Edexcel Maths Core 3

Mark Scheme Pack

2006-2013

GCE

Edexcel GCE

Core Mathematics C3 (6665)

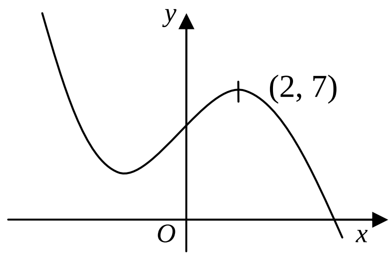
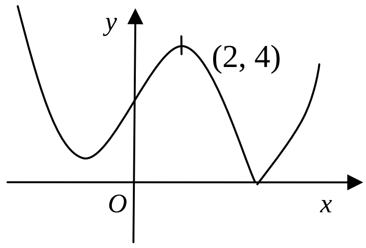
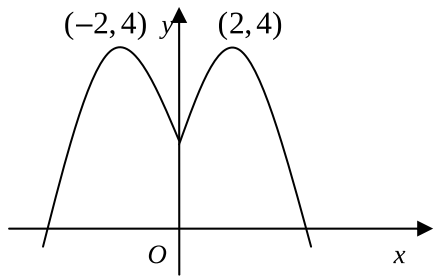
January 2006

advancing learning, changing lives

Mark Scheme (Results)

Edexcel GCE
Core Mathematics C3 (6665)

January 2006
6665 Core Mathematics C3
Mark Scheme

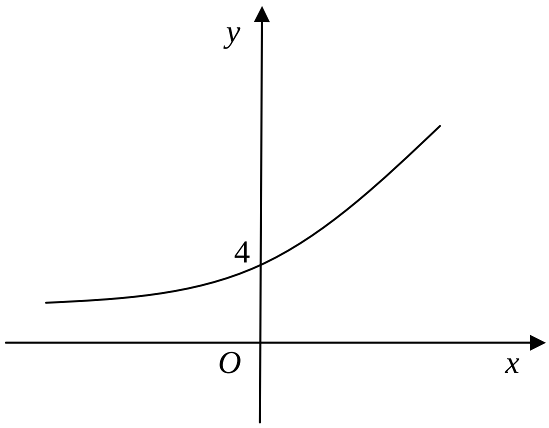
Question Number	Scheme		Marks	
1.	(a) 	Shape unchanged Point	B1 B1	 **(2)**
	(b) 	Shape Point	B1 B1	 **(2)**
	(c) 	Shape (2, 4) (−2, 4)	B1 B1 B1	 **(3)** **[7]**

Question Number	Scheme	Marks
2.	$x^2-x-2=(x-2)(x+1)$ At any stage	B1
	$\dfrac{2x^2+3x}{(2x+3)(x-2)}=\dfrac{x(2x+3)}{(2x+3)(x-2)}=\dfrac{x}{x-2}$	B1
	$\dfrac{2x^2+3x}{(2x+3)(x-2)}-\dfrac{6}{x^2-x-2}=\dfrac{x(x+1)-6}{(x-2)(x+1)}$	M1
	$=\dfrac{x^2+x-6}{(x-2)(x+1)}$	A1
	$=\dfrac{(x+3)(x-2)}{(x-2)(x+1)}$	M1 A1
	$=\dfrac{x+3}{x+1}$	A1 **(7)**
		[7]
	Alternative method	
	$x^2-x-2=(x-2)(x+1)$ At any stage	B1
	$(2x+3)$ appearing as a factor of the numerator at any stage	B1
	$\dfrac{2x^2+3x}{(2x+3)(x-2)}-\dfrac{6}{(x-2)(x+1)}=\dfrac{(2x^2+3x)(x+1)-6(2x+3)}{(2x+3)(x-2)(x+1)}$	M1
	$=\dfrac{2x^3+5x^2-9x-18}{(2x+3)(x-2)(x+1)}$ can be implied	A1
	$=\dfrac{(x-2)(2x^2+9x+9)}{(2x+3)(x-2)(x+1)}$ or $\dfrac{(2x+3)(x^2+x-6)}{(2x+3)(x-2)(x+1)}$ or $\dfrac{(x+3)(2x^2-x-6)}{(2x+3)(x-2)(x+1)}$ Any one linear factor × quadratic	M1
	$=\dfrac{(2x+3)(x-2)(x+3)}{(2x+3)(x-2)(x+1)}$ Complete factors	A1
	$=\dfrac{x+3}{x+1}$	A1 **(7)**

Question Number	Scheme	Marks
3.	$\dfrac{dy}{dx} = \dfrac{1}{x}$ accept $\dfrac{3}{3x}$	M1 A1
	At $x = 3$, $\dfrac{dy}{dx} = \dfrac{1}{3} \Rightarrow m' = -3$ Use of $mm' = -1$	M1
	$y - \ln 1 = -3(x-3)$	M1
	$y = -3x + 9$ Accept $y = 9 - 3x$	A1 **(5)**
		[5]
	$\dfrac{dy}{dx} = \dfrac{1}{3x}$ leading to $y = -9x + 27$ is a maximum of M1 A0 M1 M1 A0 =3/5	
4.	(a) (i) $\dfrac{d}{dx}\left(e^{3x+2}\right) = 3e^{3x+2} \quad \left(\text{or } 3e^{2}e^{3x}\right)$ At any stage	B1
	$\dfrac{dy}{dx} = 3x^2 e^{3x+2} + 2x e^{3x+2}$ Or equivalent	M1 A1+A1
		(4)
	(ii) $\dfrac{d}{dx}\left(\cos\left(2x^3\right)\right) = -6x^2 \sin\left(2x^3\right)$ At any stage	M1 A1
	$\dfrac{dy}{dx} = \dfrac{-18x^3 \sin\left(2x^3\right) - 3\cos\left(2x^3\right)}{9x^2}$	M1 A1 **(4)**
	Alternatively using the product rule for second M1 A1	
	$y = (3x)^{-1} \cos\left(2x^3\right)$	
	$\dfrac{dy}{dx} = -3(3x)^{-2} \cos\left(2x^3\right) - 6x^2 (3x)^{-1} \sin\left(2x^3\right)$	
	Accept equivalent unsimplified forms	
	(b) $1 = 8\cos(2y+6)\dfrac{dy}{dx}$ or $\dfrac{dx}{dy} = 8\cos(2y+6)$	M1
	$\dfrac{dy}{dx} = \dfrac{1}{8\cos(2y+6)}$	M1 A1
	$\dfrac{dy}{dx} = \dfrac{1}{8\cos\left(\arcsin\left(\dfrac{x}{4}\right)\right)} \quad \left(= (\pm)\dfrac{1}{2\sqrt{\left(16 - x^2\right)}}\right)$	M1 A1 **(5)**
		[13]

Question Number	Scheme	Marks
5.	(a) $2x^2 - 1 - \frac{4}{x} = 0$ Dividing equation by x	M1
	$x^2 = \frac{1}{2} + \frac{4}{2x}$ Obtaining $x^2 = \ldots$	M1
	$x = \sqrt{\left(\frac{2}{x} + \frac{1}{2}\right)}$ ∗ cso	A1 **(3)**
	(b) $x_1 = 1.41, x_2 = 1.39, x_3 = 1.39$	B1, B1, B1
	If answers given to more than 2 dp, penalise first time then accept awrt above.	**(3)**
	(c) Choosing $(1.3915, 1.3925)$ or a tighter interval	M1
	$\mathrm{f}(1.3915) \approx -3\times10^{-3}$, $\mathrm{f}(1.3925) \approx 7\times10^{-3}$ Both, awrt	A1
	Change of sign (and continuity) $\Rightarrow \alpha \in (1.3915, 1.3925)$	
	$\Rightarrow \alpha = 1.392$ to 3 decimal places ∗ cso	A1 **(3)** **[9]**
6.	(a) $R\cos\alpha = 12$, $R\sin\alpha = 4$	
	$R = \sqrt{(12^2 + 4^2)} = \sqrt{160}$ Accept if just written down, awrt 12.6	M1 A1
	$\tan\alpha = \frac{4}{12}$, $\Rightarrow$ $\alpha \approx 18.43°$ awrt 18.4°	M1, A1**(4)**
	(b) $\cos(x + \text{ their } \alpha) = \frac{7}{\text{their } R}$ (≈ 0.5534)	M1
	x + their $\alpha = 56.4°$ awrt 56°	A1
	$= \ldots, 303.6°$ 360° − their principal value	M1
	$x = 38.0°, 285.2°$ Ignore solutions out of range	A1, A1 **(5)**
	If answers given to more than 1 dp, penalise first time then accept awrt above.	
	(c)(i) minimum value is $-\sqrt{160}$ ft their R	B1ft
	(ii) $\cos(x + \text{ their } \alpha) = -1$	M1
	$x \approx 161.57°$ cao	A1 **(3)** **[12]**

Question Number	Scheme	Marks	
7.	(a) (i) Use of $\cos 2x = \cos^2 x - \sin^2 x$ in an attempt to prove the identity.	M1	
	$\dfrac{\cos 2x}{\cos x + \sin x} = \dfrac{\cos^2 x - \sin^2 x}{\cos x + \sin x} = \dfrac{(\cos x - \sin x)(\cos x + \sin x)}{\cos x + \sin x} = \cos x - \sin x$ * cso	A1	**(2)**
	(ii) Use of $\cos 2x = 2\cos^2 x - 1$ in an attempt to prove the identity.	M1	
	Use of $\sin 2x = 2\sin x \cos x$ in an attempt to prove the identity.	M1	
	$\frac{1}{2}(\cos 2x - \sin 2x) = \frac{1}{2}(2\cos^2 x - 1 - 2\sin x \cos x) = \cos^2 x - \cos x \sin x - \frac{1}{2}$ * cso	A1	**(3)**
	(b) $\cos\theta(\cos\theta - \sin\theta) = \frac{1}{2}$ Using (a)(i)	M1	
	$\cos^2\theta - \cos\theta\sin\theta - \frac{1}{2} = 0$		
	$\frac{1}{2}(\cos 2\theta - \sin 2\theta) = 0$ Using (a)(ii)	M1	
	$\cos 2\theta = \sin 2\theta$ *	A1	**(3)**
	(c) $\tan 2\theta = 1$	M1	
	$2\theta = \frac{\pi}{4}, \left(\frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}\right)$ any one correct value of 2θ	A1	
	$\theta = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$ Obtaining at least 2 solutions in range	M1	
	The 4 correct solutions	A1	**(4)**
	If decimals $(0.393, 1.963, 3.534, 5.105)$ or degrees $(22.5°, 112.5°, 202.5°, 292.5°)$ are given, but all 4 solutions are found, penalise one A mark only. Ignore solutions out of range.		**[12]**

Question Number	Scheme	Marks
8.	(a) $\mathrm{gf}(x) = \mathrm{e}^{2(2x+\ln 2)}$	M1
	$= \mathrm{e}^{4x}\mathrm{e}^{2\ln 2}$	M1
	$= \mathrm{e}^{4x}\mathrm{e}^{\ln 4}$	M1
	$= 4\mathrm{e}^{4x}$ Give mark at this point, cso	A1 **(4)**
	$\left(\text{Hence } \mathrm{gf} : x \mapsto 4\mathrm{e}^{4x}, \quad x \in \mathbb{R}\right)$	
	(b)  Shape and point	B1 **(1)**
	(c) Range is $\mathbb{R}_+$ Accept $\mathrm{gf}(x) > 0$, $y > 0$	B1 **(1)**
	(d) $\frac{\mathrm{d}}{\mathrm{d}x}\left[\mathrm{gf}(x)\right] = 16\mathrm{e}^{4x}$	
	$\mathrm{e}^{4x} = \frac{3}{16}$	M1 A1
	$4x = \ln\frac{3}{16}$	M1
	$x \approx -0.418$	A1 **(4)**
		[10]

Mark Scheme (Results)
January 2007

GCE

GCE Mathematics

Core Mathematics C3 (6665)

Edexcel Limited. Registered in England and Wales No. 4496750
Registered Office: One90 High Holborn, London WC1V 7BH

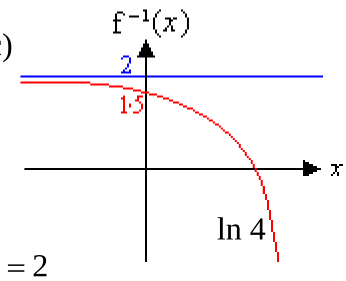
A PEARSON COMPANY

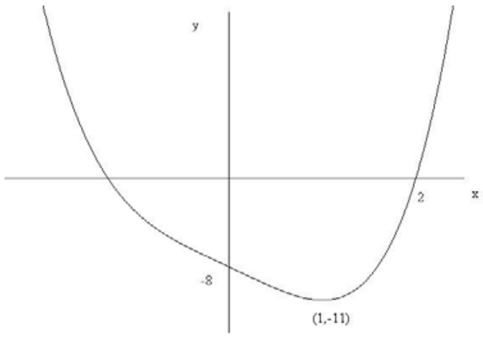
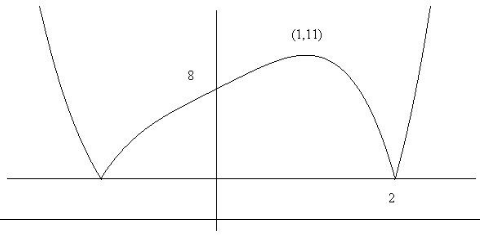
January 2007
6665 Core Mathematics C3
Mark Scheme

Question Number	Scheme		Marks
1.	(a) $\sin 3\theta = \sin(2\theta+\theta) = \sin 2\theta\cos\theta + \cos 2\theta \sin\theta$		B1
	$= 2\sin\theta\cos^2\theta + (1-2\sin^2\theta)\sin\theta$		B1 B1
	$= 2\sin\theta - 2\sin^3\theta + \sin\theta - 2\sin^3\theta$		M1
	$= 3\sin\theta - 4\sin^3\theta$ *	cso	A1 **(5)**
	(b) $\sin 3\theta = 3\times\frac{\sqrt{3}}{4} - 4\left(\frac{\sqrt{3}}{4}\right)^3 = \frac{3\sqrt{3}}{4} - \frac{3\sqrt{3}}{16} = \frac{9\sqrt{3}}{16}$ or exact equivalent		M1 A1 **(2)**
			[7]
2.	(a) $f(x) = \frac{(x+2)^2, -3(x+2)+3}{(x+2)^2}$		M1 A1, A1
	$= \frac{x^2+4x+4-3x-6+3}{(x+2)^2} = \frac{x^2+x+1}{(x+2)^2}$ *	cso	A1 **(4)**
	(b) $x^2+x+1 = \left(x+\frac{1}{2}\right)^2 + \frac{3}{4},\ > 0$ for all values of x.		M1 A1, A1 **(3)**
	(c) $f(x) = \frac{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}}{(x+2)^2}$ Numerator is positive from (b) $x \neq -2 \Rightarrow (x+2)^2 > 0$ (Denominator is positive) Hence $f(x) > 0$		B1 **(1)** **[8]**
	Alternative to (b)		
	$\frac{d}{dx}(x^2+x+1) = 2x+1 = 0 \Rightarrow x = -\frac{1}{2} \Rightarrow x^2+x+1 = \frac{3}{4}$		M1 A1
	A parabola with positive coefficient of x^2 has a minimum $\Rightarrow x^2+x+1 > 0$ Accept equivalent arguments		A1 **(3)**

Question Number	Scheme	Marks
3.	(a) $y=\dfrac{\pi}{4} \Rightarrow x=2\sin\dfrac{\pi}{4}=2\times\dfrac{1}{\sqrt{2}}=\sqrt{2} \Rightarrow P\in C$	B1 **(1)**
	Accept equivalent (reversed) arguments. In any method it must be clear that $\sin\dfrac{\pi}{4}=\dfrac{1}{\sqrt{2}}$ or exact equivalent is used.	
	(b) $\dfrac{dx}{dy}=2\cos y$ *or* $1=2\cos y\dfrac{dy}{dx}$	M1 A1
	$\dfrac{dy}{dx}=\dfrac{1}{2\cos y}$ May be awarded after substitution	M1
	$y=\dfrac{\pi}{4} \Rightarrow \dfrac{dy}{dx}=\dfrac{1}{\sqrt{2}}$ * cso	A1 **(4)**
	(c) $m'=-\sqrt{2}$	B1
	$y-\dfrac{\pi}{4}=-\sqrt{2}\left(x-\sqrt{2}\right)$	M1 A1
	$y=-\sqrt{2}x+2+\dfrac{\pi}{4}$	A1 **(4)**
		[9]
4.	(i) $\dfrac{dy}{dx}=\dfrac{\left(9+x^2\right)-x\left(2x\right)}{\left(9+x^2\right)^2}$ $\left(=\dfrac{9-x^2}{\left(9+x^2\right)^2}\right)$	M1 A1
	$\dfrac{dy}{dx}=0 \Rightarrow 9-x^2=0 \Rightarrow x=\pm 3$	M1 A1
	$\left(3,\dfrac{1}{6}\right),\ \left(-3,-\dfrac{1}{6}\right)$ Final two A marks depend on second M only	A1, A1 **(6)**
	(ii) $\dfrac{dy}{dx}=\dfrac{3}{2}\left(1+e^{2x}\right)^{\frac{1}{2}}\times 2e^{2x}$	M1 A1 A1
	$x=\dfrac{1}{2}\ln 3 \Rightarrow \dfrac{dy}{dx}=\dfrac{3}{2}\left(1+e^{\ln 3}\right)^{\frac{1}{2}}\times 2e^{\ln 3}=3\times 4^{\frac{1}{2}}\times 3=18$	M1 A1 **(5)**
		[11]

Question Number	Scheme		Marks	
5.	(a) $R^2 = \left(\sqrt{3}\right)^2 + 1^2 \Rightarrow R = 2$		M1 A1	
	$\tan\alpha = \sqrt{3} \Rightarrow \alpha = \frac{\pi}{3}$	accept awrt 1.05	M1 A1	**(4)**
	(b) $\sin\left(x + \text{ their } \alpha\right) = \frac{1}{2}$		M1	
	$x + \text{ their } \alpha = \frac{\pi}{6} \quad \left(\frac{5\pi}{6}, \frac{13\pi}{6}\right)$		A1	
	$x = \frac{\pi}{2}, \frac{11\pi}{6}$	accept awrt 1.57, 5.76	M1 A1	**(4)**
				[8]
	The use of degrees loses only one mark in this question. Penalise the first time it occurs in an answer and then ignore.			

