2=25$	M1	$(x+2)^2$ and $(y-4)^2$ seen (or implied by x^2+4x+y^2-8y)	Alternative markscheme for *f, g, c* method:
			$x^2+4x+4+y^2-8y+16-25=0$	M1	$(x\pm 2)^2+(y\pm 4)^2 = 25$	x^2+4x+y^2-8y **B1**
			$x^2+y^2+4x-8y-5=0$	A1 **[3]**	Correct equation in correct form (terms can be in any order but must have "=0")	$c = 2^2 + (\pm 4)^2 - 25$ **M1** Correct equation in correct form **A1**
10	(ii)		gradient of radius = $\frac{8-4}{-5+2}$	M1	uses $\frac{y_2-y_1}{x_2-x_1}$ (3/4 substitutions correct)	
			$= -\frac{4}{3}$	A1	Allow $\frac{4}{-3}$	
			gradient of tangent = $\frac{3}{4}$	B1FT		
			$y-8=\frac{3}{4}(x+5)$	M1	correct equation of straight line through (– 5, 8), any non-zero gradient	
			$3x-4y+47=0$	A1 **[5]**	Shows rearrangement to given equation **AG** **CWO** throughout for A1	
			Alternative by rearrangement Gradient of radius = $\frac{8-4}{-5+2}$ = $\frac{-4}{3}$ **M1* A1** Attempts to rearrange equation of line to find gradient of line = $\frac{3}{4}$ **M1dep** Multiply gradients to get -1 **B1** Check (-5, 8) lies on line **B1 (dep on both M1s)**		Alternative for equating given line to circle Substitute for *x*/*y* or attempt to get an equation in 1 variable only **M1** $k(x^2 + 10x + 25) = 0$ or $k(y^2 - 16y + 64) = 0$ **A1** Correct method to solve quadratic **M1** x = -5, y = 8 found **A1** States one root implies tangent **B1**	Alternative markscheme for implicit differentiation: **M1** Attempt at implicit diff as evidenced by $2y\frac{dy}{dx}$ term **A1ft** $2x+2y\frac{dy}{dx}+4-8\frac{dy}{dx}=0$ ft from their equation in (i) **A1** Substitution of (-5, 8) to obtain $\frac{3}{4}$ then final 2 marks as main scheme

Question			Answer	Marks	Guidance	
10	(iii)		(3 × 3) – (4 × 14) + 47 = 0	B1 **[1]**	Sufficient correct working to verify statement e.g. verifying co-ordinate as shown	Alt: showing line joining (-5, 8) to (3, 14) has same gradient etc.
10	(iv)		$\sqrt{(3-{}^{-}5)^2+(14-8)^2}$ $=10$ Area of triangle $=\frac{1}{2}\times 10\times 5$ $= 25$	M1 A1 M1 A1 **[4]**	Use of $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ for *TP* Must use their *TP* and their *CP*	Alternative method: Attempt to find area of enclosing rectangle and subtract areas of other three triangles **M1*** Correct use area of triangle formula **M1 dep** All four values correct **A1** Final answer correct **A1** (Use the same principle for any enclosing shape)

Solving a quadratic

This is particularly important to mark correctly as it can sometimes feature several times on a single examination paper. An example is usually included with the markscheme each session; this has varied slightly over the years and should be referred to every session. Consider the equation $3x^2 - 10x - 8 = 0$.

1) If the candidate attempts to solve by factorisation, their attempt when expanded must produce the **correct quadratic term** and **one other correct term** (with correct sign):

$(3x+1)(x-8) = 0$	**M1**	$3x^2$ and -8 obtained from expansion
$(3x-1)(x-3) = 0$	**M1**	$3x^2$ and $-10x$ obtained from expansion
$(3x-2)(x-4) = 0$	**M0**	only $3x^2$ term correct

2) If the candidate attempts to solve by using the formula

a) If the formula is quoted incorrectly then **M0**.
b) If the formula is quoted correctly then one **sign** slip is permitted. Substituting the wrong numerical value for *a* or *b* or *c* scores **M0**.

$\dfrac{-10 \pm \sqrt{(-10)^2 - 4\times3\times-8}}{6}$	earns **M1** (minus sign incorrect at start of formula)
$\dfrac{10 \pm \sqrt{(-10)^2 - 4\times3\times-8}}{2\times3}$	earns **M1** (8 for *c* instead of −8)
$\dfrac{-10 \pm \sqrt{(-10)^2 - 4\times3\times8}}{6}$	**M0** (2 sign errors: initial sign and *c* incorrect)
$\dfrac{10 \pm \sqrt{(-10)^2 - 4\times3\times-8}}{2\times-10}$	**M0** (2*b* on the denominator)

Notes – for equations such as $3x^2 - 10x - 8 = 0$, then $b^2 = 10^2$ would be condoned in the discriminant and would not be counted as a sign error. Repeating the sign error for *a* both occurrences in the formula would be two sign errors and score **M0**.

c) If the formula is not quoted at all, substitution must be completely correct to earn the **M1**

3) If the candidate attempts to complete the square, they must get to the “square root stage” involving ±; we are looking for evidence that the candidate knows a quadratic has two solutions.

$$3x^2 - 10x - 8 = 0$$

$$3\left(x^2 - \frac{10}{3}x\right) - 8 = 0$$

$$3\left[\left(x - \frac{5}{3}\right)^2 - \frac{25}{9}\right] - 8 = 0$$

$$\left(x - \frac{5}{3}\right)^2 = \frac{49}{9}$$

$$x - \frac{5}{3} = \pm\sqrt{\frac{49}{9}}$$

← This is where the **M1** is awarded – arithmetical errors may be condoned provided $x - \frac{5}{3}$ (or equivalent) seen or implied

If a candidate makes repeated attempts (e.g. fails to factorise and then tries the formula), mark only what you consider to be their last full attempt – see guidance later in this document.

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

Education and Learning
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

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RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced Subsidiary GCE

Unit **4721:** Core Mathematics 1

Mark Scheme for June 2012

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, OCR Nationals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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1. **Annotations and abbreviations**

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	

Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

2. **Subject-specific Marking Instructions for GCE Mathematics Pure strand**

a. Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c. The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep *’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g. Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