Question Number	Scheme		Marks	
6.	(a) $y = \ln(4-2x)$			
	$e^y = 4-2x$ leading to $x = 2 - \frac{1}{2}e^y$	Changing subject and removing ln	M1 A1	
	$y = 2 - \frac{1}{2}e^x \quad \Rightarrow \quad f^{-1} \mapsto 2 - \frac{1}{2}e^x$ ∗	cso	A1	
	Domain of f^{-1} is		B1	**(4)**
	(b) Range of f^{-1} is $f^{-1}(x) < 2$ (and $f^{-1}(x) \in$)		B1	**(1)**
	(c) 	Shape	B1	
		1.5	B1	
		ln 4	B1	
	$y = 2$		B1	**(4)**
	(d) $x_1 \approx -0.3704, \quad x_2 \approx -0.3452$	cao	B1, B1	**(2)**
	If more than 4 dp given in this part a maximum on one mark is lost. Penalise on the first occasion.			
	(e) $x_3 = -0.354\,030\,19 \ldots$			
	$x_4 = -0.350\,926\,88 \ldots$			
	$x_5 = -0.352\,017\,61 \ldots$			
	$x_6 = -0.351\,633\,86 \ldots$	Calculating to at least x_6 to at least four dp	M1	
	$k \approx -0.352$	cao	A1	**(2)**
				[13]
	Alternative to (e)			
	$k \approx -0.352$	Found in any way		
	Let $g(x) = x + \frac{1}{2}e^x$			
	$g(-0.3515) \approx +0.0003, \quad g(-0.3525) \approx -0.001$		M1	
	Change of sign (and continuity) $\Rightarrow k \in (-0.3525, -0.3515)$			
	$\Rightarrow k = -0.352$ (to 3 dp)		A1	**(2)**

Question Number	Scheme	Marks
7.	(a) $\mathrm{f}(-2) = 16 + 8 - 8\ (= 16) > 0$ $\mathrm{f}(-1) = 1 + 4 - 8\ (= -3) < 0$ Change of sign (and continuity) $\Rightarrow$ root in interval $(-2, -1)$ ft their calculation as long as there is a sign change	B1 B1 B1ft **(3)**
	(b) $\frac{\mathrm{d}y}{\mathrm{d}x} = 4x^3 - 4 = 0 \quad \Rightarrow \quad x = 1$ Turning point is $(1, -11)$	M1 A1 A1 **(3)**
	(c) $a = 2, b = 4, c = 4$	B1 B1 B1 **(3)**
	(d)  Shape ft their turning point in correct quadrant only 2 and −8	B1 B1 ft B1 **(3)**
	(e)  Shape	B1 **(1)** **[13]**

Question Number	Scheme	Marks
8.	(i) $\sec^2 x - \operatorname{cosec}^2 x = \left(1+\tan^2 x\right) - \left(1+\cot^2 x\right)$	M1 A1
	$= \tan^2 x - \cot^2 x$ ∗ cso	A1 **(3)**
	(ii)(a) $y = \arccos x \Rightarrow x = \cos y$	B1
	$x = \sin\left(\frac{\pi}{2} - y\right) \Rightarrow \arcsin x = \frac{\pi}{2} - y$	B1 **(2)**
	Accept $\arcsin x = \arcsin \cos y$	
	(b) $\arccos x + \arcsin x = y + \frac{\pi}{2} - y = \frac{\pi}{2}$	B1 **(1)**
		[6]
	Alternatives for (i)	
	$\sec^2 x - \tan^2 x = 1 = \operatorname{cosec}^2 x - \cot^2 x$	M1 A1
	Rearranging $\sec^2 x - \operatorname{cosec}^2 x = \tan^2 x - \cot^2 x$ ∗ cso	A1 **(3)**
	$\left(\text{LHS} = \frac{1}{\cos^2 x} - \frac{1}{\sin^2 x} = \frac{\sin^2 x - \cos^2 x}{\cos^2 x \sin^2 x}\right)$	
	RHS $= \frac{\sin^2 x}{\cos^2 x} - \frac{\cos^2 x}{\sin^2 x} = \frac{\sin^4 x - \cos^4 x}{\cos^2 x \sin^2 x} = \frac{\left(\sin^2 x - \cos^2 x\right)\left(\sin^2 x + \cos^2 x\right)}{\cos^2 x \sin^2 x}$	M1
	$= \frac{\sin^2 x - \cos^2 x}{\cos^2 x \sin^2 x}$	A1
	$=$ LHS ∗ or equivalent	A1 **(3)**

Mark Scheme (Results)

January 2008

GCE

GCE Mathematics (6665/01)

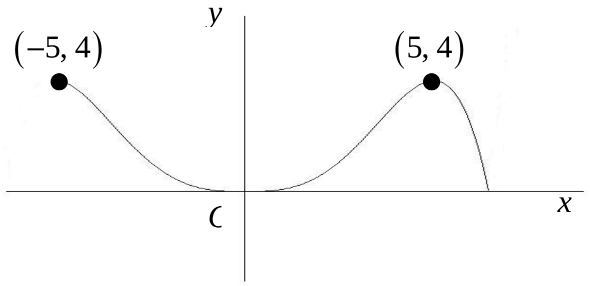
Edexcel Limited. Registered in England and Wales No. 4496750
Registered Office: One90 High Holborn, London WC1V 7BH

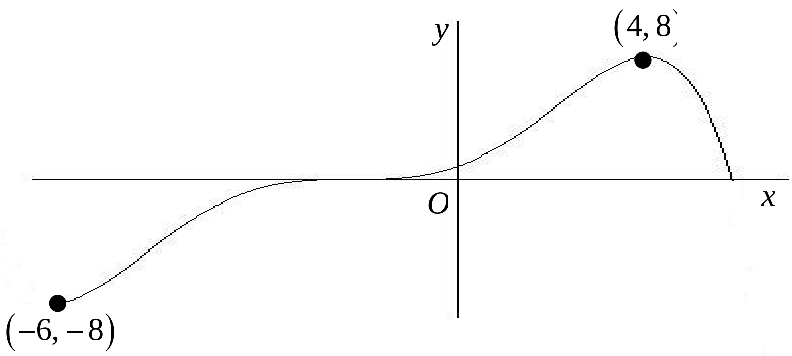
A PEARSON COMPANY

January 2008
6665 Core Mathematics C3
Mark Scheme

Question Number	Scheme	Marks
1.	$\begin{array}{r\|l} & 2x^2 \qquad -1 \\ \hline x^2-1 & 2x^4 \quad -3x^2+x+1 \\ & 2x^4 \quad -2x^2 \\ \hline & \qquad -x^2+x+1 \\ & \qquad -x^2 \qquad +1 \\ \hline & \qquad\qquad x \end{array}$	M1
	$a=2$ stated or implied	A1
	$c=-1$ stated or implied	A1
	$2x^2-1+\dfrac{x}{x^2-1}$	
	$a=2,\ b=0,\ c=-1,\ d=1,\ e=0$	
	$d=1$ and $b=0,\ e=0$ stated or implied	A1
		[4]
2.	(a)	
	$\dfrac{dy}{dx}=2e^{2x}\tan x+e^{2x}\sec^2 x$	M1 A1+A1
	$\dfrac{dy}{dx}=0 \quad \Rightarrow \quad 2e^{2x}\tan x+e^{2x}\sec^2 x=0$	M1
	$2\tan x+1+\tan^2 x=0$	A1
	$(\tan x+1)^2=0$	
	$\tan x=-1$ * cso	A1 (6)
	(b) $\left(\dfrac{dy}{dx}\right)_0=1$	M1
	Equation of tangent at $(0,0)$ is $y=x$	A1 (2)
		[8]

Question Number	Scheme		Marks	
3.	(a) $\mathrm{f}(2) = 0.38 \ \ldots$			
	$\mathrm{f}(3) = -0.39 \ \ldots$		M1	
	Change of sign (and continuity) $\Rightarrow$ root in $(2, 3)$ ✱	cso	A1	(2)
	(b) $x_1 = \ln 4.5 + 1 \approx 2.50408$		M1	
	$x_2 \approx 2.50498$		A1	
	$x_3 \approx 2.50518$		A1	(3)
	(c) Selecting $[2.5045, 2.5055]$, or appropriate tighter range, and evaluating at both ends.		M1	
	$\mathrm{f}(2.5045) \approx 6 \times 10^{-4}$			
	$\mathrm{f}(2.5055) \approx -2 \times 10^{-4}$			
	Change of sign (and continuity) $\Rightarrow$ root $\in (2.5045, 2.5055)$			
	$\Rightarrow$ root $= 2.505$ to 3 dp ✱	cso	A1	(2)
				[7]
	Note: The root, correct to 5 dp, is 2.50524			

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>4.</td><td>(a)


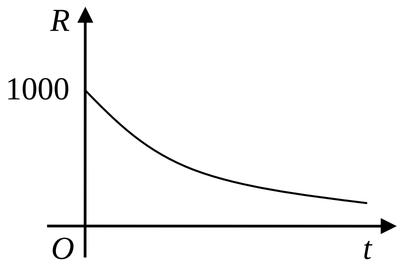
Shape
$(5, 4)$
$(-5, 4)$
(b) For the purpose of marking this paper, the graph is identical to (a)
Shape
$(5, 4)$
$(-5, 4)$
(c)


General shape – unchanged
Translation to left
$(4, 8)$
$(-6, -8)$
In all parts of this question ignore any drawing outside the domains shown in the diagrams above.</td><td>

B1
B1
B1 (3)

B1
B1
B1 (3)

B1
B1
B1
B1 (4)
[10]</td></tr>
</table>

Question Number	Scheme		Marks	
5.	(a) 1000		B1	(1)
	(b) $1000\,\mathrm{e}^{-5730c} = 500$		M1	
	$\mathrm{e}^{-5730c} = \frac{1}{2}$		A1	
	$-5730c = \ln\frac{1}{2}$		M1	
	$c = 0.000121$	cao	A1	(4)
	(c) $R = 1000\,\mathrm{e}^{-22920c} = 62.5$	Accept 62-63	M1 A1	(2)
	(d) 	Shape 1000	B1 B1	 (2) **[9]**

Question Number	Scheme	Marks
6.	(a) $\cos(2x+x)=\cos 2x\cos x-\sin 2x\sin x$	M1
	$=\left(2\cos^2 x-1\right)\cos x-\left(2\sin x\cos x\right)\sin x$	M1
	$=\left(2\cos^2 x-1\right)\cos x-2(1-\cos^2 x)\cos x$ any correct expression	A1
	$=4\cos^3 x-3\cos x$	A1 (4)
	(b)(i) $\dfrac{\cos x}{1+\sin x}+\dfrac{1+\sin x}{\cos x}=\dfrac{\cos^2 x+\left(1+\sin x\right)^2}{\left(1+\sin x\right)\cos x}$	M1
	$=\dfrac{\cos^2 x+1+2\sin x+\sin^2 x}{\left(1+\sin x\right)\cos x}$	A1
	$=\dfrac{2\left(1+\sin x\right)}{\left(1+\sin x\right)\cos x}$	M1
	$=\dfrac{2}{\cos x}=2\sec x$ * cso	A1 (4)
	(c) $\sec x=2$ *or* $\cos x=\dfrac{1}{2}$	M1
	$x=\dfrac{\pi}{3},\dfrac{5\pi}{3}$ accept awrt 1.05, 5.24	A1, A1 (3)
		[11]
7.	(a) $\dfrac{\mathrm{d}y}{\mathrm{d}x}=6\cos 2x-8\sin 2x$	M1 A1
	$\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)_0=6$	B1
	$y-4=-\dfrac{1}{6}x$ or equivalent	M1 A1 (5)
	(b) $R=\sqrt{\left(3^2+4^2\right)}=5$	M1 A1
	$\tan\alpha=\dfrac{4}{3},\ \alpha\approx 0.927$ awrt 0.927	M1 A1 (4)
	(c) $\sin\left(2x+\text{ their }\alpha\right)=0$	M1
	$x=-2.03,-0.46,1.11,2.68$	A1 A1 A1 (4)
	First A1 any correct solution; second A1 a second correct solution; third A1 all four correct and to the specified accuracy or better. Ignore the *y*-coordinate.	**[13]**

Question Number	Scheme	Marks
8.	(a) $x = 1 - 2y^3 \quad \Rightarrow \quad y = \left(\frac{1-x}{2}\right)^{\frac{1}{3}}$ *or* $\sqrt[3]{\frac{1-x}{2}}$	M1 A1 (2)
	$f^{-1} : x \mapsto \left(\frac{1-x}{2}\right)^{\frac{1}{3}}$ Ignore domain	
	(b) $gf(x) = \frac{3}{1-2x^3} - 4$	M1 A1
	$= \frac{3-4(1-2x^3)}{1-2x^3}$	M1
	$= \frac{8x^3-1}{1-2x^3}$ * cso	A1 (4)
	$gf : x \mapsto \frac{8x^3-1}{1-2x^3}$ Ignore domain	
	(c) $8x^3 - 1 = 0$ Attempting solution of numerator = 0	M1
	$x = \frac{1}{2}$ Correct answer and no additional answers	A1 (2)
	(d) $\frac{dy}{dx} = \frac{(1-2x^3) \times 24x^2 + (8x^3-1) \times 6x^2}{(1-2x^3)^2}$	M1 A1
	$= \frac{18x^2}{(1-2x^3)^2}$	A1
	Solving their numerator = 0 and substituting to find *y*.	M1
	$x = 0,\ y = -1$	A1 (5) **[13]**

Mark Scheme (Final)

January 2009

GCE

GCE Core Mathematics C3 (6665/01)

Edexcel Limited. Registered in England and Wales No. 4496750
Registered Office: One90 High Holborn, London WC1V 7BH

A PEARSON COMPANY

General Marking Guidance

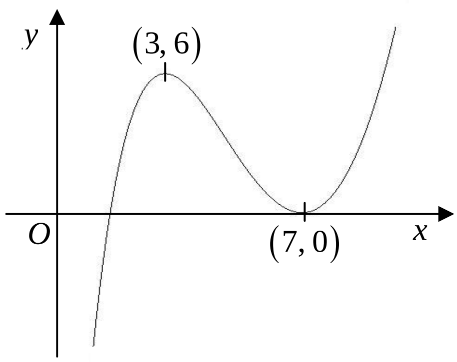
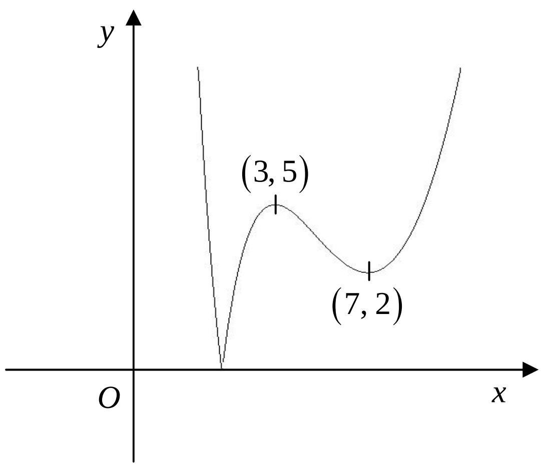
- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

January 2009
6665 Core C3
Mark Scheme

Version for Online Standardisation

Question Number	Scheme	Marks
1.	(a) $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\sqrt{(5x-1)}\right)=\dfrac{\mathrm{d}}{\mathrm{d}x}\left((5x-1)^{\frac{1}{2}}\right)$	
	$=5\times\dfrac{1}{2}(5x-1)^{-\frac{1}{2}}$	M1 A1
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=2x\sqrt{(5x-1)}+\dfrac{5}{2}x^2(5x-1)^{-\frac{1}{2}}$	M1 A1ft
	At $x=2$, $\dfrac{\mathrm{d}y}{\mathrm{d}x}=4\sqrt{9}+\dfrac{10}{\sqrt{9}}=12+\dfrac{10}{3}$	M1
	$=\dfrac{46}{3}$ Accept awrt 15.3	A1 **(6)**
	(b) $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{\sin 2x}{x^2}\right)=\dfrac{2x^2\cos 2x-2x\sin 2x}{x^4}$	M1 $\dfrac{\text{A1+A1}}{\text{A1}}$
		(4) **[10]**
	Alternative to (b)	
	$\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\sin 2x\times x^{-2}\right)=2\cos 2x\times x^{-2}+\sin 2x\times(-2)x^{-3}$	M1 A1 + A1
	$=2x^{-2}\cos 2x-2x^{-3}\sin 2x\quad\left(=\dfrac{2\cos 2x}{x^2}-\dfrac{2\sin 2x}{x^3}\right)$	A1 **(4)**

Question Number	Scheme	Marks
2.	(a) $\dfrac{2x+2}{x^2-2x-3}-\dfrac{x+1}{x-3}=\dfrac{2x+2}{(x-3)(x+1)}-\dfrac{x+1}{x-3}$	
	$=\dfrac{2x+2-(x+1)(x+1)}{(x-3)(x+1)}$	M1 A1
	$=\dfrac{(x+1)(1-x)}{(x-3)(x+1)}$	M1
	$=\dfrac{1-x}{x-3}$ Accept $-\dfrac{x-1}{x-3},\ \dfrac{x-1}{3-x}$	A1 **(4)**
	(b) $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{1-x}{x-3}\right)=\dfrac{(x-3)(-1)-(1-x)1}{(x-3)^2}$	M1 A1
	$=\dfrac{-x+3-1+x}{(x-3)^2}=\dfrac{2}{(x-3)^2}$ * cso	A1 **(3)**
		[7]
	Alternative to (a)	
	$\dfrac{2x+2}{x^2-2x-3}=\dfrac{2(x+1)}{(x-3)(x+1)}=\dfrac{2}{x-3}$	M1 A1
	$\dfrac{2}{x-3}-\dfrac{x+1}{x-3}=\dfrac{2-(x+1)}{x-3}$	M1
	$=\dfrac{1-x}{x-3}$	A1 **(4)**
	Alternatives to (b)	
	① $\mathrm{f}(x)=\dfrac{1-x}{x-3}=-1-\dfrac{2}{x-3}=-1-2(x-3)^{-1}$	
	$\mathrm{f}'(x)=(-1)(-2)(x-3)^{-2}$	M1 A1
	$=\dfrac{2}{(x-3)^2}$ * cso	A1 **(3)**
	② $\mathrm{f}(x)=(1-x)(x-3)^{-1}$	
	$\mathrm{f}'(x)=(-1)(x-3)+(1-x)(-1)(x-3)^{-2}$	M1
	$=-\dfrac{1}{x-3}-\dfrac{1-x}{(x-3)^2}=\dfrac{-(x-3)-(1-x)}{(x-3)^2}$	A1
	$=\dfrac{2}{(x-3)^2}$ *	A1 **(3)**

Question Number	Scheme		Marks	
3.	(a) 	Shape $(3, 6)$ $(7, 0)$	B1 B1 B1	 **(3)**
	(b) 	Shape $(3, 5)$ $(7, 2)$	B1 B1 B1	 **(3)** **[6]**