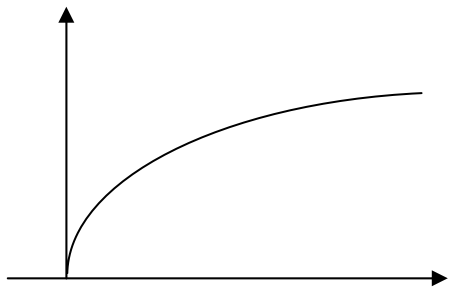
If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h. For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1			$x^3-5x^2+3x-15-(x^2+4x-x-4)$	M1	Attempt to expand both pairs of brackets	No more than one "missing term"
				A1	Expansion with at most one incorrect term (no missing terms)	**Do not allow** "invisible brackets" unless final answer correct Allow one simplified incorrect term e.g. (x^2+5x-4)
			$=x^3-6x^2-11$	A1 **[3]**	cao	
2	(i)		$\sqrt[4]{7}=7^{\frac{1}{4}}$	B1 **[1]**	Allow $7^{0.25}$, k = 0.25 etc.	
2	(ii)		$\frac{1}{7\sqrt{7}}=7^{-\frac{3}{2}}$	M1	**Clear evidence** of **correct use** of $7^a\times7^b=7^{a+b}$ or a single term $\frac{1}{7^d}=7^{-d}$	Allow $\frac{1}{7^d7^e}=(7^d7^e)^{-1}$ [not $=7^d7^{-e}$]
				A1 **[2]**	Allow -1.5, k = -1.5 etc.	
2	(iii)		$7^4\times7^{20}$ $=7^{24}$	M1 A1 **[2]**	7^{20} or 49^2 seen (or 49^{12}) Allow k = 24	$(7^2)^{10}$ is **not** good enough for M1
3	(i)		$\frac{3}{5}$	B1 **[1]**	Allow 0.6 or any equivalent fraction	Do not allow $\frac{3}{5}x$ as final answer
3	(ii)		$P\left(\frac{20}{3},0\right)$	B1	May be implied by subsequent working	Allow $x=\frac{20}{3}$ for P
			Q (0, -4)	B1	May be implied	Allow y = - 4 for Q
			$\left(\frac{\frac{20}{3}+0}{2},\frac{0+{}^{-}4}{2}\right)$	M1	Correct method to find midpoint of line	Check formula, or if formula not seen, the use of formula is correct (including correct signs) for both x and y, Can be implied by correct final answers
			$\left(\frac{10}{3},-2\right)$	A1 **[4]**	Allow exact equivalent forms, decimals must be correct to at least 2dp	**SC** If P and Q given the wrong way round but then used correctly to obtain correct final answer **B2**

Question			Answer	Marks	Guidance	
4	(i)		$2\left(x^2-10x\right)+49$ $=2\left(x-5\right)^2-50+49$ $=2\left(x-5\right)^2-1$	B1 B1 M1 A1 **[4]**	$p=2$ $\left(x-5\right)^2$ $49-2q^2$ or $\frac{49}{2}-q^2$	If p, q, r found correctly, then **ISW** slips in format. $2(x-5)^2+1$ **B1 B1 M0 A0** $2(x-5)-1$ **B1 B1 M1 A1 (BOD)** $2(x-5x)^2-1$ **B1 B0 M1 A0** $2(x^2-5)^2-1$ **B1 B0 M1 A0** $2(x+5)^2-1$ **B1 B0 M1 A1 (BOD)** $2x(x-5)^2-1$ **B0 B1M1A1**
4	(ii)		(5, –1)	B1 FT B1 FT **[2]**	ft their q (Do not allow “5x”) ft their r (Do not allow “-1y”)	If restarted then **B1 B1** for each **B0** if more than one answer given
5	(i)			M1 A1 **[2]**	Correct shape of graph in Q1 Ignore reflection in the x axis Correct graph in Q1 only	Ignore “feathering” Finite “plot” scores **M0** Need not meet origin for **M** mark Allow slight curve downwards for **M** mark but not for **A** Allow tending to horizontal
5	(ii)		Translate(d) or Translation Parallel to x-axis, (+)4 units	B1 B1 **[2]**	**Do not accept** “shift”, “move” etc. without the word translation/translate(d) For “parallel to the x axis” allow “horizontally”, “across”, “to the right”, “in the (positive) x direction”. **Do not accept** “in/on/across/up/along/to/towards the x axis”	Allow e.g. “4 units across in the positive x direction parallel to the x axis” but do not award second B1 if statements are contradictory. “Factor 4” not acceptable
5	(iii)		$y=\sqrt{\left(\frac{x}{5}\right)}$	M1 A1 **[2]**	$\sqrt{5x}$ or $\sqrt{\frac{x}{5}}$ seen **Must have** “$y=$ ” to earn A mark (do not allow “$f(x)=$”)	**SC** If doubt over whether use of square root/solidus is totally correct **B1** (Must still have “$y=$ ”) Allow $\sqrt{5}y=\sqrt{x}$ or equivalent

Question			Answer	Marks	Guidance	
6			$\frac{dy}{dx} = -12x^{-3}$	M1 A1	Attempt to differentiate (i.e. kx^{-3} seen) Correct derivative	"+ C" is A0
			When $x = 2$, $\frac{dy}{dx} = -\frac{3}{2}$	A1	Correct value of $\frac{dy}{dx}$. Allow equivalent fractions.	
			Gradient of normal = $\frac{2}{3}$	B1 FT	Follow through their **evaluated** $\frac{dy}{dx}$	Must be processed correctly
			When $x = 2$, $y = -\frac{7}{2}$	B1	Correct y coordinate, accept equivalent forms	
			$y + \frac{7}{2} = \frac{2}{3}(x - 2)$	M1	Correct equation of straight line through (2, their evaluated y), any non-zero gradient	
			$4x - 6y - 29 = 0$	A1 **[7]**	Correct equation in required form i.e. $k(4x - 6y - 29) = 0$ for integer k. **Must have** "=0".	
7			$k = x^{\frac{1}{2}}$ $k^2 - 6k + 2 = 0$	M1*	Use a substitution to obtain a quadratic with k^2, $6k$ and 2(may be implied by squaring or rooting later)	**Any sight of 4 or 36x from "squaring" original equation scores 0/6.** Alternative solution: $6\sqrt{x} = x + 2$ $36x = x^2 + 4x + 4$ Rearrange and square both sides **M1*** Correct simplified quadratic $x^2 - 32x + 4 = 0$ **A1** Method to solve quadratic **M1dep** Correct unsimplified expression **A1** Correct discriminant **A1** $16 \pm 6\sqrt{7}$ o.e. **A1** **SC** If no evidence of substitution at start and no squaring/rooting at end: Correct method for solving quadratic with $a = 1$, $b = -6$, $c = 2$ and solution simplified to $3 \pm \sqrt{7}$ **B1**
			$(k - 3)^2 - 7 = 0$	M1 dep	Correct method to solve resulting quadratic	
			$k = 3 \pm \sqrt{7}$	A1	$k = \mathbf{3 \pm \sqrt{7}}$ **or** $k = \frac{6 \pm \sqrt{28}}{2}$ **or** $k = 3 \pm \frac{\sqrt{28}}{2}$	
			$x = (3 \pm \sqrt{7})^2$	M1 M1	Recognise the need to square to obtain x Correct method for squaring $a + \sqrt{b}$ (3 or 4 term expansion)	
			$x = 16 + 6\sqrt{7}$ or $x = 16 - 6\sqrt{7}$	A1 **[6]**	Allow $16 \pm 3\sqrt{28}$ or $16 \pm 2\sqrt{63}$	