Question Number	Scheme		Marks
4.	$x = \cos(2y + \pi)$		
	$\dfrac{dx}{dy} = -2\sin(2y+\pi)$		M1 A1
	$\dfrac{dy}{dx} = -\dfrac{1}{2\sin(2y+\pi)}$	Follow through their $\dfrac{dx}{dy}$ before or after substitution	A1ft
	At $y = \dfrac{\pi}{4}$, $\dfrac{dy}{dx} = -\dfrac{1}{2\sin\frac{3\pi}{2}} = \dfrac{1}{2}$		B1
	$y - \dfrac{\pi}{4} = \dfrac{1}{2}x$		M1
	$y = \dfrac{1}{2}x + \dfrac{\pi}{4}$		A1 **(6)**
			[6]

Question Number	Scheme	Marks
5.	(a) $\mathrm{g}(x) \geq 1$	B1 **(1)**
	(b) $\mathrm{f\,g}(x) = \mathrm{f}\left(\mathrm{e}^{x^2}\right) = 3\mathrm{e}^{x^2} + \ln \mathrm{e}^{x^2}$	M1
	$= x^2 + 3\mathrm{e}^{x^2}$ ✱	A1 **(2)**
	$\left(\mathrm{fg} : x \mapsto x^2 + 3\mathrm{e}^{x^2}\right)$	
	(c) $\mathrm{fg}(x) \geq 3$	B1 **(1)**
	(d) $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(x^2 + 3\mathrm{e}^{x^2}\right) = 2x + 6x\mathrm{e}^{x^2}$	M1 A1
	$2x + 6x\mathrm{e}^{x^2} = x^2\mathrm{e}^{x^2} + 2x$	
	$\mathrm{e}^{x^2}\left(6x - x^2\right) = 0$	M1
	$\mathrm{e}^{x^2} \neq 0$, $6x - x^2 = 0$	A1
	$x = 0, 6$	A1 A1 **(6)**
		[10]

Question Number	Scheme		Marks	
6.	(a)(i) $\sin 3\theta = \sin(2\theta+\theta)$			
	$= \sin 2\theta \cos\theta + \cos 2\theta \sin\theta$			
	$= 2\sin\theta\cos\theta.\cos\theta + (1-2\sin^2\theta)\sin\theta$		M1 A1	
	$= 2\sin\theta(1-\sin^2\theta) + \sin\theta - 2\sin^3\theta$		M1	
	$= 3\sin\theta - 4\sin^3\theta \quad *$	cso	A1	**(4)**
	(ii) $8\sin^3\theta - 6\sin\theta + 1 = 0$			
	$-2\sin 3\theta + 1 = 0$		M1 A1	
	$\sin 3\theta = \frac{1}{2}$		M1	
	$3\theta = \frac{\pi}{6}, \frac{5\pi}{6}$			
	$\theta = \frac{\pi}{18}, \frac{5\pi}{18}$		A1 A1	**(5)**
	(b) $\sin 15° = \sin(60° - 45°) = \sin 60° \cos 45° - \cos 60° \sin 45°$		M1	
	$= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}}$		M1 A1	
	$= \frac{1}{4}\sqrt{6} - \frac{1}{4}\sqrt{2} = \frac{1}{4}(\sqrt{6}-\sqrt{2}) \quad *$	cso	A1	**(4)**
				[13]
	Alternatives to (b)			
	① $\sin 15° = \sin(45° - 30°) = \sin 45° \cos 30° - \cos 45° \sin 30°$		M1	
	$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$		M1 A1	
	$= \frac{1}{4}\sqrt{6} - \frac{1}{4}\sqrt{2} = \frac{1}{4}(\sqrt{6}-\sqrt{2}) \quad *$	cso	A1	**(4)**
	② Using $\cos 2\theta = 1 - 2\sin^2\theta$, $\cos 30° = 1 - 2\sin^2 15°$			
	$2\sin^2 15° = 1 - \cos 30° = 1 - \frac{\sqrt{3}}{2}$			
	$\sin^2 15° = \frac{2-\sqrt{3}}{4}$		M1 A1	
	$\left(\frac{1}{4}(\sqrt{6}-\sqrt{2})\right)^2 = \frac{1}{16}(6+2-2\sqrt{12}) = \frac{2-\sqrt{3}}{4}$		M1	
	Hence $\sin 15° = \frac{1}{4}(\sqrt{6}-\sqrt{2}) \quad *$	cso	A1	**(4)**

Question Number	Scheme		Marks
7.	(a) $\mathrm{f}'(x) = 3\mathrm{e}^x + 3x\mathrm{e}^x$		M1 A1
	$3\mathrm{e}^x + 3x\mathrm{e}^x = 3\mathrm{e}^x(1+x) = 0$		
	$x = -1$		M1 A1
	$\mathrm{f}(-1) = -3\mathrm{e}^{-1} - 1$		B1 **(5)**
	(b) $x_1 = 0.2596$		B1
	$x_2 = 0.2571$		B1
	$x_3 = 0.2578$		B1 **(3)**
	(c) Choosing $(0.257\,55, 0.257\,65)$ or an appropriate tighter interval.		M1
	$\mathrm{f}(0.257\,55) = -0.000\,379 \ldots$		
	$\mathrm{f}(0.257\,65) = 0.000\,109 \ldots$		A1
	Change of sign (and continuity) $\Rightarrow$ root $\in (0.257\,55, 0.257\,65)$ ∗	cso	A1
	($\Rightarrow x = 0.2576$, is correct to 4 decimal places)		**(3)** **[11]**
	Note: $x = 0.257\,627\,65 \ldots$ is accurate		

Question Number	Scheme		Marks	
8.	(a) $R^2 = 3^2 + 4^2$		M1	
	$R = 5$		A1	
	$\tan\alpha = \frac{4}{3}$		M1	
	$\alpha = 53\ ...\ ^\circ$	awrt 53°	A1	**(4)**
	(b) Maximum value is 5	ft their R	B1 ft	
	At the maximum, $\cos(\theta - \alpha) = 1$ *or* $\theta - \alpha = 0$		M1	
	$\theta = \alpha = 53\ ...\ ^\circ$	ft their α	A1 ft	**(3)**
	(c) $\mathrm{f}(t) = 10 + 5\cos(15t - \alpha)^\circ$			
	Minimum occurs when $\cos(15t - \alpha)^\circ = -1$		M1	
	The minimum temperature is $(10 - 5)^\circ = 5^\circ$		A1 ft	**(2)**
	(d) $15t - \alpha = 180$		M1	
	$t = 15.5$	awrt 15.5	M1 A1	**(3)**
				[12]

Mark Scheme (Results)
January 2010

GCE

Core Mathematics C3 (6665)

Edexcel Limited. Registered in England and Wales No. 4496750
Registered Office: One90 High Holborn, London WC1V 7BH

A PEARSON COMPANY

January 2010

Publications Code US022710

January 2010
6665 Core Mathematics C3
Mark Scheme

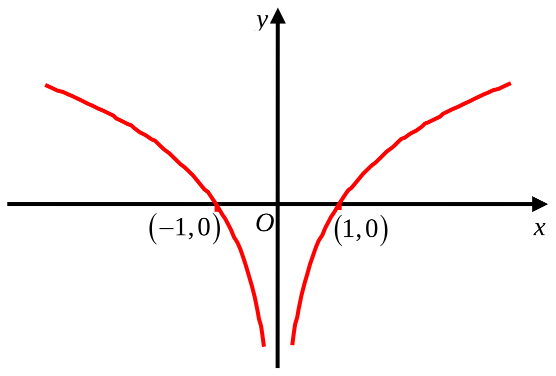
Question Number	Scheme		Marks
Q1	$\dfrac{x+1}{3x^2-3}-\dfrac{1}{3x+1}$		
	$=\dfrac{x+1}{3(x^2-1)}-\dfrac{1}{3x+1}$		
	$=\dfrac{x+1}{3(x+1)(x-1)}-\dfrac{1}{3x+1}$	$x^2-1 \rightarrow (x+1)(x-1)$ or $3x^2-3 \rightarrow (x+1)(3x-3)$ or $3x^2-3 \rightarrow (3x+3)(x-1)$ seen or implied anywhere in candidate's working.	**Award below**
	$=\dfrac{1}{3(x-1)}-\dfrac{1}{3x+1}$		
	$=\dfrac{3x+1-3(x-1)}{3(x-1)(3x+1)}$	Attempt to combine.	M1
	or $\dfrac{3x+1}{3(x-1)(3x+1)}-\dfrac{3(x-1)}{3(x-1)(3x+1)}$	Correct result.	A1
		Decide to award M1 here!!	**M1**
	$=\dfrac{4}{3(x-1)(3x+1)}$	Either $\dfrac{4}{3(x-1)(3x+1)}$ or $\dfrac{\frac{4}{3}}{(x-1)(3x+1)}$ or $\dfrac{4}{(3x-3)(3x+1)}$ or $\dfrac{4}{9x^2-6x-3}$	A1 **aef**
			[4]

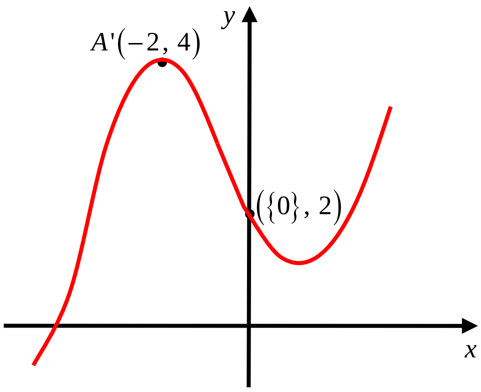
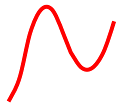
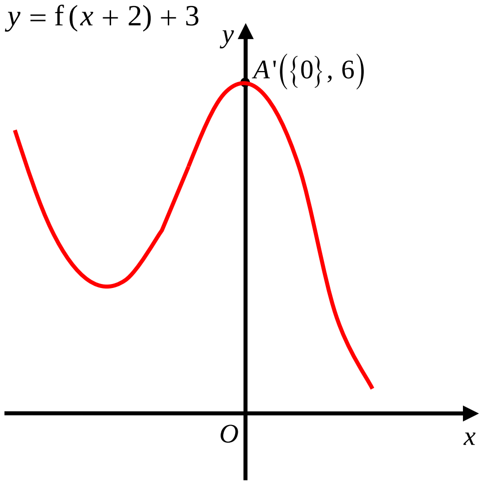
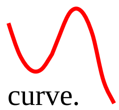
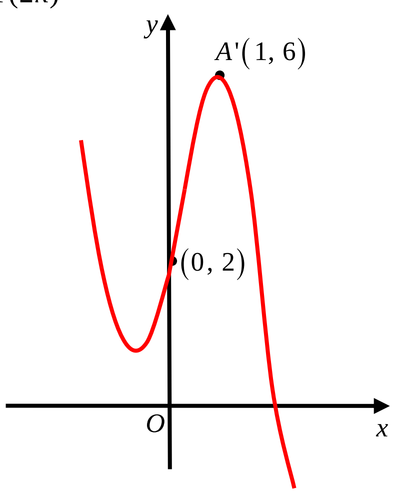
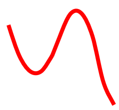
Question Number	Scheme		Marks
Q2	$\mathrm{f}(x) = x^3 + 2x^2 - 3x - 11$		
(a)	$\mathrm{f}(x) = 0 \Rightarrow x^3 + 2x^2 - 3x - 11 = 0$ $\Rightarrow x^2(x+2) - 3x - 11 = 0$	Sets $\mathrm{f}(x) = 0$ (can be implied) and takes out a factor of x^2 from $x^3 + 2x^2$, or x from $x^3 + 2x$ (slip).	M1
	$\Rightarrow x^2(x+2) = 3x + 11$ $\Rightarrow x^2 = \dfrac{3x+11}{x+2}$ $\Rightarrow x = \sqrt{\left(\dfrac{3x+11}{x+2}\right)}$	then rearranges to give the quoted result on the question paper.	A1 **AG**
			(2)
(b)	Iterative formula: $x_{n+1} = \sqrt{\left(\dfrac{3x_n + 11}{x_n + 2}\right)}$, $x_1 = 0$		
	$x_2 = \sqrt{\left(\dfrac{3(0)+11}{(0)+2}\right)}$	An attempt to substitute $x_1 = 0$ into the iterative formula. Can be implied by $x_2 = \sqrt{5.5}$ ***or 2.35*** or awrt 2.345	M1
	$x_2 = 2.34520788...$ $x_3 = 2.037324945...$	Both $x_2 =$ awrt 2.345 and $x_3 =$ awrt 2.037	A1
	$x_4 = 2.058748112...$	$x_4 =$ awrt 2.059	A1
			(3)
(c)	Let $\mathrm{f}(x) = x^3 + 2x^2 - 3x - 11 = 0$		
	$\mathrm{f}(2.0565) = -0.013781637...$ $\mathrm{f}(2.0575) = 0.0041401094...$	Choose suitable interval for x, e.g. [2.0565, 2.0575] or tighter	M1
	Sign change (and $\mathrm{f}(x)$ is continuous) therefore a root α is such that $\alpha \in (2.0565, 2.0575) \Rightarrow \alpha = 2.057$ (3 dp)	any one value awrt 1 sf	dM1
		both values correct awrt 1sf, sign change and conclusion	A1
			(3)
		As a minimum, both values must be correct to 1 sf, candidate states "change of sign, hence root".	
			[8]

Question Number	Scheme	Marks
Q3 (a)	$5\cos x - 3\sin x = R\cos(x+\alpha)$, $R > 0$, $0 < x < \frac{\pi}{2}$	
	$5\cos x - 3\sin x = R\cos x\cos\alpha - R\sin x\sin\alpha$	
	Equate $\cos x$: $5 = R\cos\alpha$ Equate $\sin x$: $3 = R\sin\alpha$	
	$R = \sqrt{5^2+3^2}$; $= \sqrt{34}$ $\{= 5.83095..\}$ — $R^2 = 5^2 + 3^2$	M1;
	$\sqrt{34}$ or awrt 5.8	A1
	$\tan\alpha = \frac{3}{5} \Rightarrow \alpha = 0.5404195003...^c$ — $\tan\alpha = \pm\frac{3}{5}$ or $\tan\alpha = \pm\frac{5}{3}$ or $\sin\alpha = \pm\frac{3}{\text{their } R}$ or $\cos\alpha = \pm\frac{5}{\text{their } R}$	M1
	$\alpha =$ awrt 0.54 or $\alpha =$ awrt 0.17π or $\alpha = \dfrac{\pi}{\text{awrt } 5.8}$	A1
	Hence, $5\cos x - 3\sin x = \sqrt{34}\cos(x + 0.5404)$	
		(4)
(b)	$5\cos x - 3\sin x = 4$	
	$\sqrt{34}\cos(x+0.5404) = 4$	
	$\cos(x+0.5404) = \dfrac{4}{\sqrt{34}}$ $\{= 0.68599...\}$ — $\cos(x \pm \text{their } \alpha) = \dfrac{4}{\text{their } R}$	M1
	$(x+0.5404) = 0.814826916...^c$ — For applying $\cos^{-1}\left(\dfrac{4}{\text{their } R}\right)$	M1
	$x = 0.2744...^c$ — awrt 0.27^c	A1
	$(x+0.5404) = 2\pi - 0.814826916...^c$ $\{= 5.468358...^c\}$ — $2\pi -$ their 0.8148	ddM1
	$x = 4.9279...^c$ — awrt 4.93^c	A1
	Hence, $x = \{0.27, 4.93\}$	
		(5)
		[9]

Part (b): If there are any EXTRA solutions inside the range $0 \le x < 2\pi$, then withhold the final accuracy mark if the candidate would otherwise score all 5 marks. Also ignore EXTRA solutions outside the range $0 \le x < 2\pi$.

Question Number	Scheme		Marks
Q4 (i)	$y = \dfrac{\ln(x^2+1)}{x}$		
	$u = \ln(x^2+1) \quad \Rightarrow \quad \dfrac{du}{dx} = \dfrac{2x}{x^2+1}$	$\ln(x^2+1) \rightarrow \dfrac{\text{something}}{x^2+1}$	M1
		$\ln(x^2+1) \rightarrow \dfrac{2x}{x^2+1}$	A1
	Apply quotient rule: $\left\{\begin{array}{ll} u = \ln(x^2+1) & v = x \\ \dfrac{du}{dx} = \dfrac{2x}{x^2+1} & \dfrac{dv}{dx} = 1 \end{array}\right\}$		
	$\dfrac{dy}{dx} = \dfrac{\left(\dfrac{2x}{x^2+1}\right)(x) - \ln(x^2+1)}{x^2}$	Applying $\dfrac{xu' - \ln(x^2+1)v'}{x^2}$ correctly.	M1
		Correct differentiation with correct bracketing but allow recovery.	A1
			(4)
	$\left\{\dfrac{dy}{dx} = \dfrac{2}{(x^2+1)} - \dfrac{1}{x^2}\ln(x^2+1)\right\}$	{Ignore subsequent working.}	
(ii)	$x = \tan y$		
	$\dfrac{dx}{dy} = \sec^2 y$	$\tan y \rightarrow \sec^2 y$ or an attempt to differentiate $\dfrac{\sin y}{\cos y}$ using either the quotient rule or product rule.	M1*
		$\dfrac{dx}{dy} = \sec^2 y$	A1
	$\dfrac{dy}{dx} = \dfrac{1}{\sec^2 y} \quad \{= \cos^2 y\}$	Finding $\dfrac{dy}{dx}$ by reciprocating $\dfrac{dx}{dy}$.	dM1*
	$\dfrac{dy}{dx} = \dfrac{1}{1+\tan^2 y}$	For writing down or applying the identity $\sec^2 y = 1 + \tan^2 y$, which must be applied/stated completely in y.	dM1*
	Hence, $\dfrac{dy}{dx} = \dfrac{1}{1+x^2}$, (as required)	For the correct proof, leading on from the previous line of working.	A1 **AG**
			(5)
			[9]

Question Number	Scheme	Marks
Q5	$y = \ln\lvert x \rvert$ 	
	Right-hand branch in quadrants 4 and 1. Correct shape.	B1
	Left-hand branch in quadrants 2 and 3. Correct shape.	B1
	Completely correct sketch and both $(-1, \{0\})$ and $(1, \{0\})$	B1
		(3)
		[3]

Question Number	Scheme	Marks
Q6 (i)	$y = \mathrm{f}(-x) + 1$	
		
	Shape of  and must have a maximum in quadrant 2 and a minimum in quadrant 1 or on the positive y-axis.	B1
	Either $(\{0\}, 2)$ or $A'(-2, 4)$	B1
	Both $(\{0\}, 2)$ and $A'(-2, 4)$	B1
		(3)
(ii)	$y = \mathrm{f}(x + 2) + 3$	
		
	Any translation  of the original curve.	B1
	The ***translated maximum*** has either x-coordinate of 0 (can be implied) or y-coordinate of 6.	B1
	The translated curve has maximum $(\{0\}, 6)$ and is in the correct position on the Cartesian axes.	B1
		(3)
(iii)	$y = 2\mathrm{f}(2x)$	
		