Question			Answer	Marks	Guidance	
8	(i)		$\frac{dy}{dx} = 4x^3 + 32$ $4x^3 + 32 = 0$ $x = -2$ $y = -48$	M1 A1 M1 A1 A1 FT **[5]**	Attempt to differentiate (one term correct) Completely correct Sets their $\frac{dy}{dx} = 0$ (can be implied) Correct value for x (**not** ±2) **www** Correct value of y for *their* single non-zero value of x	"+ C" is A0 e.g. (2, 80), (4, 384), (− 4, 128), (8, 4352), (− 8, 3840)
8	(ii)		$\frac{d^2y}{dx^2} = 12x^2$ When $x = -2$, $\frac{d^2y}{dx^2} > 0$ so minimum pt	M1 A1 **[2]**	Correct method for determining nature of a stationary point – see right hand column Fully correct for $x = -2$ only	e.g. evaluating second derivate at x = "−2" and stating a conclusion Evaluating $\frac{dy}{dx}$ either side of x = "−2" Evaluating y either side of x = "−2"
8	(iii)		$x > -2$	B1 **FT** **[1]**	ft from single x value in **(i)** consistent with **(ii)**	**Do not accept** $x \geq -2$
9	(i)		Area of tile = $4x(x + 3)$ $4x(x + 3) < 112$ $4x^2 + 12x - 112 < 0$ $4(x + 7)(x - 4) < 0$ $-7 < x < 4$ $\therefore\ 0 < x < 4$	B1 B1 √ M1 M1 A1 A1 **[6]**	Correct expression for area of rectangle (may be unsimplified) Correct inequality for their expression Correct method to solve a three term quadratic Chooses correct region for the quadratic inequality i.e. lower root < x < higher root (May be implied by correct final answer) Restricts range to positive values of x **CWO**	Correct alternative forms for factorised inequality include: $(x + 7)(4x - 16) < 0$ $(4x + 28)(x - 4) < 0$ $(2x + 14)(2x - 8) < 0$ etc. **Do not allow ≤ for final A mark**
9	(ii)		Perimeter = $4y + (y + 3) + 2y + y + 2y + 3$ $20 < 10y + 6 < 54$ $1.4 < y < 4.8$	M1 A1 B1 FT M1 A1 **[5]**	**Clear** attempt to add lengths of all **6** edges Correct perimeter simplified to $10y + 6$ seen Correct inequalities for their expression Solving 2 linear equations or inequalities dealing with all 3 terms Accept "$1.4 < y$, $y < 4.8$", "$1.4 < y$ ***and*** $y < 4.8$" but **NOT** "$1.4 < y$ ***or*** $y < 4.8$".	**Allow < or ≤ throughout part (ii)** Can still be unsimplified here **Do not ISW** if contradictory incorrect form follows correct answer

Question			Answer	Marks	Guidance	
10	(i)		Centre (5, –2) Radius = 5 Diameter = 10	B1 M1 A1 **[3]**	 5 or $\sqrt{25}$ soi	
10	(ii)		Gradient of line = $\frac{2-^-2}{7-5}(=2)$ $y-2=2(x-7)$ or y–⁻2 = 2 (*x*–5) $y=2x-12$	M1 A1 M1 A1 **[4]**	uses $\frac{y_2-y_1}{x_2-x_1}$ with their centre correct equation of straight line through (7, 2) or their centre, any non-zero gradient o.e. 3 term equation	3/4 substitutions correct Allow other points on the line e.g. mid-point is (6,0)
10	(iii)		$\sqrt{(7-5)^2+(2-^-2)^2}$ $=\sqrt{20}$ $\sqrt{20}<5$ so *P* lies inside the circle	M1 A1 B1 FT **[3]**	Use of $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ with their centre Compares their length *CP* with their radius and states consistent conclusion. **Both lengths must be mentioned.**	3/4 substitutions correct**. Must have** square root as length specifically asked for. **SC** If **M0**, award for **B1** for finding $CP^2 = 20$ and stating 20 < 25 and concluding inside **www**
10	(iv)		$(x-5)^2+(2x+2)^2(=25)$ $(x-5)^2+(2x+2)^2=25$ $x^2-10x+25+4x^2+8x+4=25$ $5x^2-2x+4=0$ $b^2-4ac=4-(4\times5\times4)$ $b^2-4ac<0$ so no real roots	M1* A1 A1 M1dep A1 **[5]**	Substitute for *x*/*y* or attempt to eliminate one of the variables Correct unsimplified equation (= 0 can be implied) Obtain correct 3 term quadratic Attempt to determine whether equation has real roots with consistent conclusion regarding roots/intersection Fully justified statement that line and circle do not meet **www**	If *x* eliminated, $5y^2-4y+16=0$ If the discriminant is evaluated, this must be –76 (from the quadratic in *x*) or –304 (from the quadratic in *y*) for full marks.

Allocation of method mark for solving a quadratic

e.g. $2x^2 - 5x - 18 = 0$

1) If the candidate attempts to solve by factorisation, their attempt when expanded must produce the **correct quadratic term** and **one other correct term** (with correct sign):

$(2x+2)(x-9)=0$	**M1**	$2x^2$ and -18 obtained from expansion
$(2x+3)(x-4)=0$	**M1**	$2x^2$ and $-5x$ obtained from expansion
$(2x-9)(x-2)=0$	**M0**	only $2x^2$ term correct

2) If the candidate attempts to solve by using the formula

a) If the formula is quoted incorrectly then **M0**.

b) If the formula is quoted correctly then one **sign** slip is permitted. Substituting the wrong numerical value for a or b or c scores **M0**

$\dfrac{-5 \pm \sqrt{(-5)^2 - 4\times 2\times -18}}{2\times 2}$ earns **M1** (minus sign incorrect at start of formula)

$\dfrac{5 \pm \sqrt{(-5)^2 - 4\times 2\times 18}}{2\times 2}$ earns **M1** (18 for *c* instead of −18)

$\dfrac{-5 \pm \sqrt{(-5)^2 - 4\times 2\times 18}}{2\times 2}$ **M0** (2 sign errors: initial sign and *c* incorrect)

$\dfrac{5 \pm \sqrt{(-5)^2 - 4\times 2\times -18}}{2\times -5}$ **M0** (2*b* on the denominator)

Notes – for equations such as $2x^2 - 5x - 18 = 0$, then $b^2 = 5^2$ would be condoned in the discriminant and would not be counted as a sign error. Repeating the sign error for *a* in both occurrences in the formula would be two sign errors and score **M0**.

c) If the formula is not quoted at all, substitution must be completely correct to earn the **M1**

3) If the candidate attempts to complete the square, they must get to the "square root stage" involving ±; we are looking for evidence that the candidate knows a quadratic has two solutions!

$$2x^2 - 5x - 18 = 0$$

$$2\left(x^2 - \frac{5}{2}x\right) - 18 = 0$$

$$2\left[\left(x - \frac{5}{4}\right)^2 - \frac{25}{16}\right] - 18 = 0$$

$$\left(x - \frac{5}{4}\right)^2 = \frac{169}{16}$$

$$x - \frac{5}{4} = \pm\sqrt{\frac{169}{16}}$$

← This is where the **M1** is awarded – arithmetical errors may be condoned provided $x - \frac{5}{4}$ seen or implied

If a candidate makes repeated attempts (e.g. fails to factorise and then tries the formula), mark only what you consider to be their last full attempt.

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CB1 2EU

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Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

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GCE

Mathematics

Advanced Subsidiary GCE

Unit **4721:** Core Mathematics 1

Mark Scheme for January 2013

Oxford Cambridge and RSA Examinations

Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	

Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*/DM1	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
ft or √	Follow through

Subject-specific Marking Instructions for GCE Mathematics Pure strand

a. Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c. The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.
Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (eg 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g. Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

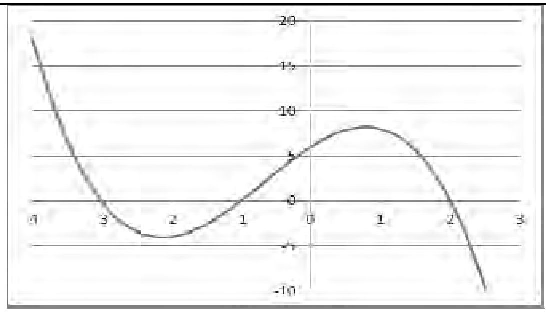
If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h. For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1	(i)		$\dfrac{6 \pm \sqrt{(-6)^2 - 4\times1\times-2}}{2\times1}$	M1	Valid attempt to use quadratic formula	**No marks for attempting to factorise**
			$= \dfrac{6 \pm \sqrt{44}}{2}$	A1		
			$= 3 \pm \sqrt{11}$	A1	Both roots correct and simplified	
			OR: $(x-3)^2 - 9 - 2 = 0$ $x - 3 = \pm\sqrt{11}$	M1 A1	Correct method to complete square	Must get to $(x - 3)$ and ± stage for the M mark, constants combined correctly gets A1
			$x = 3 \pm \sqrt{11}$	A1 **[3]**	Rearranged to correct form **cao**	
1	(ii)		$\dfrac{dy}{dx} = 2x - 6$	B1		
			$= -16$	B1 **[2]**	**www**	
2	(i)		$n = 0$	**B1** **[1]**	Allow 3^0	
2	(ii)		$\dfrac{1}{t^3} = 64$ (or 4^3)	M1	or $t^3 = \dfrac{1}{64}$ or $64t^3 = 1$ or $\left(\dfrac{1}{t}\right)^3 = 64$	Allow embedded
			$t = \dfrac{1}{4}$	A1 **[2]**	4^{-1} is **A0** $t = \pm\dfrac{1}{4}$ is **A0**	4^{-1} **www** alone implies **M1 A0**
2	(iii)		$2p^2 = 8$	M1	or $8p^6 = 8^3$. Allow $2p^{\frac{6}{3}} = 8$ for **M1**	If not 512, evidence of 8 × 8 × 8 needed.
			$p = 2$	A1	**www**	**SC** Spotted **B1** for 2, **B1** for -2, **B1** for justifying exactly 2 solutions
			or $p = -2$	A1 **[3]**	**www**	**SC** $8p^2 = 8$, $p = \pm 1$ **B1**

Question			Answer	Marks	Guidance	
3	(i)			B1 B1 B1 **[3]**	-ve cubic with 3 distinct roots (0, 6) labelled or indicated on y-axis – seen elsewhere not enough (-3, 0), (-1, 0) and (2, 0) labelled or indicated on x-axis and no other x-intercepts.	Must not stop at x-axis. Condone errors in curvature at the extremes unless extra turning point(s)/root(s) clearly implied. **Must have a curve for 2nd and 3rd marks** Do not allow final B1 if shown as repeated root(s)
3	(ii)		Reflection in the y axis	B1 B1 **[2]**	Not mirrored/flipped etc. or $x = 0$. No/through/along etc. Must be "in". Cannot get 2nd B1 without some indication of a reflection e.g. flip etc. Do not **ISW** if contradictory statement seen	**Alt** Stretch (scale) factor –1 **B1** parallel to the x axis for **B1** **Must be a single transformation for any marks**
4	(i)		$2x^2 - 3x - 5 = \dfrac{-10x-11}{2}$ $4x^2 + 4x + 1 = 0$ $(2x + 1)(2x + 1) = 0$ $x = -\dfrac{1}{2}$ $y = -3$	*M1 A1 DM1 A1 A1 **[5]**	Substitute for x/y or attempt to get an equation in 1 variable only Obtain correct 3 term quadratic – could be a multiple e.g. $2x^2 + 2x + 0.5 = 0$ Correct method to solve resulting 3 term quadratic	or $10x + 2(2x^2 - 3x - 5) + 11 = 0$ If x is eliminated, expect $k(8y^2 + 48y + 72) = 0$ **SC** If DM0 and $x = -\dfrac{1}{2}$ spotted **B1** for x value, **B1** for y value **B1** justifying only one root
4	(ii)		Line is a tangent to the curve	B1√ **[1]**	**Must be consistent with their answers to their quadratic in (i).** **1 repeated root –** indicates one point. Accept tangent, meet at, intersect, touch etc. but do not accept cross **2 roots –** indicates meet at two points **0 roots –** indicates do not meet. Do not accept "do not cross"	**Follow through from their solution to (i)**

Question			Answer	Marks	Guidance	
5	**(i)**		$5x^2 + 17x - 12 - 3(x^2 - 4x + 4)$	M1	Attempt to expand both pairs of brackets	
				A1	$5x^2 + 17x - 12$ **and** $x^2 - 4x + 4$ soi ; may be unsimplified, no more than one incorrect term, no "extra" terms at all. No "invisible brackets"	
			$= 2x^2 + 29x - 24$	A1	$2x^2 + 29x - 24$	**ISW** if they then put expression equal to zero and go on to "solve"
				[3]		
5	**(ii)**		$-5x^2 + 2kx^2 + 6x^2$	M1	Correct method to multiply out 3 brackets or correctly identify all x^2 terms	No more than 8 terms, but ignore sign errors/accuracy of non x^2 terms
				A1	All x^2 terms correct, no extras	
			$k = -2$	A1		
				[3]		