	Shape of  with a minimum in quadrant 2 and a maximum in quadrant 1.	B1
	Either $(\{0\}, 2)$ or $A'(1, 6)$	B1
	Both $(\{0\}, 2)$ and $A'(1, 6)$	B1
		(3)
		[9]

Question Number	Scheme		Marks
Q7 (a)	$y = \sec x = \dfrac{1}{\cos x} = (\cos x)^{-1}$		
	$\dfrac{dy}{dx} = -1(\cos x)^{-2}(-\sin x)$	$\dfrac{dy}{dx} = \pm\left((\cos x)^{-2}(\sin x)\right)$	M1
		$-1(\cos x)^{-2}(-\sin x)$ or $(\cos x)^{-2}(\sin x)$	A1
	$\dfrac{dy}{dx} = \left\{\dfrac{\sin x}{\cos^2 x}\right\} = \underline{\underline{\left(\dfrac{1}{\cos x}\right)\left(\dfrac{\sin x}{\cos x}\right)}} = \underline{\underline{\sec x \tan x}}$	Convincing proof. Must see both <u>underlined steps.</u>	A1 **AG**
			(3)
(b)	$y = e^{2x}\sec 3x$		
	$\left\{\begin{array}{ll} u = e^{2x} & v = \sec 3x \\ \dfrac{du}{dx} = 2e^{2x} & \dfrac{dv}{dx} = 3\sec 3x \tan 3x \end{array}\right\}$	Seen or implied. Either $e^{2x} \to 2e^{2x}$ or $\sec 3x \to 3\sec 3x \tan 3x$	M1
		Both $e^{2x} \to 2e^{2x}$ and $\sec 3x \to 3\sec 3x \tan 3x$	A1
	$\dfrac{dy}{dx} = 2e^{2x}\sec 3x + 3e^{2x}\sec 3x \tan 3x$	Applies $vu' + uv'$ correctly for their u, u', v, v'	M1
		$2e^{2x}\sec 3x + 3e^{2x}\sec 3x \tan 3x$	A1 **isw**
			(4)
(c)	Turning point $\Rightarrow \dfrac{dy}{dx} = 0$		
	Hence, $e^{2x}\sec 3x(2 + 3\tan 3x) = 0$	Sets their $\dfrac{dy}{dx} = 0$ and factorises (or cancels) out at least e^{2x} from at least two terms.	M1
	{Note $e^{2x} \neq 0$, $\sec 3x \neq 0$, so $2 + 3\tan 3x = 0$, }		
	giving $\tan 3x = -\frac{2}{3}$	$\tan 3x = \pm k$; $k \neq 0$	M1
	$\Rightarrow 3x = -0.58800 \Rightarrow x = \{a\} = -0.19600...$	Either awrt -0.196^c or awrt -11.2°	A1
	Hence, $y = \{b\} = e^{2(-0.196)}\sec(3 \times -0.196)$		
	$= 0.812093... = 0.812$ (3sf)	0.812	A1 **cao**
			(4)
			[11]

Part (c): If there are any EXTRA solutions for x (or a) inside the range $-\frac{\pi}{6} < x < \frac{\pi}{6}$, ie. $-0.524 < x < 0.524$ or ANY EXTRA solutions for y (or b)**,** (for these values of x) then withhold the final accuracy mark.
Also ignore EXTRA solutions outside the range $-\frac{\pi}{6} < x < \frac{\pi}{6}$, ie. $-0.524 < x < 0.524$.

Question Number	Scheme		Marks
Q8	$\text{cosec}^2 2x - \cot 2x = 1, \quad (\text{eqn } *) \qquad 0 \le x \le 180^{\circ}$		
	Using $\text{cosec}^2 2x = 1 + \cot^2 2x$ gives $1 + \cot^2 2x - \cot 2x = 1$	Writing down or using $\text{cosec}^2 2x = \pm 1 \pm \cot^2 2x$ or $\text{cosec}^2\theta = \pm 1 \pm \cot^2\theta$.	M1
	$\underline{\cot^2 2x - \cot 2x} = 0 \quad$ or $\quad \cot^2 2x = \cot 2x$	For either $\underline{\cot^2 2x - \cot 2x}\ \{= 0\}$ or $\cot^2 2x = \cot 2x$	A1
	$\cot 2x(\cot 2x - 1) = 0 \quad$ or $\quad \cot 2x = 1$	Attempt to factorise or solve a quadratic (See rules for factorising quadratics) or cancelling out $\cot 2x$ from both sides.	dM1
	$\cot 2x = 0 \quad$ or $\quad \cot 2x = 1$	Both $\cot 2x = 0$ and $\cot 2x = 1$.	A1
	$\cot 2x = 0 \Rightarrow (\tan 2x \to \infty) \Rightarrow 2x = 90, 270$ $\Rightarrow x = 45, 135$ $\cot 2x = 1 \Rightarrow \tan 2x = 1 \Rightarrow 2x = 45, 225$ $\Rightarrow x = 22.5, 112.5$	Candidate attempts to divide at least one of their principal angles by 2. This will be usually implied by seeing $x = 22.5$ resulting from $\cot 2x = 1$.	ddM1
	Overall, $x = \{22.5, 45, 112.5, 135\}$	***Both*** $x = 22.5$ and $x = 112.5$ ***Both*** $x = 45$ and $x = 135$	A1 B1
			[7]

If there are any EXTRA solutions inside the range $0 \le x \le 180^{\circ}$ and the candidate would otherwise score FULL MARKS then withhold the final accuracy mark (the sixth mark in this question).
Also ignore EXTRA solutions outside the range $0 \le x \le 180^{\circ}$.

Question Number	Scheme		Marks
Q9 (i)(a)	$\ln(3x-7)=5$		
	$e^{\ln(3x-7)}=e^5$	Takes e of both sides of the equation. This can be implied by $3x-7=e^5$.	M1
	$3x-7=e^5 \Rightarrow x=\dfrac{e^5+7}{3}\ \{=51.804...\}$	Then rearranges to make x the subject.	dM1
		Exact answer of $\dfrac{e^5+7}{3}$.	A1
			(3)
(b)	$3^x e^{7x+2}=15$		
	$\ln\left(3^x e^{7x+2}\right)=\ln 15$	Takes ln (or logs) of both sides of the equation.	M1
	$\ln 3^x+\ln e^{7x+2}=\ln 15$	Applies the addition law of logarithms.	M1
	$x\ln 3+7x+2=\ln 15$	$x\ln 3+7x+2=\ln 15$	A1 oe
	$x(\ln 3+7)=-2+\ln 15$	Factorising out at least two x terms on one side and collecting number terms on the other side.	ddM1
	$x=\dfrac{-2+\ln 15}{7+\ln 3}\ \{=0.0874...\}$	***Exact answer*** of $\dfrac{-2+\ln 15}{7+\ln 3}$	A1 oe
			(5)
(ii) (a)	$f(x)=e^{2x}+3,\ x\in\mathbb{R}$		
	$y=e^{2x}+3 \Rightarrow y-3=e^{2x}$ $\Rightarrow \ln(y-3)=2x$	Attempt to make x (or swapped y) the subject	M1
	$\Rightarrow \frac{1}{2}\ln(y-3)=x$	Makes e^{2x} the subject and takes ln of both sides	M1
	Hence $\underline{f^{-1}(x)=\frac{1}{2}\ln(x-3)}$	$\underline{\frac{1}{2}\ln(x-3)}$ or $\underline{\ln\sqrt{(x-3)}}$ or $\underline{f^{-1}(y)=\frac{1}{2}\ln(y-3)}$ (see appendix)	<u>A1</u> **cao**
	$f^{-1}(x)$: Domain: $\underline{x>3}$ or $\underline{(3,\infty)}$	Either $\underline{x>3}$ or $\underline{(3,\infty)}$ or $\underline{\text{Domain} > 3}$.	B1
			(4)
(b)	$g(x)=\ln(x-1),\ x\in\mathbb{R},\ x>1$		
	$fg(x)=e^{2\ln(x-1)}+3\ \{=(x-1)^2+3\}$	An attempt to put function g into function f.	M1
		$e^{2\ln(x-1)}+3$ or $(x-1)^2+3$ or x^2-2x+4.	A1 isw
	$fg(x)$: Range: $\underline{y>3}$ or $\underline{(3,\infty)}$	Either $\underline{y>3}$ or $\underline{(3,\infty)}$ or $\underline{\text{Range} > 3}$ or $\underline{fg(x)>3}$.	B1
			(3)
			[15]

Further copies of this publication are available from
Edexcel Publications, Adamsway, Mansfield, Notts, NG18 4FN

Telephone 01623 467467
Fax 01623 450481

Email publications@linneydirect.com

Order Code US022710 January 2010

For more information on Edexcel qualifications, please visit www.edexcel.com/quals

Edexcel Limited. Registered in England and Wales no.4496750
Registered Office: One90 High Holborn, London, WC1V 7BH

edexcel

advancing learning, changing lives

Mark Scheme (Results)

January 2011

GCE

GCE Core Mathematics C3 (6665) Paper 1

Edexcel Limited. Registered in England and Wales No. 4496750
Registered Office: One90 High Holborn, London WC1V 7BH

A PEARSON COMPANY

Edexcel is one of the leading examining and awarding bodies in the UK and throughout the world. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers.

Through a network of UK and overseas offices, Edexcel's centres receive the support they need to help them deliver their education and training programmes to learners.

For further information, please call our GCE line on 0844 576 0025, our GCSE team on 0844 576 0027, or visit our website at www.edexcel.com.

If you have any subject specific questions about the content of this Mark Scheme that require the help of a subject specialist, you may find our Ask The Expert email service helpful.

Ask The Expert can be accessed online at the following link:

http://www.edexcel.com/Aboutus/contact-us/

January 2011

Publications Code US026238

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

   - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - ∗ The answer is printed on the paper
   - The second mark is dependent on gaining the first mark

January 2011
Core Mathematics C3 6665
Mark Scheme

Question Number	Scheme		Marks
1. **(a)**	$7\cos x - 24\sin x = R\cos(x+\alpha)$		
	$7\cos x - 24\sin x = R\cos x\cos\alpha - R\sin x\sin\alpha$		
	Equate $\cos x$: $7 = R\cos\alpha$ Equate $\sin x$: $24 = R\sin\alpha$		
	$R = \sqrt{7^2 + 24^2}\,; = 25$	$R = 25$	B1
	$\tan\alpha = \frac{24}{7} \Rightarrow \alpha = 1.287002218...^c$	$\tan\alpha = \frac{24}{7}$ or $\tan\alpha = \frac{7}{24}$ awrt 1.287	M1 A1
	Hence, $7\cos x - 24\sin x = 25\cos(x + 1.287)$		(3)
(b)	Minimum value $= \underline{-25}$	-25 or $-R$	B1ft (1)
(c)	$7\cos x - 24\sin x = 10$		
	$25\cos(x + 1.287) = 10$		
	$\cos(x + 1.287) = \frac{10}{25}$	$\cos(x \pm \text{their } \alpha) = \frac{10}{(\text{their } R)}$	M1
	$\text{PV} = 1.159279481...^c$ or $66.42182152...^\circ$	For applying $\cos^{-1}\left(\frac{10}{\text{their } R}\right)$	M1
	So, $x + 1.287 = \{1.159279...^c, 5.123906...^c, 7.442465...^c\}$	either 2π + or − their PV^c or 360° + or − their PV°	M1
	gives, $x = \{3.836906..., 6.155465...\}$	awrt 3.84 OR 6.16 awrt 3.84 AND 6.16	A1 A1
			(5) **[9]**

Question Number	Scheme		Marks
2.			
(a)	$\dfrac{4x-1}{2(x-1)} - \dfrac{3}{2(x-1)(2x-1)}$		
	$= \dfrac{(4x-1)(2x-1)-3}{2(x-1)(2x-1)}$	An attempt to form a single fraction	M1
	$= \dfrac{8x^2-6x-2}{\{2(x-1)(2x-1)\}}$	Simplifies to give a correct quadratic numerator over a correct quadratic denominator	A1 aef
	$= \dfrac{2(x-1)(4x+1)}{\{2(x-1)(2x-1)\}}$	An attempt to factorise a 3 term quadratic numerator	M1
	$= \dfrac{4x+1}{2x-1}$		A1 (4)
(b)	$f(x) = \dfrac{4x-1}{2(x-1)} - \dfrac{3}{2(x-1)(2x-1)} - 2, \quad x > 1$		
	$f(x) = \dfrac{(4x+1)}{(2x-1)} - 2$		
	$= \dfrac{(4x+1)-2(2x-1)}{(2x-1)}$	An attempt to form a single fraction	M1
	$= \dfrac{4x+1-4x+2}{(2x-1)}$		
	$= \dfrac{3}{(2x-1)}$	Correct result	A1 * (2)
(c)	$f(x) = \dfrac{3}{(2x-1)} = 3(2x-1)^{-1}$		
	$f'(x) = 3(-1)(2x-1)^{-2}(2)$	$\pm k(2x-1)^{-2}$	M1
			A1 aef
	$f'(2) = \dfrac{-6}{9} = -\dfrac{2}{3}$	Either $\dfrac{-6}{9}$ or $-\dfrac{2}{3}$	A1
			(3) **[9]**

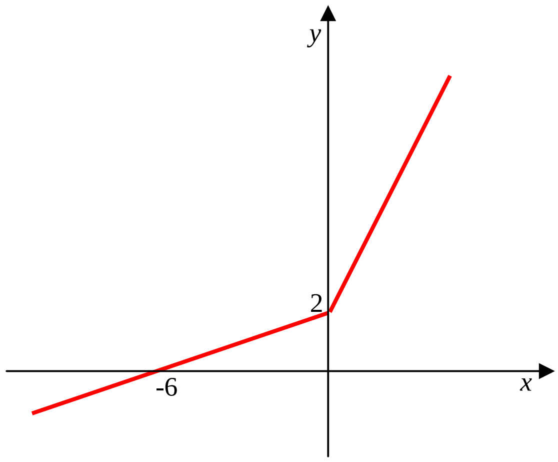
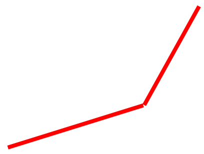
Question Number	Scheme		Marks
3.	$2\cos 2\theta = 1 - 2\sin\theta$		
	$2\left(1 - 2\sin^2\theta\right) = 1 - 2\sin\theta$	Substitutes either $1 - 2\sin^2\theta$ or $2\cos^2\theta - 1$ or $\cos^2\theta - \sin^2\theta$ for $\cos 2\theta$.	M1
	$2 - 4\sin^2\theta = 1 - 2\sin\theta$		
	$4\sin^2\theta - 2\sin\theta - 1 = 0$	Forms a "quadratic in sine" = 0	M1(*)
	$\sin\theta = \dfrac{2 \pm \sqrt{4 - 4(4)(-1)}}{8}$	Applies the quadratic formula See notes for alternative methods.	M1
	PVs: $\alpha_1 = 54^\circ$ or $\alpha_2 = -18^\circ$	Any one correct answer	A1
	$\theta = \{54, 126, 198, 342\}$	180-their pv	dM1(*)
		All four solutions correct.	A1
			[6]

Question Number	Scheme		Marks
4.			
(a)	$\theta = 20 + A\mathrm{e}^{-kt}$ (eqn *)		
	$\{t = 0, \theta = 90 \Rightarrow\}$ $90 = 20 + A\mathrm{e}^{-k(0)}$	Substitutes $t = 0$ and $\theta = 90$ into eqn *	M1
	$90 = 20 + A \Rightarrow \underline{A = 70}$	$\underline{A = 70}$	A1
			(2)
(b)	$\theta = 20 + 70\mathrm{e}^{-kt}$		
	$\{t = 5, \theta = 55 \Rightarrow\}$ $55 = 20 + 70\mathrm{e}^{-k(5)}$ $\frac{35}{70} = \mathrm{e}^{-5k}$	Substitutes $t = 5$ and $\theta = 55$ into eqn * and rearranges eqn * to make $\mathrm{e}^{\pm 5k}$ the subject.	M1
	$\ln\left(\frac{35}{70}\right) = -5k$	Takes 'lns' and proceeds to make '$\pm 5k$' the subject.	dM1
	$-5k = \ln\left(\frac{1}{2}\right)$		
	$-5k = \ln 1 - \ln 2 \Rightarrow -5k = -\ln 2 \Rightarrow \underline{k = \frac{1}{5}\ln 2}$	Convincing proof that $k = \frac{1}{5}\ln 2$	A1 *
			(3)
(c)	$\theta = 20 + 70\mathrm{e}^{-\frac{1}{5}t\ln 2}$		
	$\frac{\mathrm{d}\theta}{\mathrm{d}t} = -\frac{1}{5}\ln 2.(70)\mathrm{e}^{-\frac{1}{5}t\ln 2}$	$\pm \alpha\,\mathrm{e}^{-kt}$ where $k = \frac{1}{5}\ln 2$ $-14\ln 2\,\mathrm{e}^{-\frac{1}{5}t\ln 2}$	M1 A1 oe
	When $t = 10$, $\frac{\mathrm{d}\theta}{\mathrm{d}t} = -14\ln 2\,\mathrm{e}^{-2\ln 2}$		
	$\frac{\mathrm{d}\theta}{\mathrm{d}t} = -\frac{7}{2}\ln 2 = -2.426015132...$		
	Rate of decrease of $\theta = 2.426\ ^{\circ}C/\mathrm{min}$ (3 dp.)	awrt ± 2.426	A1
			(3) **[8]**

Question Number	Scheme		Marks
5. **(a)**	Crosses x-axis $\Rightarrow \mathrm{f}(x) = 0 \Rightarrow (8-x)\ln x = 0$		
	Either $(8-x) = 0$ or $\ln x = 0 \Rightarrow x = 8, 1$	Either one of {x}=1 OR x={8}	B1
	Coordinates are $A(1, 0)$ and $B(8, 0)$.	Both $A(1, \{0\})$ and $B(8, \{0\})$	B1
			(2)
(b)	Apply product rule: $\begin{cases} u = (8-x) & v = \ln x \\ \frac{\mathrm{d}u}{\mathrm{d}x} = -1 & \frac{\mathrm{d}v}{\mathrm{d}x} = \frac{1}{x} \end{cases}$	$vu' + uv'$	M1
	$\mathrm{f}'(x) = -\ln x + \frac{8-x}{x}$	Any one term correct	A1
		Both terms correct	A1
			(3)
(c)	$\mathrm{f}'(3.5) = 0.032951317...$ $\mathrm{f}'(3.6) = -0.058711623...$ Sign change (and as $\mathrm{f}'(x)$ is continuous) therefore the x-coordinate of Q lies between 3.5 and 3.6.	Attempts to evaluate **both** $\mathrm{f}'(3.5)$ and $\mathrm{f}'(3.6)$	M1
		both values correct to at least 1 sf, sign change and conclusion	A1
			(2)
(d)	At Q, $\mathrm{f}'(x) = 0 \Rightarrow -\ln x + \frac{8-x}{x} = 0$	Setting $\mathrm{f}'(x) = 0$.	M1
	$\Rightarrow -\ln x + \frac{8}{x} - 1 = 0$	Splitting up the numerator and proceeding to x=	M1
	$\Rightarrow \frac{8}{x} = \ln x + 1 \Rightarrow 8 = x(\ln x + 1)$		
	$\Rightarrow x = \frac{8}{\ln x + 1}$ (as required)	For correct proof. No errors seen in working.	A1
			(3)

Question Number	Scheme		Marks
(e)	Iterative formula: $x_{n+1} = \dfrac{8}{\ln x_n + 1}$		
	$x_1 = \dfrac{8}{\ln(3.55) + 1}$	An attempt to substitute $x_0 = 3.55$ into the iterative formula. Can be implied by $x_1 = 3.528(97)...$	M1
	$x_1 = 3.528974374...$ $x_2 = 3.538246011...$ $x_3 = 3.534144722...$	Both $x_1 =$ awrt 3.529 and $x_2 =$ awrt 3.538	A1
	$x_1 = 3.529,\ x_2 = 3.538,\ x_3 = 3.534,$ to 3 dp.	x_1 , x_2 , x_3 all stated correctly to 3 dp	A1 (3) **[13]**

Question Number	Scheme		Marks
6.			
(a)	$y = \dfrac{3-2x}{x-5} \Rightarrow y(x-5) = 3-2x$	Attempt to make *x* (or swapped *y*) the subject	M1
	$xy - 5y = 3 - 2x$		
	$\Rightarrow xy + 2x = 3 + 5y \Rightarrow x(y+2) = 3 + 5y$	Collect *x* terms together and factorise.	M1
	$\Rightarrow x = \dfrac{3+5y}{y+2} \quad \therefore \mathrm{f}^{-1}(x) = \underline{\dfrac{3+5x}{x+2}}$	$\underline{\dfrac{3+5x}{x+2}}$	A1 oe
			(3)
(b)	Range of g is $-9 \le \mathrm{g}(x) \le 4$ or $-9 \le y \le 4$	<u>Correct Range</u>	B1 (1)
(c)		Deduces that $\mathrm{g}(2)$ is 0. Seen or implied.	M1
	$\mathrm{g\,g}(2) = \mathrm{g}(0) = -6$, from sketch.	-6	A1 (2)
(d)	$\mathrm{fg}(8) = \mathrm{f}(4)$	Correct order g followed by f	M1
	$= \dfrac{3-4(2)}{4-5} = \dfrac{-5}{-1} = \underline{5}$	5	A1
			(2)

Question Number	Scheme		Marks
(e)(ii)		Correct shape 	B1
		Graph goes through $(\{0\},\ 2)$ and $(-6,\ \{0\})$ which are marked.	B1
			(4)
(f)	Domain of g^{-1} is $-9 \le x \le 4$	Either correct answer or a follow through from part (b) answer	B1 $\sqrt{}$
			(1) **[13]**

edexcel

Question Number	Scheme		Marks
7			
(a)	$y = \dfrac{3 + \sin 2x}{2 + \cos 2x}$		
	Apply quotient rule: $\begin{Bmatrix} u = 3 + \sin 2x & v = 2 + \cos 2x \\ \dfrac{du}{dx} = 2\cos 2x & \dfrac{dv}{dx} = -2\sin 2x \end{Bmatrix}$		
	$\dfrac{dy}{dx} = \dfrac{2\cos 2x(2 + \cos 2x) - -2\sin 2x(3 + \sin 2x)}{(2 + \cos 2x)^2}$	Applying $\dfrac{vu' - uv'}{v^2}$	M1
		Any one term correct on the numerator	A1
		Fully correct (unsimplified).	A1
	$= \dfrac{4\cos 2x + 2\cos^2 2x + 6\sin 2x + 2\sin^2 2x}{(2 + \cos 2x)^2}$		
	$= \dfrac{4\cos 2x + 6\sin 2x + 2(\cos^2 2x + \sin^2 2x)}{(2 + \cos 2x)^2}$		
	$= \dfrac{4\cos 2x + 6\sin 2x + 2}{(2 + \cos 2x)^2}$ (as required)	For correct proof with an understanding that $\cos^2 2x + \sin^2 2x = 1$. No errors seen in working.	A1*
			(4)
(b)	When $x = \frac{\pi}{2}$, $y = \dfrac{3 + \sin \pi}{2 + \cos \pi} = \dfrac{3}{1} = 3$	$y = 3$	B1
	At $\left(\frac{\pi}{2}, 3\right)$, $\mathrm{m}(\mathbf{T}) = \dfrac{6\sin \pi + 4\cos \pi + 2}{(2 + \cos \pi)^2} = \dfrac{-4+2}{1^2} = -2$	$\mathrm{m}(\mathbf{T}) = -2$	B1
	Either **T**: $y - 3 = -2(x - \frac{\pi}{2})$ or $y = -2x + c$ and $3 = -2\left(\frac{\pi}{2}\right) + c \Rightarrow c = 3 + \pi$;	$y - y_1 = m(x - \frac{\pi}{2})$ with 'their TANGENT gradient' and their y_1; or uses $y = mx + c$ with 'their TANGENT gradient';	M1
	T: $y = -2x + (\pi + 3)$	$y = -2x + \pi + 3$	A1
			(4) **[8]**

Question Number	Scheme		Marks
8.			
(a)	$y = \sec x = \dfrac{1}{\cos x} = (\cos x)^{-1}$		
	$\dfrac{dy}{dx} = -1(\cos x)^{-2}(-\sin x)$	Writes $\sec x$ as $(\cos x)^{-1}$ and gives $\dfrac{dy}{dx} = \pm\left((\cos x)^{-2}(\sin x)\right)$	M1
		$-1(\cos x)^{-2}(-\sin x)$ or $(\cos x)^{-2}(\sin x)$	A1
	$\dfrac{dy}{dx} = \left\{\dfrac{\sin x}{\cos^2 x}\right\} = \underline{\underline{\left(\dfrac{1}{\cos x}\right)\left(\dfrac{\sin x}{\cos x}\right)}} = \underline{\underline{\sec x \tan x}}$	Convincing proof. Must see both $\underline{\underline{\text{underlined steps.}}}$	A1 **AG**
			(3)
(b)	$x = \sec 2y, \quad y \neq (2n+1)\frac{\pi}{4}, n \in \mathbb{Z}.$		
	$\dfrac{dx}{dy} = 2\sec 2y \tan 2y$	$K \sec 2y \tan 2y$	M1
		$2\sec 2y \tan 2y$	A1
			(2)
(c)	$\dfrac{dy}{dx} = \dfrac{1}{2\sec 2y \tan 2y}$	Applies $\dfrac{dy}{dx} = \dfrac{1}{\left(\frac{dx}{dy}\right)}$	M1
	$\dfrac{dy}{dx} = \dfrac{1}{2x \tan 2y}$	Substitutes x for $\sec 2y$.	M1
	$1 + \tan^2 A = \sec^2 A \Rightarrow \tan^2 2y = \sec^2 2y - 1$	Attempts to use the identity $1 + \tan^2 A = \sec^2 A$	M1
	So $\tan^2 2y = x^2 - 1$		
	$\dfrac{dy}{dx} = \dfrac{1}{2x\sqrt{(x^2-1)}}$	$\dfrac{dy}{dx} = \dfrac{1}{2x\sqrt{(x^2-1)}}$	A1
			(4)
			[9]

Further copies of this publication are available from
Edexcel Publications, Adamsway, Mansfield, Notts, NG18 4FN

Telephone 01623 467467
Fax 01623 450481

Email publications@linneydirect.com

Order Code US026238 January 2011

For more information on Edexcel qualifications, please visit www.edexcel.com/quals

Edexcel Limited. Registered in England and Wales no.4496750
Registered Office: One90 High Holborn, London, WC1V 7BH

Mark Scheme (Results)

January 2012

GCE Core Mathematics C3 (6665) Paper 1

Edexcel is one of the leading examining and awarding bodies in the UK and throughout the world. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers.

Through a network of UK and overseas offices, Edexcel's centres receive the support they need to help them deliver their education and training programmes to learners.

For further information, please call our GCE line on 0844 576 0025, our GCSE team on 0844 576 0027, or visit our website at www.edexcel.com.

If you have any subject specific questions about the content of this Mark Scheme that require the help of a subject specialist, you may find our **Ask The Expert** email service helpful.

Ask The Expert can be accessed online at the following link:

http://www.edexcel.com/Aboutus/contact-us/

January 2012

Publications Code US030310

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate’s response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⎣ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

General Principals for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. <u>Factorisation</u>

$(x^2 + bx + c) = (x + p)(x + q), \text{ where } |pq| = |c|$, leading to x = ...

$(ax^2 + bx + c) = (mx + p)(nx + q), \text{ where } |pq| = |c| \text{ and } |mn| = |a|$, leading to x = …

2. <u>Formula</u>

Attempt to use <u>correct</u> formula (with values for a, b and c), leading to x = …

3. <u>Completing the square</u>

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c, \quad q \neq 0$, leading to x = …

Method marks for differentiation and integration:

1. <u>Differentiation</u>

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. <u>Integration</u>

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

<u>Method mark</u> for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.

Where the formula is <u>not</u> quoted, the method mark can be gained by implication from <u>correct</u> working with values, but may be lost if there is any mistake in the working.

Question No	Scheme	Marks
1	(a) $\frac{d}{dx}(\ln(3x)) \to \frac{B}{x}$ for any constant B Applying vu'+uv' , $\ln(3x) \times 2x + x$ (b) Applying $\frac{vu'-uv\prime}{v^2}$ $\frac{x^3 \times 4cos(4x) - \sin(4x) \times 3x^2}{x^6}$ $= \frac{4xcos(4x) - 3\sin(4x)}{x^4}$	M1 M1, A1 A1 (4) M1 A1+A1 A1 A1 (5) (9 MARKS)

(a) M1 Differentiates the $\ln(3x)$ term to $\frac{B}{x}$. Note that $\frac{1}{3x}$ is fine for this mark.

M1 Applies the product rule to $x^2\ln(3x)$. If the rule is quoted it must be correct.
There must have been some attempt to differentiate both terms.
If the rule is **not quoted (or implied by their working)** only accept answers of the form
$\ln(3x) \times Ax + x^2 \times \frac{B}{x}$ where A and B are non- zero constants

A1 One term correct and simplified, either $2xln(3x)$or x. $\ln 3x^{2x}$ and $\ln(3x)\,2x$ are acceptable forms

A1 Both terms correct and simplified on the same line. $2xln(3x) + x$, $\ln(3x) \times 2x + x$, $x(2ln3x + 1)$ oe

(b) M1 Applies the quotient rule. A version of this appears in the formula booklet. If the formula is quoted it must be correct. There must have been some attempt to differentiate both terms.
If the formula is **not quoted (nor implied by their working)** only accept answers of the form

$$\frac{x^3 \times \pm Acos(4x) - \sin(4x) \times Bx^2}{(x^3)^2 \text{ or } x^6 \text{ or } x^5 \text{ or } x^9} \text{ with } B > 0$$

A1 Correct first term on numerator $x^3 \times 4cos(4x)$

A1 Correct second term on numerator $-\sin(4x) \times 3x^2$

A1 Correct denominator x^6, the $(x^3)^2$ needs to be simplified

A1 Fully correct simplified expression $\frac{4xcos(4x) - 3\sin(4x)}{x^4}$, $\frac{cos(4x)4x - \sin(4x)3}{x^4}$ oe .
Accept $4x^{-3}\cos(4x) - 3x^{-4}\sin(4x)$ oe

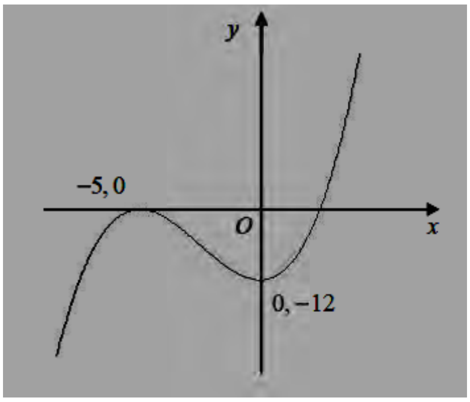
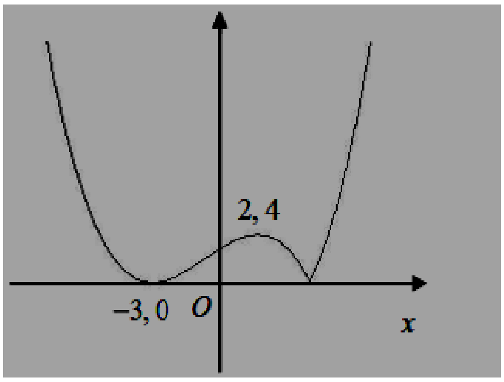
Alternative method using the product rule.

M1,A1 **Writes $\frac{sin(4x)}{x^3}$ as** $\sin(4x) \times x^{-3}$ and applies the product rule. They will score both of these marks or neither of them. If the formula is quoted it must be correct. There must have been some attempt to differentiate both terms. If the formula is **not quoted (nor implied by their working)** only accept answers of the form $x^{-3} \times Acos(4x) + \sin(4x) \times \pm Bx^{-4}$

A1 One term correct, either $x^{-3} \times 4cos(4x)$ or $\sin(4x) \times -3x^{-4}$

A1 Both terms correct,Eg. $x^{-3} \times 4cos(4x) + \sin(4x) \times -3x^{-4}$.

A1 Fully correct expression. $4x^{-3}cos(4x) - 3x^{-4}\sin(4x)$ or $4cos(4x)x^{-3} - 3\sin(4x)x^{-4}$ oe
The negative must have been dealt with for the final mark.

Question No	Scheme		Marks
2	(a)  	Shape x coordinates correct y coordinates correct	B1 B1 B1 (3)
	(b)  	Shape Max at (2,4) Min at (-3,0)	B1 B1 B1 (3)
			6 marks

(a)

B1 Shape unchanged. The positioning of the curve is not significant for this mark. The right hand section of the curve does not have to cross *x* axis.

B1 The x- coordinates of P' and Q' are -5 and 0 respectively. This is for translating the curve 2 units left. The minimum point Q' must be on the y axis. Accept if -5 is marked on the x axis for P' with Q' on the *y* axis (marked -12).

B1 The y- coordinates of P' and Q' are 0 and -12 respectively. This is for the stretch ×3 parallel to the *y* axis. The maximum P' must be on the *x* axis. Accept if -12 is marked on the y axis for Q' with P' on the x axis (marked -5)

(b)

B1 The curve below the x axis reflected in the x axis and the curve above the x axis is unchanged. Do not accept if the curve is clearly rounded off with a zero gradient at the x axis but allow small curvature issues. Use the same principles on the lhs- do not accept if this is a cusp.

B1 Both the x- and y- coordinates of Q', (2,4) given correctly and associated with the maximum point in the first quadrant. To gain this mark there must be a graph and it must only have one maximum.
Accept as 2 and 4 marked on the correct axes or in the script as long as there is no ambiguity.

B1 Both the x- and y- coordinates of P', (-3,0) given correctly and associated with the minimum point in the second quadrant. To gain this mark there must be a graph. Tolerate two cusps if this mark has been lost earlier in the question. Accept (0, -3) marked on the correct axis.

Question No	Scheme	Marks
3	(a) 20 (mm^2)	**B1** **(1)** **M1**

	(b) $'40' = 20\ e^{1.5t} \quad \rightarrow \quad e^{1.5t} = c$ $e^{1.5t} = \frac{40}{20} = \ (2)$ Correct order $1.5t = ln'2' \rightarrow \quad t = \frac{lnc}{1.5}$ $t = \frac{ln2}{1.5} = (\ awrt\ 0.46)$ 12.28 or 28 (minutes}	**A1** **M1** **A1** **A1** **(5)** **(6 marks)**

(a)

B1 Sight of 20 relating to the value of A at t=0. Do not worry about (incorrect) units. Accept its sight in (b)

(b)

M1 Substitutes A=40 or twice their answer to (a) **and** proceeds to $e^{1.5t} = constant$. Accept non numerical answers.

A1 $e^{1.5t} = \frac{40}{20}\ or\ 2$

M1 Correct ln work to find t. Eg $e^{1.5t} = constant \rightarrow 1.5t = \ln(constant) \rightarrow t =$
The order must be correct**.** Accept non numerical answers**. See below for correct alternatives**

A1 Achieves either $\frac{\ln(2)}{1.5}$ or awrt 0.46 2sf

A1 Either 12:28 or 28 (minutes). Cao

Alternatives in (b)

Alt 1- taking ln's of both sides on line 1

M1 Substitutes A=40, or twice (a) takes ln's of both sides **and** proceeds to $\ln('40') = ln20 + lne^{1.5t}$

A1 $\ln(40) = ln20 + 1.5t$

M1 Make t the subject with correct *ln* work.

$$\ln('40') - ln20 = 1.5t\ \ or\ \ \ln\left(\frac{'40'}{20}\right) = 1.5t \rightarrow\ t =$$

A1,A1 are the same

Alt 2- trial and improvement-hopefully seen rarely

M1 Substitutes t= 0.46 and t=0.47 into $20e^{1.5t}$ to obtain A at both values. Must be to at least 2dp but you may accept tighter interval but the interval must span the correct value of 0.46209812

A1 Obtains A(0.46)=39.87 AND A(0.47)=40.47 or equivalent

M1 Substitutes t=0.462 and t=0.4625 into $40e^{1.5t}$

A1 Obtains A(0.462)=39.99 AND A(0.4625)=40.02 or equivalent and states t=0.462 (3sf)

A1 AS ABOVE

No working leading to fully correct accurate answer (3sf or better) send/escalate to team leader

Question No	Scheme	Marks
4	$\left(\frac{dx}{dy}\right) = 2sec^2\left(y+\frac{\pi}{12}\right)$	M1,A1
	substitute $y=\frac{\pi}{4}$ into their $\frac{dx}{dy} = 2sec^2\left(\frac{\pi}{4}+\frac{\pi}{12}\right) = 8$	M1, A1
	When $y=\frac{\pi}{4}$. x=2√3 awrt 3.46	B1
	$\left(y-\frac{\pi}{4}\right) = their\ m(x - their\ 2\sqrt{3})$	M1
	$\left(y-\frac{\pi}{4}\right) = -8(x-2\sqrt{3})$ oe	A1 (7 marks)

M1 For differentiation of $2\tan\left(\boldsymbol{y}+\frac{\pi}{12}\right) \rightarrow 2sec^2\left(\boldsymbol{y}+\frac{\pi}{12}\right)$. There is no need to identify this with $\frac{dx}{dy}$

A1 For correctly writing $\frac{dx}{dy} = 2sec^2\left(y+\frac{\pi}{12}\right)$ or $\frac{dy}{dx} = \frac{1}{2sec^2\left(y+\frac{\pi}{12}\right)}$

M1 Substitute $y=\frac{\pi}{4}$ into their $\frac{dx}{dy}$. Accept if $\frac{dx}{dy}$ is inverted and $y=\frac{\pi}{4}$ substituted into $\frac{dy}{dx}$.