Question			Answer	Marks	Guidance	
6	(i)		$\frac{p-7}{-4-{}^{-}2}$ or $\frac{7-p}{-2-{}^{-}4}$	M1	uses $\frac{y_2-y_1}{x_2-x_1}$ (at least 3out of 4 correct)	**Alternative method:** Equation of line through one of the given points with gradient 4 **M1** Substitutes **other point** into their equation **M1**
			$\frac{p-7}{-4-{}^{-}2}=4$ or $\frac{7-p}{-2-{}^{-}4}=4$	A1	Correct, unsimplified **equation**	Obtains $p = -1$ (Accept $y = -1$)**A1**
			$p = -1$	A1 **[3]**		**Note:** Other "informal" methods can score full marks provided **www**
6	(ii)		$\frac{{}^{-}2+6}{2}=m,\quad \frac{7+q}{2}=5$	M1	Correct method (may be implied by one correct coordinate)	Use the same marking principle for candidates who add/subtract half the difference to an end point or use similar triangles or other valid "informal" methods.
			$m = 2$	A1		
			$q = 3$	A1 **[3]**		
6	(iii)		$\sqrt{(-2-d)^2+(7-3)^2}$	*M1	Correct method to find line length/square of line length using Pythagoras' theorem (at least 3out of 4 correct)	
			$d^2 + 4d + 20 = 52$	B1	$\left(2\sqrt{13}\right)^2 = 52$ or $2\sqrt{13}=\sqrt{52}$	
			$d^2 + 4d - 32 = 0$ $(d + 8)(d - 4) = 0$	DM1	Correct method to solve 3 term quadratic, must involve their "52"	**SC: B1** for each value of d found or "spotted" from correct working
			$d = -8$ or 4	A1 **[4]**		**Note:** Other "informal" methods can score full marks provided **www**

Question			Answer	Marks	Guidance	
7	(i)		$y = 9x^5$	M1	Obtain kx^5	If individual terms are differentiated then **M0A0B0**
				A1	Correct expression for y ($9x^5$)	
			$\frac{dy}{dx} = 45x^4$	B1 ft **[3]**	Follow through from their single kx^n, $n \neq 0$. Must be simplified.	$\frac{3x^2 + x^4}{x}$ is **not** a misread **M0A0B0**
7	(ii)		$y = x^{\frac{1}{3}}$	B1	$\sqrt[3]{x} = x^{\frac{1}{3}}$	
				B1	$kx^{-\frac{2}{3}}$	**SC** $\sqrt[3]{x} = x^{-\frac{1}{3}}$ differentiated to
			$\frac{dy}{dx} = \frac{1}{3}x^{-\frac{2}{3}}$	B1 **[3]**	$\frac{1}{3}x^{-\frac{2}{3}}$. Allow $0.\dot{3}$ (not finite)	$-\frac{1}{3}x^{-\frac{4}{3}}$ **B1**
7	(iii)		$y = \frac{1}{2}x^{-3}$	M1	kx^{-4} seen	
			$\frac{dy}{dx} = -\frac{3}{2}x^{-4}$	A1 **[2]**		
8			$(3k-1)^2 - 4 \times k \times -4$	*M1	Attempts $b^2 - 4ac$ or an equation or inequality involving b^2 and $4ac$. Must involve k^2 in first term (but no x anywhere). If $b^2 - 4ac$ not stated, must be clear attempt.	Must be working with the discriminant explicitly and not only as part of the quadratic formula. Allow $\sqrt{b^2 - 4ac}$ for first **M1 A1**
			$= 9k^2 + 10k + 1$	A1	Correct discriminant, simplified to 3 terms	
			$9k^2 + 10k + 1 < 0$	M1	States discriminant < 0 or $b^2 < 4ac$.	Can be awarded at any stage. Doesn't need first M1. No square root here.
			$(9k + 1)(k + 1) < 0$	DM1	Correct method to find roots of a three term quadratic	
			$-1, -\frac{1}{9}$	A1	Both values of k correct	
			$-1 < k < -\frac{1}{9}$	M1	Chooses "inside region" of inequality	Allow correct region for their inequality
				A1 **[7]**	Allow "$k < -\frac{1}{9}$ **and** $k > -1$" etc. must be strict inequalities for A mark	Do not allow "$k < -\frac{1}{9}$ **or** $k > -1$";

Question			Answer	Marks	Guidance	
9	(i)		Centre (1, −5) $(x-1)^2+(y+5)^2-19-1-25=0$ $(x-1)^2+(y+5)^2=45$ Radius $=\sqrt{45}$	B1 M1 A1 **[3]**	Correct centre Correct method to find r^2 Correct radius. **Do not allow if wrong centre used in calculation of radius.**	 $r^2=(\pm5)^2+(\pm1)^2+19$ for the M mark **A0** if $\pm\sqrt{45}$
9	(ii)		$7^2+(-2)^2-14-20-19$ $=0$	B1 **[1]**	Substitution of coordinates into equation of circle in any form or use of Pythagoras' theorem to calculate the distance of (7, -2) from C	**No follow through** for this part as **AG. Must be consistent–** do not allow finding the distance as $\sqrt{45}$ if no/wrong radius found in 9(i).
9	(iii)		gradient of radius = $\frac{-5-(-2)}{1-7}$ or $\frac{-2-(-5)}{7-1}$ $=\frac{1}{2}$ gradient of tangent $=-2$ $y+2=-2(x-7)$ $2x+y-12=0$	M1 A1√ B1√ M1 A1 **[5]**	uses $\frac{y_2-y_1}{x_2-x_1}$ with **their C** (3/4 correct) Follow through from their C, allow unsimplified single fraction e.g. $\frac{-3}{-6}$ Follow through from their gradient, even if M0 scored. Allow $\frac{-1}{\text{their fraction}}$ **B1** correct equation of straight line through (7, −2), any non-zero numerical gradient oe 3 term equation in correct form i.e. $k(2x+y-12)=0$ where k is an integer **cao**	**Follow through from 9(i) until final mark.** If (−1,5) is used for C, then expect Gradient of radius = $\frac{5-(-2)}{-1-7}=-\frac{7}{8}$ Gradient of tangent = $\frac{8}{7}$ <u>Alternative markscheme for implicit differentiation</u>: **M1** Attempt at implicit diff as evidenced by $2y\frac{dy}{dx}$ term **A1** $2x+2y\frac{dy}{dx}-2+10\frac{dy}{dx}=0$ **A1** Substitution of (7, −2) to obtain gradient of tangent = −2 Then **M1 A1** as main scheme

Question			Answer	Marks	Guidance	
10			$\frac{dy}{dx} = x^2 - 9x^{-2}$	B1	x^2 from differentiating first term	
				M1	kx^{-2}	
				A1	$-9x^{-2}$ (no + c)	
			Gradient of line = 8	B1		
			$x^2 - 9x^{-2} = 8$	M1	Equate their $\frac{dy}{dx}$ to 8 (or their gradient of line, if clear)	**Note:** If equated to +/-1/8 then M0 but the next M1 and its dependencies are available
			$x^4 - 8x^2 - 9 = 0$ $k^2 - 8k - 9 = 0$	*M1	Use a correct substitution to obtain a 3 term quadratic or factorise into 2 brackets each containing x^2	If no substitution stated and treated as a quadratic (e.g. quadratic formula), no more marks until square rooting seen
			$(k-9)(k+1) = 0$	DM1	Correct method to solve 3 term quadratic – **dependent on previous M1**	**SC:** If spotted after first five marks- (3, 12) **B1** (-3, -12) **B1** Justifies exactly two solutions **B3**
			$k = 9$ (don't need $k = -1$)	A1	No extras	
			$x = 3,\ -3$	DM1	Attempt to find x by square rooting – accept one value	
			$y = 12,\ -12$	A1 **[10]**	No extras	