A1 $\frac{dx}{dy}$ = 8 or $\frac{dy}{dx}=\frac{1}{8}$ oe

B1 Obtains the value of x=2√3 corresponding to y= $\frac{\pi}{4}$. Accept awrt 3.46

M1 This mark requires **all of the necessary elements for** finding **a numerical equation** of the **normal.**
Either Invert their value of $\frac{dx}{dy}$, to find $\frac{dy}{dx}$, then use $m_1\times m_2$=-1 to find the numerical gradient of the normal
Or use their numerical value of $-\frac{dx}{dy}$
Having done this then use $\left(y-\frac{\pi}{4}\right) = their\ m(x - their\ 2\sqrt{3})$
The 2√3 could appear as awrt 3.46, the $\frac{\pi}{4}$ as awrt 0.79,
This cannot be awarded for finding the equation of a tangent.
Watch for candidates who correctly use $\left(x - their\ 2\sqrt{3}\right) = -their\ numerical\ \frac{dy}{dx}\left(y-\frac{\pi}{4}\right)$
If they use 'y=mx+c' it must be a full method to find c.

A1 Any correct form of the answer. It does not need to be simplified and the question does not ask for an exact answer.
$\left(y-\frac{\pi}{4}\right) = -8(x-2\sqrt{3})$, $\frac{y-\frac{\pi}{4}}{x-2\sqrt{3}} = -8$, $y=-8x+\frac{\pi}{4}+16\sqrt{3}$, y=-8x+ (awrt) 28.5

Alternatives using arctan (first 3 marks)

M1 Differentiates $y = \arctan\left(\frac{x}{2}\right) - \frac{\pi}{12}$ to get $\frac{1}{1+(\frac{x}{2})^2} \times constant$. Don't worry about the lhs

A1 Achieves $\frac{dy}{dx} = \frac{1}{1+(\frac{x}{2})^2} \times \frac{1}{2}$

M1 This method mark requires *x* to be found, which then needs to be substituted into $\frac{dy}{dx}$

The rest of the marks are then the same.

Or implicitly (first 2 marks)

M1 Differentiates implicitly to get $1 = 2\,sec^2\left(y + \frac{\pi}{12}\right) \times \frac{dy}{dx}$

A1 Rearranges to get $\frac{dy}{dx}$ or $\frac{dx}{dy}$ in terms of y

The rest of the marks are the same

Or by compound angle identities

$x = 2\tan\left(y + \frac{\pi}{12}\right) = \frac{2tany + 2\tan\left(\frac{\pi}{12}\right)}{1 - tany\ tan\frac{\pi}{12}}$ oe

M1 Differentiates using quotient rule-see question 1 in applying this. Additionally the tany **must** have been differentiated to $sec^2 y$. There is no need to assign to $\frac{dx}{dy}$

A1 The correct answer for $\frac{dx}{dy} = \frac{\left(1 - tany\ tan\frac{\pi}{12}\right) \times 2sec^2 y - \left(2tany + 2\tan\left(\frac{\pi}{12}\right)\right) \times -sec^2 y tan\frac{\pi}{12}}{(1 - tany\ tan\frac{\pi}{12})^2}$

The rest of the marks are as the main scheme

Question No	Scheme	Marks
5.	Uses the identity $cot^2(3\theta) = cosec^2(3\theta) - 1$ *in* $2cot^2(3\theta) = 7cosec(3\theta) - 5$	**M1**

	$$2cosec^2(3\theta) - 7cosec(3\theta) + 3 = 0$$	**A1**
	$$(2cosec3\theta - 1)(cosec3\theta - 3) = 0$$	**dM1**
	$$cosec3\theta = 3$$	**A1**
	$$\theta = \frac{invsin(\frac{1}{3})}{3}, \ \frac{19.5°}{3} = awrt\ 6.5°$$	**ddM1, A1**
	$$\theta = \frac{180° - invsin(\frac{1}{3})}{3}, 53.5°$$ Correct 2nd value	**ddM1,A1**
	$$\theta = \frac{360° + invsin(\frac{1}{3})}{3}$$ Correct 3rd value	**ddM1**
	All 4 correct answers awrt 6.5^0,53.5^0,$126.5\ ^0$or 173.5^0	**A1** **(10 marks)**

M1 Uses the substitution $cot^2(3\theta) = \pm 1 \pm cosec^2(3\theta)$ to produce a quadratic equation in *cosec*(3θ)
Accept 'invisible' brackets in which $2cot^2(3\theta)$ is replaced by $2cosec^2(3\theta) - 1$
A (longer) but acceptable alternative is to convert everything to *sin*(3θ).
For this to be scored $cot^2 3\theta$ must be replaced by $\frac{cos^2(3\theta)}{sin^2(3\theta)}$, *cosec*$(3\theta)$ must be replaced by $\frac{1}{\sin 3\theta}$.
An attempt must be made to multiply by $sin^2(3\theta)$ and finally $cos^2(3\theta)$replaced by $= \pm 1 \pm sin^2(3\theta)$

A1 A correct equation (=0) written or implied by working is obtained. Terms must be collected together on one side of the equation. The usual alternatives are
$2cosec^2(3\theta) - 7cosec(3\theta) + 3 = 0$ or $3sin^2(3\theta) - 7sin(3\theta) + 2 = 0$

dM1 Either an attempt to factorise a 3 term quadratic **in *cosec*(3θ) or *sin*(3θ)** with the usual rules
Or use of a correct formula to produce a solution in *cosec*(3θ) or *sin*(3θ)

A1 Obtaining the correct value of $cosec(3\theta) = 3$ or $\sin(3\theta) = \frac{1}{3}$. Ignore other values

ddM1 Correct method to produce the principal value of θ. It is dependent upon the two M's being scored.
Look for $\theta = \frac{invsin(their\ \frac{1}{3})}{3}$

A1 Awrt 6.5

ddM1 Correct method to produce a secondary value. This is dependent upon the candidate having scored the first 2 M's. Usually you look for $\frac{180 - \text{their } 19.5}{3}$ or $\frac{360 + \text{their } 19.5}{3}$ or $\frac{540 - \text{their } 19.5}{3}$
Note 180-their 6.5 must be marked correct BUT 360+their 6.5 is incorrect

A1 Any other correct answer. Awrt 6.5,53.5,126.5 or 173.5

ddM1 Correct method to produce a THIRD value. This is dependent upon the candidate having scored the first 2 M's . See above for alternatives

A1 All 4 correct answers awrt 6.5,53.5,126.5 or 173.5 and no extras inside the range. Ignore any answers outside the range.

Radian answers: awrt 0.11, 0.93, 2.21, 3.03. Accuracy must be to 2dp.
Lose the first mark that could have been scored. Fully correct radian answer scores 1,1,1,1,1,0,1,1,1,1=9 marks
Candidates cannot mix degrees and radians for method marks.

Special case: Some candidates solve the equation in $cosec(\theta\ or\ x)$, $\sin(\theta\ or\ x)$ to produce $cosec(\theta\ or\ x) = 3$

$$\sin(\theta\ or\ x) = \frac{1}{3}.$$

Question No	Scheme	Marks
6	(a) f(0.8) = 0.082, f(0.9)= -0.089 Change of sign $\Rightarrow$ root (0.8,0.9)	M1 A1 **(2)**
	(b) $$f'(x) = 2x - 3 - \sin\left(\frac{1}{2}x\right)$$ Sets $f'(x) = 0 \Rightarrow \quad x = \frac{3+\sin\left(\frac{1}{2}x\right)}{2}$	 M1 A1 M1A1* **(4)**
	(c) Sub x_0=2 into $x_{n+1} = \frac{3+\sin\left(\frac{1}{2}x_n\right)}{2}$ x_1=awrt 1.921, x_2=awrt 1.91(0) and x_3=awrt 1.908	M1 A1,A1 **(3)**
	(d) [1.90775,1.90785] f'(1.90775)=-0.00016.. AND f'(1.90785)= 0.0000076.. Change of sign $\Rightarrow$ x=1.9078	M1 M1 A1 **(3)** **(12 marks)**

(a)

M1 Calculates both f(0.8) and f(0.9). Evidence of this mark could be, either, seeing both '*x*' substitutions written out in the expression, or, one value correct to 1 sig fig, or the appearance of incorrect values of f(0.8)=awrt 0.2 or f(0.9)=awrt 0.1 from use of degrees

A1 This requires both values to be correct as well as a reason and a conclusion.
Accept f(0.8)= 0.08 truncated or rounded (2dp) or 0.1 rounded (1dp) and f(0.9)=-0.08 truncated or rounded as -0.09 (2dp) or -0.1(1dp)
Acceptable reasons are change of sign, <0 >0, +ve –ve, f(0.8)f(0.9)<0. Acceptable conclusion is hence root or $\square$

(b)

M1 Attempts to differentiate f(x). Seeing any of 2x,3 or ±Asin(½x) is sufficient evidence.

A1 f'(x) correct. Accept $\frac{dy}{dx} = 2x - 3 - \sin\left(\frac{1}{2}x\right)$

M1 Sets their f'(x)=0 and proceeds to x=….. You must be sure that they are setting what they think is f'(x)=0.
Accept $2x = 3 + \sin\left(\frac{1}{2}x\right)$ going to x=..only if f'(x) =0 is stated first

A1 * $x = \frac{3+\sin\left(\frac{1}{2}x\right)}{2}$. This is a given answer so don't accept just the sight of this answer. It is cso

(c) M1 Substitutes x_0=2 into $x_{n+1} = \frac{3+\sin\left(\frac{1}{2}x_n\right)}{2}$. Evidence of this mark could be awrt 1.9 or 1.5 (from degrees)

A1 x_1=awrt 1.921

A1 x_2=awrt 1.91(0) and x_3=awrt 1.908

(d) **Continued iteration is not acceptable for this part. Question states 'By choosing a suitable interval…'**

M1 Chooses the interval [1.90775,1.90785] or tighter containing the root= 1.907845522

M1 Calculates f'(1.90775) and f'(1.90785) or tighter with at least one correct, rounded or truncated
f'(1.90775)=-0.0001 truncated or awrt -0.0002 rounded
f'(1.90785)= 0.000007 truncated or awrt 0.000008 rounded

Accept versions of g(*x*)-*x* where g(*x*) $= \frac{3+\sin\left(\frac{1}{2}x\right)}{2}$.
When x= 1.90775, $g(x) - x = 8 \times 10^{-5}$ rounded and truncated
When x= 1.90785, $g(x) - x = -3 \times 10^{-6}$ truncated or $= -4 \times 10^{-6}$ rounded

A1 Both values correct, rounded or truncated, a valid reason (see part a) and a minimal conclusion (see part a). Saying hence root is acceptable. There is no need to refer to the 'turning point'.

Question No	Scheme	Marks
7	(a) $2x^2+7x-4=(2x-1)(x+4)$	B1
	$\dfrac{3(x+1)}{(2x-1)(x+4)}-\dfrac{1}{(x+4)}=\dfrac{3(x+1)-(2x-1)}{(2x-1)(x+4)}$	M1
	$=\frac{x+4}{(2x-1)(x+4)}$	M1
	$=\frac{1}{2x-1}$	A1*
		(4)
	(b) $y=\frac{1}{2x-1}\Rightarrow y(2x-1)=1\Rightarrow 2xy-y=1$	
	$2xy=1+y\Rightarrow x=\dfrac{1+y}{2y}$	M1M1
	$y\ OR\ f^{-1}(x)=\dfrac{1+x}{2x}$	A1
		(3)
	(c) x>0	B1
		(1)
	(d) $\frac{1}{2\ln(x+1)-1}=\frac{1}{7}$	M1
	$\ln(x+1)=4$	A1
	$x=e^4-1$	M1A1
		(4)
		12 Marks

(a)

B1 Factorises the expression $2x^2+7x-4=(2x-1)(x+4)$. This may not be on line 1

M1 Combines the two fractions to form a single fraction with a common denominator. Cubic denominators are fine for this mark. Allow slips on the numerator but one must have been adapted. Allow 'invisible' brackets. Accept two separate fractions with the same denominator. Amongst many possible options are

Correct $\frac{3(x+1)-(2x-1)}{(2x-1)(x+4)}$,Invisible bracket $\frac{3x+1-2x-1}{(2x-1)(x+4)}$,

Cubic and separate $\frac{3(x+1)(x+4)}{(2x^2+7x-4)(x+4)} - \frac{2x^2+7x-4}{(2x^2+7x-4)(x+4)}$

M1 Simplifies the (now) single fraction to one with a linear numerator divided by a quadratic factorised denominator. Any cubic denominator must have been fully factorised (check first and last terms) and cancelled with terms on a fully factorised numerator (check first and last terms).

A1* Cso. This is a given solution and it must be fully correct. All bracketing/algebra must have been correct. You can however accept $\frac{x+4}{(2x-1)(x+4)}$ going to $\frac{1}{2x-1}$ without the need for 'seeing' the cancelling

For example $\frac{3(x+1)-2x-1}{(2x-1)(x+4)} = \frac{x+4}{(2x-1)(x+4)} = \frac{1}{2x-1}$ scores B1,M1,M1,A0. Incorrect line leading to solution.

Whereas $\frac{3(x+1)-(2x-1)}{(2x-1)(x+4)} = \frac{x+4}{(2x-1)(x+4)} = \frac{1}{2x-1}$ scores B1,M1,M1,A1

(b)

M1 This is awarded for an attempt to make x or a swapped y the subject of the formula. The minimum criteria is that they start by multiplying by (2*x*-1) and finish with x= or swapped y=. Allow 'invisible' brackets.

M1 For applying the order of operations correctly. Allow maximum of one 'slip'. Examples of this are

$y = \frac{1}{2x-1} \to y(2x-1) = 1 \to 2x - 1 = \frac{1}{y} \to x = \frac{\frac{1}{y} \pm 1}{2}$ (allow slip on sign)

$y = \frac{1}{2x-1} \to y(2x-1) = 1 \to 2xy - y = 1 \to 2xy = 1 \pm y \to x = \frac{1 \pm y}{2y}$ (allow slip on sign)

$y = \frac{1}{2x-1} \to 2x - 1 = \frac{1}{y} \to 2x = \frac{1}{y} + 1 \to x = \frac{1}{2y} + 1$ (allow slip on ÷2)

A1 Must be written in terms of x but can be $y = \frac{1+x}{2x}$ or equivalent inc $y = \frac{\frac{1}{x}+1}{2}, y = \frac{x^{-1}+1}{2}, y = \frac{1}{2x} + \frac{1}{2}$

(c)

B1 Accept x>0, (0,∞), domain is all values more than 0. **Do not accept** x≥ 0 , y>0, [0, ∞], $f^{-1}(x) > 0$

(d)

M1 Attempt to write down fg(x) and set it equal to 1/7.
The order must be correct but accept incorrect or lack of bracketing. Eg $\frac{1}{2lnx+1-1} = \frac{1}{7}$

A1 Achieving correctly the line $\ln(x+1) = 4$. Accept also $ln(x+1)^2 = 8$

M1 Moving from $\ln(x \pm A) = c \quad A \neq 0 \ to \ x =$ The ln work must be correct
Alternatively moving from $ln(x+1)^2 = c$ to $x = \cdots$
Full solutions to calculate *x* leading from $gf(x) = \frac{1}{7}$, that is $\ln\left(\frac{1}{2x-1} + 1\right) = \frac{1}{7}$ can score this mark.

A1 Correct answer only $= e^4 - 1$. Accept $e^4 - e^0$

Question No	Scheme	Marks
8	(a) $\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)} = \frac{sinAcosB+cosAsinB}{cosAcosB-sinAsinB}$	M1A1
	$= \frac{\frac{sinA}{cosA}+\frac{sinB}{cosB}}{1-\frac{sinAsinB}{cosAcosB}}$ $(\div cosAcosB)$	M1

		$= \dfrac{tanA + tanB}{1 - tanAtanB}$	A1 *
			(4)
	(b)	$\tan\left(\theta + \frac{\pi}{6}\right) = \dfrac{tan\theta + tan\frac{\pi}{6}}{1 - tan\theta tan\frac{\pi}{6}}$	M1
		$= \dfrac{tan\theta + \frac{1}{\sqrt{3}}}{1 - tan\theta\frac{1}{\sqrt{3}}}$	M1
		$= \dfrac{\sqrt{3}tan\theta + 1}{\sqrt{3} - tan\theta}$	A1 *
			(3)
	(c)	$\tan\left(\theta + \frac{\pi}{6}\right) = \tan(\pi - \theta).$	M1
		$\left(\theta + \frac{\pi}{6}\right) = (\pi - \theta)$	dM1
		$\theta = \frac{5}{12}\pi$	ddM1 A1
		$\tan\left(\theta + \frac{\pi}{6}\right) = \tan(2\pi - \theta)$	dddM1
		$\theta = \frac{11}{12}\pi$	A1
			(6)
			(13 MARKS)

(a)

M1 Uses the identity **{** $\tan(A + B) = \frac{\sin(A+B)}{\cos(A+B)}\} = \frac{sinAcosB \pm cosAsinB}{cosAcosB \mp sinAsinB}$. Accept incorrect signs for this. Just the right hand side is acceptable.

A1 Fully correct statement in terms of cos and sin { $\tan(A + B)\} = \frac{sinAcosB + cosAsinB}{cosAcosB - sinAsinB}$

M1 Divide **both** numerator and denominator by cosAcosB. This can be stated or implied by working. If implied you must have seen at least one term modified on both the numerator and denominator.