More Additional Guidance for Q10

If curve equated to line and before differentiating:

First four marks **B1 M1 A1 B1** available as main scheme
Then **M0** for equating as this not been explicitly done
Allow the **M1** for the substitution
DM1 for quadratic as main scheme (dependent on a correct substitution)
A0 for the 9 (as follows wrong working)
DM1 for square rooting (dependent on a correct substitution)
A0 for the co-ordinates (as follows wrong working). Max mark **7/10**

Allocation of method mark for solving a quadratic

e.g. $2x^2 - 5x - 18 = 0$

1) If the candidate attempts to solve by factorisation, their attempt when expanded must produce the **correct quadratic term** and **one other correct term** (with correct sign):

$(2x+2)(x-9)=0$	**M1**	$2x^2$ and -18 obtained from expansion
$(2x+3)(x-4)=0$	**M1**	$2x^2$ and $-5x$ obtained from expansion
$(2x-9)(x-2)=0$	**M0**	only $2x^2$ term correct

2) If the candidate attempts to solve by using the formula

a) If the formula is quoted incorrectly then **M0**.

b) If the formula is quoted correctly then one **sign** slip is permitted. Substituting the wrong numerical value for a or b or c scores **M0**

$\dfrac{-5 \pm \sqrt{(-5)^2 - 4\times 2\times -18}}{2\times 2}$	earns **M1** (minus sign incorrect at start of formula)
$\dfrac{5 \pm \sqrt{(-5)^2 - 4\times 2\times 18}}{2\times 2}$	earns **M1** (18 for *c* instead of −18)
$\dfrac{-5 \pm \sqrt{(-5)^2 - 4\times 2\times 18}}{2\times 2}$	**M0** (2 sign errors: initial sign and *c* incorrect)
$\dfrac{5 \pm \sqrt{(-5)^2 - 4\times 2\times -18}}{2\times -5}$	**M0** (2*b* on the denominator)

Notes – for equations such as $2x^2 - 5x - 18 = 0$, then $b^2 = 5^2$ would be condoned in the discriminant and would not be counted as a sign error. Repeating the sign error for *a* in both occurrences in the formula would be two sign errors and score **M0**.

c) If the formula is not quoted at all, substitution must be completely correct to earn the **M1**

3) If the candidate attempts to complete the square, they must get to the "square root stage" involving ±; we are looking for evidence that the candidate knows a quadratic has two solutions!

$$2x^2 - 5x - 18 = 0$$

$$2\left(x^2 - \frac{5}{2}x\right) - 18 = 0$$

$$2\left[\left(x - \frac{5}{4}\right)^2 - \frac{25}{16}\right] - 18 = 0$$

$$\left(x - \frac{5}{4}\right)^2 = \frac{169}{16}$$

$$x - \frac{5}{4} = \pm\sqrt{\frac{169}{16}}$$

This is where the **M1** is awarded – arithmetical errors may be condoned provided $x - \frac{5}{4}$ seen or implied

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1 Hills Road
Cambridge
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OCR Customer Contact Centre

Education and Learning
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

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GCE

Mathematics

Advanced Subsidiary GCE

Unit **4721:** Core Mathematics 1

Mark Scheme for June 2013

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

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A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

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Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

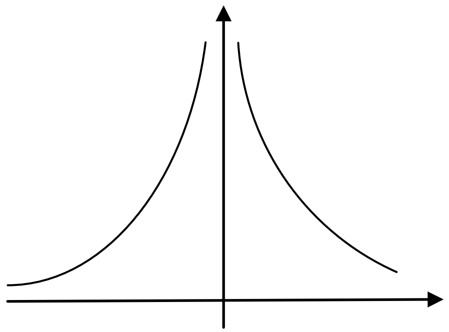
NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

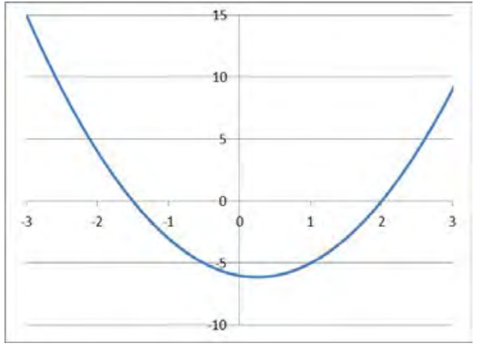
Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1	**(i)**		$4\sqrt{45}$ $=12\sqrt{5}$	M1 A1 **[2]**	or $4\sqrt{5}\sqrt{3}\times\sqrt{3}$ (not just $4\sqrt{5\times3}\times\sqrt{3}$) or $\sqrt{720}$ or $\sqrt{240}\times\sqrt{3}$ or better Correctly simplified answer	For method mark, makes a correct start to manipulate the expression i.e. at least combines two parts correctly or splits one part correctly
1	**(ii)**		$\frac{20\sqrt{5}}{5}=4\sqrt{5}$	B1 **[1]**	**cao**, do not allow unsimplified, do not allow if clearly from wrong working	
1	**(iii)**		$5\sqrt{5}$	B1 **[1]**	**cao www**, do not allow unsimplified, do not allow if clearly from wrong working	
2			$k = x^3$ $8k^2 + 7k - 1 = 0$ $(8k - 1)(k + 1) = 0$ $k = \frac{1}{8},\ k = -1$ $x = \frac{1}{2},\ x = -1$	M1* DM1 * A1 M1 A1 **[5]**	Use a substitution to obtain a quadratic or factorise into 2 brackets each containing x^3 Correct method to solve a quadratic Both values of k correct Attempt to cube root at least one value to obtain x Both values of x correct and no other values	**No marks** if whole equation cube rooted etc. **No marks** if straight to formula with no evidence of substitution at start and no cube rooting/cubing at end. **Spotted solutions:** If **M0 DMO** or **M1 DM0** **SC B1** $x = -1$ **www** **SC B1** $x = \frac{1}{2}$ **www** (Can then get 5/5 if both found **www** and exactly two solutions justified)