A1* This is a given solution. The last two principal's reports have highlighted lack of evidence in such questions. Both sides of the identity must be seen or implied. Eg lhs=
The minimum expectation for full marks is

$$\tan(A+B)=\frac{\sin(A+B)}{\cos(A+B)}=\frac{sinAcosB+cosAsinB}{cosAcosB-sinAsinB}=\frac{\frac{sinA}{cosA}+\frac{sinB}{cosB}}{1-\frac{sinAsinB}{cosAcosB}}=\frac{tanA+tanB}{1-tanAtanB}$$

The solution $\tan(A+B)=\frac{\sin(A+B)}{\cos(A+B)}=\frac{sinAcosB+cosAsinB}{cosAcosB-sinAsinB}=\frac{tanA+tanB}{1-tanAtanB}$ scores M1A1M0A0

The solution $\tan(A+B)=\frac{\sin(A+B)}{\cos(A+B)}=\frac{sinAcosB+cosAsinB}{cosAcosB-sinAsinB}\ (\div cosAcosB)=\frac{tanA+tanB}{1-tanAtanB}$ scores M1A1M1A0

(b)

M1 An attempt to use part (a) with A=θ and B=$\frac{\pi}{6}$. Seeing $\frac{tan\theta+tan\frac{\pi}{6}}{1-tan\theta tan\frac{\pi}{6}}$ is enough evidence. Accept sign slips

M1 Uses the identity $\tan\left(\frac{\pi}{6}\right)=\frac{1}{\sqrt{3}}\ or\ \frac{\sqrt{3}}{3}$ in the rhs of the identity on both numerator and denominator

A1* cso. This is a given solution. Both sides of the identity must be seen. All steps must be correct with no unreasonable jumps. Accept

$$\tan\left(\theta+\frac{\pi}{6}\right)=\frac{tan\theta+tan\frac{\pi}{6}}{1-tan\theta tan\frac{\pi}{6}}=\frac{tan\theta+\frac{1}{\sqrt{3}}}{1-tan\theta\frac{1}{\sqrt{3}}}=\frac{\sqrt{3}tan\theta+1}{\sqrt{3}-tan\theta}$$

However the following is only worth 2 out of 3 as the last step is an unreasonable jump without further explanation
.

$$\tan\left(\theta+\frac{\pi}{6}\right)=\frac{tan\theta+tan\frac{\pi}{6}}{1-tan\theta tan\frac{\pi}{6}}=\frac{tan\theta+\frac{\sqrt{3}}{3}}{1-tan\theta\frac{\sqrt{3}}{3}}=\frac{\sqrt{3}tan\theta+1}{\sqrt{3}-tan\theta}$$

(c)

M1 Use the given identity in (b) to obtain $\tan\left(\theta+\frac{\pi}{6}\right)=\tan(\pi-\theta)$. Accept sign slips

dM1 Writes down an equation that will give one value of θ , usually $\theta+\frac{\pi}{6}=\pi-\theta$. This is dependent upon the first M mark. Follow through on slips

ddM1 Attempts to solve their equation in θ. It must end θ= and the first two marks must have been scored.

A1 Cso θ=$\frac{5}{12}\pi$ or $\frac{11}{12}\pi$

dddM1 Writes down an equation that would produce a second value of θ . Usually $\theta+\frac{\pi}{6}=2\pi-\theta$

A1 cso θ= $\frac{5}{12}\pi$ $(accept\ \frac{\pi}{2.4})$and $\frac{11}{12}\pi$ with no extra solutions in the range. Ignore extra solutions outside the range.

Note that under this method one correct solution would score 4 marks. A small number of candidates find the second solution only. They would score 1,1,1,1,0,0

Alternative to (a) starting from rhs

M1 Uses correct identities for both *tan*A and *tan*B in the rhs expression. Accept only errors in signs

A1 $\frac{tanA+tanB}{1-tanAtanB}=\frac{\frac{sinA}{cosA}+\frac{sinB}{cosB}}{1-\frac{sinAsinB}{cosAcosB}}$

M1 Multiplies both numerator and denominator by *cos*A*cos*B. This can be stated or implied by working. If implied you must have seen at least one term modified on both the numerator and denominator

A1 This is a given answer. Correctly completes proof. All three expressions must be seen or implied.

$$\frac{sinAcosB+cosAsinB}{cosAcosB-sinAsinB}=\frac{\sin(A+B)}{\cos(A+B)}=\tan(A+B)$$

Alternative to (a) starting from both sides

The usual method can be marked like this

M1 Uses correct identities for both *tan*A and *tan*B in the rhs expression. Accept only errors in signs

A1 $\frac{tanA+tanB}{1-tanAtanB}=\frac{\frac{sinA}{cosA}+\frac{sinB}{cosB}}{1-\frac{sinAsinB}{cosAcosB}}$

M1 Multiplies both numerator and denominator by *cos*A*cos*B. This can be stated or implied by working. If implied you must have seen at least one term modified on both the numerator and denominator

A1 Completes proof. Starting now from the lhs writes $\tan(A+B)=\frac{\sin(A+B)}{\cos(A+B)}=\frac{sinAcosB+cosAsinB}{cosAcosB-sinAsinB}$

And then states that the lhs is equal to the rhs **Or** hence proven. There must be a statement of closure

Alternative to (b) from sin and cos

M1 Writes $\tan\left(\theta+\frac{\pi}{6}\right)=\frac{\sin(\theta+\frac{\pi}{6})}{\cos(\theta+\frac{\pi}{6})}=\frac{sin\theta cos\frac{\pi}{6}+cos\theta sin\frac{\pi}{6}}{cos\theta cos\frac{\pi}{6}-sin\theta sin\frac{\pi}{6}}$

M1 Uses the identities $\sin\left(\frac{\pi}{6}\right)=\frac{1}{2}$ and $cos\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}$ oe in the rhs of the identity on both numerator and denominator and divides both numerator and denominator by $\cos\theta$ to produce an identity in $\tan\theta$

A1 As in original scheme

<u>Alternative solution for c.</u> Starting with $1+\sqrt{3}tan\theta=(\sqrt{3}-\tan\theta)\,tan(\pi-\theta)$

Let $\tan\theta=t$

$$1+\sqrt{3}t=(\sqrt{3}-t)(-t)$$

$$t^2-2\sqrt{3}t-1=0$$

$$t=\frac{2\sqrt{3}\pm\sqrt{(12+4)}}{2}$$

$$=\sqrt{3}\pm2$$

Must find an exact surd

$$\theta = \frac{5\pi}{12}, \frac{11\pi}{12}$$

Accept the use of a calculator for the A marks as long as there is an exact surd for the solution of the quadratic and exact answers are given.

M1 Starting with $1+\sqrt{3}tan\theta = (\sqrt{3} - \tan\theta)\,tan(\pi - \theta)$ expand $\tan(\pi - \theta)$ by the correct compound angle identity (or otherwise) and substitute tan π=0 to produce an equation in $\tan\theta$

dM1 Collect terms and produce a 3 term quadratic in $\tan\theta$

ddM1 Correct use of quadratic formula to produce exact solutions to $\tan\theta$. All previous marks must have been scored.

dddM1 All 3 previous marks must have been scored. This is for producing two exact values for θ

A1 One solution $\frac{5}{12}\pi$ $(accept\ \frac{\pi}{2.4})$ or $\frac{11}{12}\pi$

A1 Both solutions $\frac{5}{12}\pi$ $(accept\ \frac{\pi}{2.4})$ and $\frac{11}{12}\pi$ and no extra solutions inside the range. Ignore extra solutions outside the range.

Special case: Watch for candidates who write $tan(\pi - \theta) = tan(\pi) - tan(\theta) = -tan(\theta)$ and proceed correctly. They will lose the first mark but potentially can score the others.

Solutions in degrees

Apply as before. Lose the first correct mark that would have been scored-usually 75^0

Further copies of this publication are available from
Edexcel Publications, Adamsway, Mansfield, Notts, NG18 4FN

Telephone 01623 467467
Fax 01623 450481
Email publication.orders@edexcel.com

Order Code US030310 January 2012

For more information on Edexcel qualifications, please visit
www.edexcel.com/quals

Pearson Education Limited. Registered company number 872828
with its registered office at Edinburgh Gate, Harlow, Essex CM20 2JE

Llywodraeth Cynulliad Cymru
Welsh Assembly Government

Rewarding Learning

Mark Scheme (Results)

January 2013

GCE Core Mathematics – C3 (6665/01)

ALWAYS LEARNING

PEARSON

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications come from Pearson, the world's leading learning company. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk for our BTEC qualifications.
Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

If you have any subject specific questions about this specification that require the help of a subject specialist, you can speak directly to the subject team at Pearson.
Their contact details can be found on this link: www.edexcel.com/teachingservices.

You can also use our online Ask the Expert service at www.edexcel.com/ask. You will need an Edexcel username and password to access this service.

Pearson: helping people progress, everywhere
Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

January 2013
Publications Code US034365

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
- Unless indicated in the mark scheme a correct answer with no working should gain full marks for that part of the question.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate's response may differ from the final mark scheme.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper
- $\lfloor$ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but incorrect answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. The maximum mark allocation for each question/part question(item) is set out in the marking grid and you should allocate a score of ‘0’ or ‘1’ for each mark, or “trait”, as shown:

	0	1
aM		●
aA	●	
bM1		●
bA1	●	
bB	●	
bM2		●
bA2		●

January 2013
6665 Core Mathematics C3
Mark Scheme

Question Number	Scheme	Marks
1.	(a) $-32=(2w-3)^5 \Rightarrow w=\frac{1}{2}$ oe	M1A1
		(2)
	(b) $\frac{dy}{dx}=5\times(2x-3)^4\times 2$ or $10(2x-3)^4$	M1A1
	When $x=\frac{1}{2}$, Gradient = 160	M1
	Equation of tangent is $'160'=\frac{y-(-32)}{x-'\frac{1}{2}'}$ oe	dM1
	$y=160x-112$ cso	A1
		(5)
		(7 marks)

(a) M1 Substitute ***y***=-32 into $y=(2w-3)^5$ **and** proceed to *w*=….. [Accept positive sign used of *y*, ie *y*=+32]

A1 Obtains w or $x=\frac{1}{2}$ oe with no incorrect working seen. Accept alternatives such as 0.5.

Sight of just the answer would score both marks as long as no incorrect working is seen.

(b) M1 Attempts to differentiate $y=(2x-3)^5$ using the chain rule.

Sight of $\pm A(2x-3)^4$ where *A* is a non- zero constant is sufficient for the method mark.

A1 A correct (un simplified) form of the differential.

Accept $\frac{dy}{dx}=5\times(2x-3)^4\times 2$ or $\frac{dy}{dx}=10(2x-3)^4$

M1 This is awarded for an attempt to find the gradient of the tangent to the curve at *P*

Award for substituting their numerical value to part (a) into their differential to find the numerical gradient of the tangent

dM1 Award for a correct method to find an equation of the tangent to the curve at *P*. It is dependent upon the previous M mark being awarded.

$$\text{Award for } '\textit{their } 160'=\frac{y-(-32)}{x-\textit{their }'\frac{1}{2}'}$$

If they use $y=mx+c$ it must be a full method, using m= ‘their 160’, their ‘$\frac{1}{2}$’ and -32.

An attempt must be seen to find *c*=…

A1 cso $y = 160x - 112$. The question is specific and requires the answer in this form. You may isw in this question after a correct answer.

Question Number	Scheme	Marks
2.	(a) $0 = e^{x-1} + x - 6 \Rightarrow x = \ln(6-x) + 1$	M1A1* (2)
	(b) Sub $x_0 = 2$ into $x_{n+1} = \ln(6 - x_n) + 1 \Rightarrow x_1 = 2.3863$ AWRT 4 dp. $x_2 = 2.2847$ $x_3 = 2.3125$	M1, A1 A1 (3)
	(c) Chooses interval [2.3065,2.3075]	M1
	g(2.3065)=-0.0002(7), g(2.3075)=0.004(4)	dM1
	Sign change , hence root (correct to 3dp)	A1 (3) **(8 marks)**

(a) M1 Sets g(x)=0,and using correct *ln* work, makes the x of the e^{x-1} term the subject of the formula.
Look for $e^{x-1} + x - 6 = 0 \Rightarrow e^{x-1} = \pm 6 \pm x \Rightarrow x = \ln(\pm 6 \pm x) \pm 1$
Do not accept $e^{x-1} = 6 - x$ **without firstly seeing** $e^{x-1} + x - 6 = 0$ **or a statement that g(x)=0 $\Rightarrow$**
A1* cso. $x = \ln(6-x) + 1$ **Note that this is a given answer (and a proof).**
'Invisible' brackets are allowed for the M but not the A
Do not accept recovery from earlier errors for the A mark. The solution below scores 0 marks.
$0 = e^{x-1} + x - 6 \Rightarrow \quad 0 = x - 1 + \ln(x-6) \Rightarrow x = \ln(6-x) + 1$

(b) M1 Sub $x_0 = 2$ into $x_{n+1} = \ln(6 - x_n) + 1$ to produce a numerical value for x_1.
Evidence for the award could be any of $\ln(6-2)+1$, $\ln 4 + 1$, 2.3….. or awrt 2.4
A1 Answer correct to 4 dp $x_1 = 2.3863$.
The subscript is not important. Mark as the first value given/found.
A1 Awrt 4 dp. $x_2 = 2.2847$ *and* $x_3 = 2.3125$
The subscripts are not important. Mark as the second and third values given/found

(c) M1 Chooses the interval [2.3065,2.3075] or smaller containing the root 2.306558641
dM1 Calculates g(2.3065) and g(2.3075) with at least one of these correct to 1sf.
The answers can be rounded or truncated
g(2.3065) = -0.0003 rounded, g(2.3065)= -0.0002 truncated
g(2.3075) = (+) 0.004 rounded and truncated
A1 Both values correct (rounded or truncated),
A reason which could include change of sign, >0 <0, g(2.3065)× g(2.3075) <0
AND a minimal conclusion such as hence root, α=2.307 or $\square$
Do not accept continued iteration as question demands an interval to be chosen.

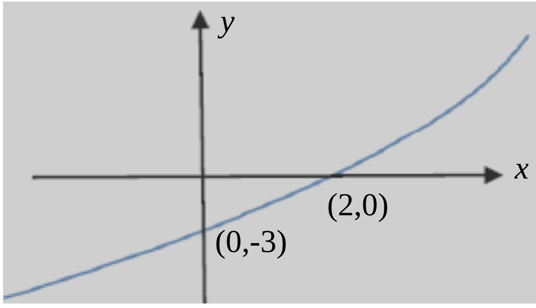
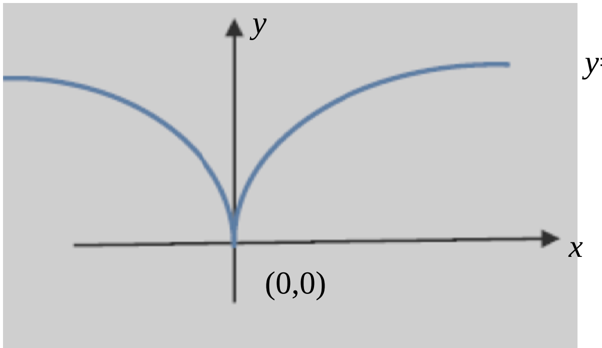
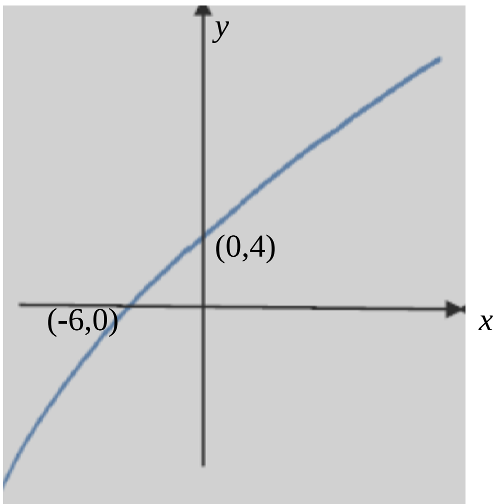
Alternative solution to (a) working backwards

M1 Proceeds from $x = \ln(6-x) + 1$ using correct exp work to …….=0
A1 **Arrives correctly** at $e^{x-1} + x - 6 = 0$ **and** makes a statement to the effect that this is g(x)=0

Alternative solution to (c) using $\mathrm{f}(x) = \ln(6-x) + 1 - x$ **{Similarly** $\mathrm{h}(x) = x - 1 - \ln(6-x)$ **}**

M1 Chooses the interval [2.3065,2.3075] or smaller containing the root 2.306558641
dM1 Calculates f(2.3065) and f(2.3075) with at least 1 correct rounded or truncated
f(2.3065) = 0.000074. Accept 0.00007 rounded or truncated. Also accept 0.0001

$f(2.3075) = -\ 0.0011$.. Accept -0.001 rounded or truncated

Question Number	Scheme	Marks
3.	(a) ff(-3)= f(0),=2	M1,A1
		(2)
	(b)  $y = f^{-1}(x)$	
	Shape	B1
	(0,-3) and (2,0)	B1
		(2)
	(c)  y=f(|x|)-2	
	Shape	B1
	(0,0)	B1
		(2)
	(d) 	
	Shape	B1
	(-6,0) or (0,4)	B1
	(-6,0) and (0,4)	B1
		(3)
		(9 marks)

(a) M1 A full method of finding ff(-3). f(0) is acceptable but f(-3)=0 is not.

Accept a solution obtained from two substitutions into the equation $y = \frac{2}{3}x + 2$ as the line passes through both points. Do not allow for $y = \ln(x+4)$, which only passes through one of the points.

A1 Cao ff(-3)=2. Writing down 2 on its own is enough for both marks provided no incorrect working is seen.

(b)

B1 For the correct shape. Award this mark for an increasing function in quadrants 3, 4 and 1 only. Do not award if the curve bends back on itself or has a clear minimum

B1 This is independent to the first mark and for the graph passing through (0,-3) and (2, 0)

Accept -3 and 2 marked on the correct axes.
Accept (-3,0) and (0,2) instead of (0,-3) and (2,0) as long as they are on the correct axes
Accept P'=(0,-3), Q'=(2,0) stated elsewhere as long as P'and Q' are marked in the correct place on the graph
There must be a graph for this to be awarded

(c)

B1 Award for a correct shape 'roughly' symmetrical about the *y*- axis. It must have a cusp and a gradient that 'decreases' either side of the cusp. Do not award if the graph has a clear maximum

B1 (0,0) lies on their graph. Accept the graph passing through the origin without seeing (0, 0) marked

(d) B1 Shape. The position is not important. The gradient should be always positive but decreasing There should not be a clear maximum point.

B1 The graph passes through (0,4) **or** (-6,0). See part (b) for allowed variations

B1 The graph passes through (0,4) **and** (-6,0). See part (b) for allowed variations

Question Number	Scheme	Marks
4.	(a) $R^2 = 6^2 + 8^2 \Rightarrow R = 10$	M1A1
	$\tan\alpha = \frac{8}{6} \Rightarrow \alpha = \text{awrt } 0.927$	M1A1
		(4)
	(b)(i) $\text{p}(x) = \frac{4}{12 + 10\cos(\theta - 0.927)}$	
	$\text{p}(x) = \frac{4}{12 - 10}$	M1
	Maximum = 2	A1
		(2)
	(b)(ii) $\theta - \text{'}\mathit{their}\ \alpha\text{'} = \pi$	M1
	$\theta = \text{awrt } 4.07$	A1
		(2)
		(8 marks)

(a) M1 Using Pythagoras' Theorem with 6 and 8 to find *R*. Accept $R^2 = 6^2 + 8^2$

If α has been found first accept $R = \pm\frac{8}{\sin\text{'}\alpha\text{'}}$ or $R = \pm\frac{6}{\cos\text{'}\alpha\text{'}}$

A1 $R = 10$. Many candidates will just write this down which is fine for the 2 marks. Accept ± 10 but not -10

M1 For $\tan\alpha = \pm\frac{8}{6}$ or $\tan\alpha = \pm\frac{6}{8}$

If *R* is used then only accept $\sin\alpha = \pm\frac{8}{R}$ or $\cos\alpha = \pm\frac{6}{R}$

A1 $\alpha = \text{awrt } 0.927$. Note that 53.1^0 is A0

(b) Note that (b)(i) and (b)(ii) can be marked together

(i) M1 Award for $\text{p}(x) = \frac{4}{12 - \text{'}R\text{'}}$.