Question			Answer	Marks	Guidance	
3	**(i)**		$f(x) = 6x^{-2} + 2x$ $f'(x) = -12x^{-3} + 2$	M1 A1 B1 **[3]**	kx^{-3} obtained by differentiation $-12x^{-3}$ $2x$ correctly differentiated to 2	**ISW** incorrect simplification after correct expression
3	**(ii)**		$f''(x) = 36x^{-4}$	M1 A1 **[2]**	Attempt to differentiate **their (i)** i.e. at least one term "correct" Fully correct **cao** No follow through for **A** mark	Allow constant differentiated to zero **ISW** incorrect simplification after correct expression
4	**(i)**		$3(x^2 + 3x) + 10$ $= 3\left(x + \frac{3}{2}\right)^2 - \frac{27}{4} + 10$ $= 3\left(x + \frac{3}{2}\right)^2 + \frac{13}{4}$	B1 M1 A1 **[3]**	$\left(x + \frac{3}{2}\right)^2$ $10 - 3p^2$ or $\frac{10}{3} - p^2$ Allow $p = \frac{3}{2}$, $q = \frac{13}{4}$ **A1 www**	If p, q found correctly, then **ISW** slips in format. $3(x + 1.5)^2 - 3.25$ **B1 M0 A0** $3(x + 1.5) + 3.25$ **B1 M1 A1 (BOD)** $3(x + 1.5x)^2 + 3.25$ **B0 M1 A0** $3(x^2 + 1.5)^2 + 3.25$ **B0 M1 A0** $3(x - 1.5)^2 + 3.25$ **B0 M1 A1 (BOD)** $3x(x + 1.5)^2 + 3.25$ **B0M1A0**
4	**(ii)**		$\left(-\frac{3}{2}, \frac{13}{4}\right)$	B1 B1 **[2]**	FT i.e. – their p FT i.e. their q	If restarted e.g. by differentiation: Correct method to find x value of minimum point **M1** Fully correct answer **www A1**
4	**(iii)**		$9^2 - (4 \times 3 \times 10)$ $= -39$	M1 A1 **[2]**	Uses $b^2 - 4ac$ Ignore >0, <0 etc. **ISW** comments about number of roots	Use of $\sqrt{b^2 - 4ac}$ is **M0** unless recovered

Question			Answer	Marks	Guidance	
5	**(i)**			B1 B1 **[2]**	Excellent curve for $y = \frac{2}{x^2}$ in either quadrant Excellent curve for $y = \frac{2}{x^2}$ in other quadrant and no more. **SC B1** Reasonably correct curves in 1st and 2nd quadrants and no more	**N.B. Ignore 'feathering' now that answers are scanned.** For Excellent: Correct shape, not touching axes, asymptotes clearly the axes. Allow slight movement away from asymptote at one end but not more. Not finite. For **SC B1**, graph must not touch axes more than twice.
5	**(ii)**		$y = \frac{2}{(x+5)^2}$	M1 A1 **[2]**	$\frac{2}{(x+5)^2}$ or $\frac{2}{(x-5)^2}$ seen Fully correct, must include "y =" or "f(x) ="	
5	**(iii)**		Stretch scale factor $\frac{1}{2}$ parallel to y-axis	B1 B1 **[2]**	Or "stretched" etc; **do not accept** squashed, compressed etc. **oe** e.g. scale factor $1/\sqrt{2}$ parallel to x-axis	**0/2** if more than one type of transformation mentioned **ISW** non-contradictory statements For "parallel to the y-axis" allow "vertically", "up", "in the (positive) y direction". **Do not accept** "in/on/ across/up/along/to/towards the y-axis"
6	**(i)**		Centre (0, −4) $x^2 + (y + 4)^2 - 16 - 24 = 0$ Radius = $\sqrt{40}$	B1 M1 A1 **[3]**	 $(y \pm 4)^2 - 4^2$ seen (or implied by correct answer) Do not allow **A** mark from $(y - 4)^2$	 Or attempt at $r^2 = f^2 + g^2 - c$ **A0** for $\pm\sqrt{40}$
6	**(ii)**		(−2, −10)	B1FT B1FT **[2]**	FT through centre given in **(i)** FT through centre given in **(i)**	i.e. (their $2x - 2$, their $2y - 2$) Apply same scheme if equation of diameter found and attempt to solve simultaneously; no marks until a correct value of x/y found.

Question			Answer	Marks	Guidance	
7	**(i)**		$8x < -1$ $x < -\frac{1}{8}$	B1 B1 **[2]**	soi, allow $-8x > 1$ but not just $8x + 1 < 0$ Correct working only, allow $-\frac{1}{8} > x$ Do not allow $\frac{1}{-8}$	Allow ≤ or ≥ for first mark Do not **ISW** if contradictory Do not allow ≤ or ≥
7	**(ii)**		$2x^2 - 10x \le 0$ $2x(x-5) \le 0$ $0 \le x \le 5$	M1* DM1* A1 DM1* A1 **[5]**	Expand brackets and rearrange to collect all terms on one side Correct method to find roots of resulting quadratic 0, 5 seen as roots – could be on sketch graph Chooses "inside region" for their roots of their **resulting** quadratic (not the original) Do **not** accept strict inequalities for final mark	No more than one incorrect term Allow $(2x + 0)(x - 5)$ **Do not allow (2*x* – 4)(*x* – 3),** this is the original expression. Dependent on first M1 only Allow "$x \ge 0$, $x \le 5$" , "$x \ge 0$ **and** $x \le 5$" but do not allow "$x \ge 0$ **or** $x \le 5$"
8			Midpoint of AB is $\left(\frac{-2+3}{2}, \frac{6+-8}{2}\right)$ $\left(\frac{1}{2}, -1\right)$ Gradient of given line = $\frac{1}{3}$ Gradient of $l = -3$ $y + 1 = -3\left(x - \frac{1}{2}\right)$ $6x + 2y - 1 = 0$	M1 A1 B1 B1FT M1 A1 A1 **[7]**	Correct method to find midpoint – can be implied by one correct value Must be stated or used – just rearranging the equation is not sufficient Use of $m_1 m_2 = -1$ (may be implied), allow for any initial non-zero numerical gradient Correct equation for line, any non-zero numerical gradient, through their $\left(\frac{1}{2}, -1\right)$ Correct equation in any three-term form $k(6x + 2y - 1) = 0$ for integer k **www**	**NB – "correct" answer can be found with wrong mid-pt. Check working thoroughly.** Must include "= 0"

Question			Answer	Marks	Guidance	
9	(i)		$(2x+3)(x-2)=0$	M1	Correct method to find roots	
			$x=-\frac{3}{2},\ x=2$	A1	Correct roots	
				B1	Reasonably symmetrical positive quadratic curve, must cross x axis	
				B1	y intercept $(0, -6)$ only	Indicated on graph or clearly stated, but there must be a curve
				B1	Good curve, with correct roots indicated and min point in 4th quadrant (not on axis)	Only allow final B1 if curve is clearly intended to be a quadratic symmetrical about min point in 4th quadrant
				[5]		
9	(ii)		$\frac{dy}{dx}=4x-1=0$	M1	Attempt to find x coordinate of vertex by differentiating and equating/comparing to zero, completing the square, finding the mid-point of their roots oe	**SC** Award **B1 (FT)** for $x<0$ if clearly from their graph
			Vertex when $x=\frac{1}{4}$	A1	cao	**NB Look for solution to 9ii done in the space below 9i graph**
			$x<\frac{1}{4}$	A1 FT	$x <$ their vertex, allow $\le$	
				[3]		
9	(iii)		$2x^2-x-6=4$	M1	Set quadratic expression equal to 4	
			$2x^2-x-10=0$ $(2x-5)(x+2)=0$	M1	Correct method to solve **resulting** three term quadratic	**Not** $2x^2-x-6=0$ with no use of 4
			$x=\frac{5}{2},\ x=-2$	A1	Must have both solutions – no mark for one spotted root	
			Distance $PQ = 4\frac{1}{2}$	B1FT	FT from their x values found from their resulting quadratic, provided $y=4$	Allow $\frac{9}{2}$ oe, but do **not** accept unsimplified expressions like $\sqrt{\frac{81}{4}}$
				[4]		