A1 Cao $\text{p}(x)_{\max} = 2$.

The answer is acceptable for both marks as long as no incorrect working is seen

(ii) M1 For setting $\theta - \text{'}\mathit{their}\ \alpha\text{'} = \pi$ and proceeding to θ=..

If working exclusively in degrees accept $\theta - \text{'}\mathit{their}\ \alpha\text{'} = 180$

Do not accept mixed units

A1 $\theta = \text{awrt } 4.07$. If the final A mark in part (a) is lost for 53.1, then accept awrt 233.1

Question Number	Scheme	Marks
5.	(i)(a) $\dfrac{dy}{dx} = 3x^2 \times \ln 2x + x^3 \times \dfrac{1}{2x} \times 2$	M1A1A1
	$= 3x^2 \ln 2x + x^2$	
		(3)
	(i)(b) $\dfrac{dy}{dx} = 3\boldsymbol{(x + \sin 2x)}^2 \times (1 + 2\cos 2x)$	**B1**M1A1
		(3)
	(ii) $\dfrac{dx}{dy} = -\mathrm{cosec}^2 y$	M1A1
	$\dfrac{dy}{dx} = -\dfrac{1}{\mathrm{cosec}^2 y}$	M1
	Uses $\mathrm{cosec}^2 y = 1 + \cot^2 y$ and $x = \cot y$ in $\dfrac{dy}{dx}$ or $\dfrac{dx}{dy}$ to get an expression in x	
	$\dfrac{dy}{dx} = -\dfrac{1}{\mathrm{cosec}^2 y} = -\dfrac{1}{1 + \cot^2 y} = -\dfrac{1}{1 + x^2}$ cso	M1, A1*
		(5)
		(11 marks)

(i)(a) M1 Applies the product rule vu'+uv' to $x^3 \ln 2x$.

If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, with terms written out u=…,u'=….,v=….,v'=….followed by their vu'+uv') then only accept answers of the form

$$Ax^2 \times \ln 2x + x^3 \times \frac{B}{x} \qquad \text{where } A, B \text{ are constants} \neq 0$$

A1 One term correct, either $3x^2 \times \ln 2x$ or $x^3 \times \dfrac{1}{2x} \times 2$

A1 Cao. $\dfrac{dy}{dx} = 3x^2 \times \ln 2x + x^3 \times \dfrac{1}{2x} \times 2$. The answer does not need to be simplified.

For reference the simplified answer is $\dfrac{dy}{dx} = 3x^2 \ln 2x + x^2 = x^2(3\ln 2x + 1)$

(i)(b) B1 Sight of $(x + \sin 2x)^2$

M1 For applying the chain rule to $(x + \sin 2x)^3$. If the rule is quoted it must be correct. If it is not quoted possible forms of evidence could be sight of $C(x + \sin 2x)^2 \times (1 \pm D\cos 2x)$ where C and D are non- zero constants.

Alternatively accept $u = x + \sin 2x$, u'= followed by $Cu^2 \times$ their u'

Do not accept $C(x + \sin 2x)^2 \times 2\cos 2x$ unless you have evidence that this is their u'

Allow 'invisible' brackets for this mark, ie. $C(x + \sin 2x)^2 \times 1 \pm D\cos 2x$

A1 Cao $\dfrac{dy}{dx} = 3(x + \sin 2x)^2 \times (1 + 2\cos 2x)$. There is no requirement to simplify this.

You may ignore subsequent working (isw) after a correct answer in part (i)(a) and (b)

(ii) M1 Writing the derivative of cot***y*** as -cosec^2***y***. It must be in terms of *y*

A1 $\frac{dx}{dy} = -\text{cosec}^2 y$ or $1 = -\text{cosec}^2 y \frac{dy}{dx}$. Both lhs and rhs must be correct.

M1 Using $\frac{dy}{dx} = \frac{1}{dx/dy}$

M1 Using $\text{cosec}^2 y = 1 + \cot^2 y$ **and** $x = \cot y$ to get $\frac{dy}{dx}$ or $\frac{dx}{dy}$ just in terms of *x*.

A1 cso $\frac{dy}{dx} = -\frac{1}{1+x^2}$

Alternative to (a)(i) when ln(2x) is written lnx+ln2

M1 Writes $x^3 \ln 2x$ as $x^3 \ln 2 + x^3 \ln x$.
Achieves Ax^2 for differential of $x^3 \ln 2$ and applies the product rule vu'+uv' to $x^3 \ln x$.

A1 Either $3x^2 \times \ln 2 + 3x^2 \ln x$ or $x^3 \times \frac{1}{x}$

A1 A correct (un simplified) answer. Eg $3x^2 \times \ln 2 + 3x^2 \ln x + x^3 \times \frac{1}{x}$

Alternative to 5(ii) using quotient rule

M1 Writes $\cot y$ as $\frac{\cos y}{\sin y}$ and applies the quotient rule, a form of which appears in the formula book. If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, meaning terms are written out u=…,u'=….,v=….,v'=….followed by their $\frac{vu'-uv'}{v^2}$) only accept answers of the form $\frac{\sin y \times \pm \sin y - \cos y \times \pm \cos y}{(\sin y)^2}$

A1 Correct un simplified answer with both lhs and rhs correct.

$$\frac{dx}{dy} = \frac{\sin y \times -\sin y - \cos y \times \cos y}{(\sin y)^2} = \{-1 - \cot^2 y\}$$

M1 Using $\frac{dy}{dx} = \frac{1}{dx/dy}$

M1 Using $\sin^2 y + \cos^2 y = 1$, $\frac{1}{\sin^2 y} = \text{cosec}^2 y$ and $\text{cosec}^2 y = 1 + \cot^2 y$ to get $\frac{dy}{dx}$ or $\frac{dx}{dy}$ in *x*

A1 cso $\frac{dy}{dx} = -\frac{1}{1+x^2}$

Alternative to 5(ii) using the chain rule, first two marks

M1 Writes $\cot y$ as $(\tan y)^{-1}$ and applies the chain rule (or quotient rule).

Accept answers of the form $-(\tan y)^{-2} \times \sec^2 y$

A1 Correct un simplified answer with both lhs and rhs correct.

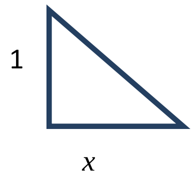
$$\frac{\mathrm{d}x}{\mathrm{d}y} = -(\tan y)^{-2} \times \sec^2 y$$

Alternative to 5(ii) using a triangle – last M1

M1 Uses triangle with $\tan y = \dfrac{1}{x}$ to find siny

and get $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ or $\dfrac{\mathrm{d}x}{\mathrm{d}y}$ just in terms of x

$$x = \cot y \Rightarrow \tan y = \frac{1}{x}$$



$$\sin y = \frac{1}{\sqrt{1+x^2}}$$

Question Number	Scheme	Marks
6.	(i) $(\sin 22.5+\cos 22.5)^2=\sin^2 22.5+\cos^2 22.5+......$	M1
	$=\sin^2 22.5+\cos^2 22.5+2\sin 22.5\cos 22.5$	
	States or uses $\sin^2 22.5+\cos^2 22.5=1$	B1
	Uses $2\sin x\cos x=\sin 2x \Rightarrow 2\sin 22.5\cos 22.5=\sin 45$	M1
	$(\sin 22.5+\cos 22.5)^2=1+\sin 45$	A1
	$=1+\frac{\sqrt{2}}{2}$ or $1+\frac{1}{\sqrt{2}}$ cso	A1
		(5)
	(ii) (a) $\cos 2\theta+\sin\theta=1\Rightarrow 1-2\sin^2\theta+\sin\theta=1$	M1
	$\sin\theta-2\sin^2\theta=0$	
	$2\sin^2\theta-\sin\theta=0$ or $k=2$	A1*
		(2)
	(b) $\sin\theta(2\sin\theta-1)=0$	M1
	$\sin\theta=0,\quad \sin\theta=\frac{1}{2}$	A1
	Any two of 0,30,150,180	B1
	All four answers 0,30,150,180	A1
		(4)
		(11 marks)

(i) M1 Attempts to expand $(\sin 22.5+\cos 22.5)^2$. Award if you see $\sin^2 22.5+\cos^2 22.5+......$

There must be > two terms. Condone missing brackets ie $\sin 22.5^2+\cos 22.5^2+......$

B1 Stating or using $\sin^2 22.5+\cos^2 22.5=1$. Accept $\sin 22.5^2+\cos 22.5^2=1$ as the intention is clear. Note that this may also come from using the double angle formula

$$\sin^2 22.5+\cos^2 22.5=\left(\frac{1-\cos 45}{2}\right)+\left(\frac{1+\cos 45}{2}\right)=1$$

M1 Uses $2\sin x\cos x=\sin 2x$ to write $2\sin 22.5\cos 22.5$ as sin 45 or sin(2×22.5)

A1 Reaching the intermediate answer $1+\sin 45$

A1 Cso $1+\frac{\sqrt{2}}{2}$ or $1+\frac{1}{\sqrt{2}}$. Be aware that both 1.707 and $\frac{2+\sqrt{2}}{2}$ can be found by using a calculator for 1+sin45. Neither can be accepted on their own without firstly seeing one of the two answers given above. **Each stage should be shown as required by the mark scheme.**

Note that if the candidates use $(\sin\theta+\cos\theta)^2$ they can pick up the first M and B marks, but no others until they use $\theta=22.5$. All other marks then become available.

(iia) M1 Substitutes $\cos 2\theta=1-2\sin^2\theta$ in $\cos 2\theta+\sin\theta=1$ to produce an equation in $\sin\theta$ only.

It is acceptable to use $\cos 2\theta=2\cos^2\theta-1$ *or* $\cos^2\theta-\sin^2\theta$ as long as the $\cos^2\theta$ is subsequently replaced by $1-\sin^2\theta$

A1* Obtains the correct simplified equation in $\sin\theta$.

$\sin\theta-2\sin^2\theta=0$ or $\sin\theta=2\sin^2\theta$ must be written in the form $2\sin^2\theta-\sin\theta=0$ as required by the question. Also accept $k=2$ as long as no incorrect working is seen.

(iib) M1 Factorises or divides by $\sin\theta$. For this mark $1='k'\sin\theta$ is acceptable. If they have a 3 TQ in $\sin\theta$ this can be scored for correct factorisation

A1 **Both** $\sin\theta=0$, and $\sin\theta=\frac{1}{2}$

B1 Any two answers from 0, 30, 150, 180.

A1 All four answers 0, 30, 150, 180 with no extra solutions inside the range. Ignore solutions outside the range.

Question Number	Scheme	Marks
6.alt 1	(i) $(\sin 22.5+\cos 22.5)^2=\sin^2 22.5+\cos^2 22.5+......$	M1
	$=\sin^2 22.5+\cos^2 22.5+2\sin 22.5\cos 22.5$	
	States or uses $\sin^2 22.5+\cos^2 22.5=1$	B1
	Uses $2\sin x\cos x=2\sqrt{\dfrac{1-\cos 2x}{2}}\sqrt{\dfrac{\cos 2x+1}{2}}\Rightarrow\sqrt{1-\cos 45}\sqrt{1+\cos 45}$	M1
	$=\sqrt{1-\cos^2 45}$	A1
	Hence $(\sin 22.5+\cos 22.5)^2=1+\dfrac{\sqrt{2}}{2}$ or $1+\dfrac{1}{\sqrt{2}}$	A1
		(5)

Question Number	Scheme	Marks
6.alt 2	(i) Uses Factor Formula $(\sin 22.5+\sin 67.5)^2=\left(2\sin 45\cos 22.5\right)^2$	M1,A1
	Reaching the stage $=2\cos^2 22.5$	B1
	Uses the double angle formula $=2\cos^2 22.5=1+\cos 45$	M1
	$=1+\dfrac{\sqrt{2}}{2}$ or $1+\dfrac{1}{\sqrt{2}}$	A1
		(5)

Question Number	Scheme	Marks
6.alt 3	(i) Uses Factor Formula $(\cos 67.5+\cos 22.5)^2=\left(2\cos 45\cos 22.5\right)^2$	M1,A1
	Reaching the stage $=2\cos^2 22.5$	B1
	Uses the double angle formula $=2\cos^2 22.5=1+\cos 45$	M1
	$=1+\dfrac{\sqrt{2}}{2}$ or $1+\dfrac{1}{\sqrt{2}}$	A1
		(5)

Question Number	Scheme	Marks
7.	(a) $\dfrac{2}{x+2}+\dfrac{4}{x^2+5}-\dfrac{18}{(x+2)(x^2+5)}=\dfrac{2(x^2+5)+4(x+2)-18}{(x+2)(x^2+5)}$	M1A1
	$=\dfrac{2x(x+2)}{(x+2)(x^2+5)}$	M1
	$=\dfrac{2x}{(x^2+5)}$	A1*
		(4)
	(b) $\mathrm{h}'(x)=\dfrac{(x^2+5)\times 2-2x\times 2x}{(x^2+5)^2}$	M1A1
	$\mathrm{h}'(x)=\dfrac{10-2x^2}{(x^2+5)^2}$ cso	A1
		(3)
	(c) Maximum occurs when $\mathrm{h}'(x)=0\Rightarrow 10-2x^2=0\Rightarrow x=..$	M1
	$\Rightarrow x=\sqrt{5}$	A1
	When $x=\sqrt{5}\Rightarrow \mathrm{h}(x)=\dfrac{\sqrt{5}}{5}$	M1,A1
	Range of h(x) is $0\le h(x)\le\dfrac{\sqrt{5}}{5}$	A1ft
		(5) **(12 marks)**

(a) M1 Combines the three fractions to form a single fraction with a common denominator.
Allow errors on the numerator but at least one must have been adapted.
Condone 'invisible' brackets for this mark.
Accept three separate fractions with the same denominator.
Amongst possible options allowed for this method are

$\dfrac{2x^2+5+4x+2-18}{(x+2)(x^2+5)}$ Eg 1 An example of 'invisible' brackets

$\dfrac{2(x^2+5)}{(x+2)(x^2+5)}+\dfrac{4}{(x+2)(x^2+5)}-\dfrac{18}{(x+2)(x^2+5)}$ Eg 2An example of an error (on middle term), 1st term has been adapted

$\dfrac{2(x^2+5)^2(x+2)+4(x+2)^2(x^2+5)-18(x^2+5)(x+2)}{(x+2)^2(x^2+5)^2}$ Eg 3 An example of a correct fraction with a different denominator

A1 Award for a correct un simplified fraction with the correct (lowest) common denominator.

$\dfrac{2(x^2+5)+4(x+2)-18}{(x+2)(x^2+5)}$

Accept if there are three separate fractions with the correct (lowest) common denominator.

Eg $\dfrac{2(x^2+5)}{(x+2)(x^2+5)}+\dfrac{4(x+2)}{(x+2)(x^2+5)}-\dfrac{18}{(x+2)(x^2+5)}$

Note, Example 3 would score M1A0 as it does not have the correct lowest common denominator

M1 There must be a single denominator. Terms must be collected on the numerator. A factor of (x+2) must be taken out of the numerator and then cancelled with one in the denominator. The cancelling may be assumed if the term 'disappears'

A1* Cso $\dfrac{2x}{(x^2+5)}$ This is a given solution and this mark should be withheld if there are any errors

(b) M1 Applies the quotient rule to $\dfrac{2x}{(x^2+5)}$, a form of which appears in the formula book.

If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, meaning terms are written out u=…,u'=….,v=….,v'=….followed by their $\dfrac{vu'-uv'}{v^2}$) then only accept answers of the form

$$\frac{(x^2+5)\times A-2x\times Bx}{(x^2+5)^2} \quad \text{where } A,B>0$$

A1 Correct unsimplified answer $\mathrm{h}'(x)=\dfrac{(x^2+5)\times 2-2x\times 2x}{(x^2+5)^2}$

A1 $\mathrm{h}'(x)=\dfrac{10-2x^2}{(x^2+5)^2}$ The correct simplified answer. Accept $\dfrac{2(5-x^2)}{(x^2+5)^2}$ $\dfrac{-2(x^2-5)}{(x^2+5)^2}$, $\dfrac{10-2x^2}{(x^4+10x^2+25)}$

DO NOT ISW FOR PART (b). INCORRECT SIMPLIFICATION IS A0

(c) M1 Sets their h'(x)=0 and proceeds with a correct method to find x. There must have been an attempt to differentiate. Allow numerical errors but do not allow solutions from 'unsolvable' equations.

A1 Finds the correct x value of the maximum point $x=\sqrt{5}$. Ignore the solution $x=-\sqrt{5}$ but withhold this mark if other positive values found.

M1 Substitutes their answer into their h'(x)=0 in h(x) to determine the maximum value

A1 Cso-the maximum value of h(x) = $\dfrac{\sqrt{5}}{5}$. Accept equivalents such as $\dfrac{2\sqrt{5}}{10}$ **but not** 0.447

A1ft Range of h(x) is $0\le \mathrm{h}(x)\le \dfrac{\sqrt{5}}{5}$. Follow through on their maximum value if the M's have been scored. Allow $0\le y\le \dfrac{\sqrt{5}}{5}$, $0\le Range\le \dfrac{\sqrt{5}}{5}$, $\left[0,\dfrac{\sqrt{5}}{5}\right]$ but not $0\le x\le \dfrac{\sqrt{5}}{5}$, $\left(0,\dfrac{\sqrt{5}}{5}\right)$

If a candidate attempts to work out $h^{-1}(x)$ in (b) and does all that is required for (b) in (c), then allow. Do not allow $h^{-1}(x)$ to be used for h'(x) in part (c). For this question (b) and (c) can be scored together.

Alternative to (b) using the product rule

M1 Sets $\mathrm{h}(x)=2x(x^2+5)^{-1}$ and applies the product rule vu'+uv' with terms being 2x and $(x^2+5)^{-1}$ If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, meaning terms are written out u=…,u'=….,v=….,v'=….followed by their vu'+uv') then only accept answers of the form

$$(x^2+5)^{-1}\times A+2x\times \pm Bx(x^2+5)^{-2}$$

A1 Correct un simplified answer $(x^2+5)^{-1}\times 2+2x\times -2x(x^2+5)^{-2}$

A1 The question asks for h'(x) to be put in its simplest form. Hence in this method the terms need to be combined to form a single correct expression.

For a correct simplified answer accept

$$h'(x)=\frac{10-2x^2}{(x^2+5)^2}=\frac{2(5-x^2)}{(x^2+5)^2}=\frac{-2(x^2