Question			Answer	Marks	Guidance	
10	(i)		$y = -x^3 - 3x^2 + 4x - kx + k$	M1 A1	Attempt to multiply out brackets Can be unsimplified	Must have $\pm x^3$ and 5 or 6 terms
			$\frac{dy}{dx} = -3x^2 - 6x + 4 - k$	M1 A1	Attempt to differentiate **their** expansion (**M0** if signs have changed throughout)	**<u>If using product rule:</u>** Clear attempt at correct rule **M1*** Differentiates both parts correctly **A1** Expand brackets of both parts ***DM1**
			When $x = -3$, $\frac{dy}{dx} = 0$	M1* DM1*	Sets $\frac{dy}{dx} = 0$ Substitutes $x = -3$ into their $\frac{dy}{dx} = 0$	
			$-27 + 18 + 4 - k = 0$ $k = -5$	A1 **[7]**	**www**	Then as main scheme
10	(ii)		$\frac{d^2y}{dx^2} = -6x - 6$	M1	Evaluates second derivative at $x = -3$ or other fully correct method	**Alternate valid methods include:** 1) Evaluating gradient at either side of −3 2) Evaluating y at either side of −3 3) Finding other turning point and stating "negative cubic so min before max"
			When $x = -3$, $\frac{d^2y}{dx^2}$ is positive so min point	A1 **[2]**	No incorrect working seen in this part i.e. if second derivate is evaluated, it must be 12. (Ignore errors in k value)	
10	(iii)		$-3x^2 - 6x + 9 = 9$	M1	Sets their gradient function from (i) (or from a restart) to 9	Allow first **M** even if k not found but look out for correct answer from wrong working.
			$3x(x + 2) = 0$ $x = 0$ or $x = -2$	A1	Correct x-values	**<u>SEE NEXT PAGE FOR ALTERNATIVE METHODS</u>**
			When $x = 0$, $y = -9$ for line $y = -5$ for curve	M1	One of their x-values substituted into both curve and line/substituted into one and verified to be on the other	**Note:** Putting a value into $x^3 + 3x^2 - 4 = 0$ (where the line and curve meet) is equivalent
			When $x = -2$, $y = -27$ for line $y = -27$ for curve	M1	Conclusion that $x = -2$ is the correct value **<u>or</u>** Second x-value substituted into both curve and line/verified as above	<u>If curve equated to line before differentiating:</u>
			$x = -2$, $y = -27$	A1 **[5]**	$x = -2$, $y = -27$ **www *(Check k correct)***	**M0 A0**, can get **M1M1** but **A0 ww** Maximum mark **2/5**

Question			Answer	Marks	Guidance
10	(iii)		Alternative method Attempt to solve equations of curve and tangent simultaneously **and** uses valid method to establish at least one root of the resulting cubic ($x^3 + 3x^2 - 4 = 0$ oe) **M1** All roots found **A1** Either 1) States $x = -2$ is repeated root so tangent **M2** (If double root found but not explicitly stated that repeated root implies tangent then **M0** but **B1** if (−2, −27) found) Or 2) Substitutes one x value into their gradient function to determine if equal to gradient of the line **M1** Substitutes other x value into their gradient function to determine if equal to gradient of the line **or** conclusion that −2 is the correct one **M1** $x = -2$, $y = -27$ **A1 www** **SC** Trial and Improvement Finds at least one value at which the gradient of the curve is 9 **B1** Verifies on both line and curve **B1 2/5**		

APPENDIX 1

Allocation of method mark for solving a quadratic

e.g. $2x^2 - x - 6 = 0$

1) If the candidate attempts to solve by factorisation, their attempt when expanded must produce the **correct quadratic term** and **one other correct term** (with correct sign):

$(2x - 3)(x + 2)$	**M1**	$2x^2$ and -6 obtained from expansion
$(2x - 3)(x + 1)$	**M1**	$2x^2$ and $-x$ obtained from expansion
$(2x + 3)(x + 2)$	**M0**	only $2x^2$ term correct

2) If the candidate attempts to solve by using the formula

a) If the formula is quoted incorrectly then **M0**.

b) If the formula is quoted correctly then one **sign** slip is permitted. Substituting the wrong numerical value for a or b or c scores **M0**

$$\frac{-1 \pm \sqrt{(-1)^2 - 4\times 2\times -6}}{2\times 2}$$ earns **M1** (minus sign incorrect at start of formula)

$$\frac{1 \pm \sqrt{(-1)^2 - 4\times 2\times 6}}{2\times 2}$$ earns **M1** (6 for c instead of -6)

$$\frac{-1 \pm \sqrt{(-1)^2 - 4\times 2\times 6}}{2\times 2}$$ **M0** (2 sign errors: initial sign and c incorrect)

$$\frac{1 \pm \sqrt{(-1)^2 - 4\times 2\times -6}}{2\times -6}$$ **M0** ($2c$ on the denominator)

Notes – for equations such as $2x^2 - x - 6 = 0$, then $b^2 = 1^2$ would be condoned in the discriminant and would not be counted as a sign error. Repeating the sign error for a in both occurrences in the formula would be two sign errors and score **M0**.

c) If the formula is not quoted at all, substitution must be completely correct to earn the **M1**

3) If the candidate attempts to complete the square, they must get to the “square root stage” involving ±; we are looking for evidence that the candidate knows a quadratic has two solutions!

$$2x^2 - x - 6 = 0$$

$$2\left(x^2 - \frac{1}{2}x\right) - 6 = 0$$

$$2\left[\left(x - \frac{1}{4}\right)^2 - \frac{1}{16}\right] - 6 = 0$$

$$\left(x - \frac{1}{4}\right)^2 = \frac{49}{16}$$

$$x - \frac{1}{4} = \pm\sqrt{\frac{49}{16}}$$

This is where the **M1** is awarded – arithmetical errors may be condoned provided $x - \frac{1}{4}$ seen or implied

If a candidate makes repeated attempts (e.g. fails to factorise and then tries the formula), mark only what you consider to be their last full attempt

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

Education and Learning
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk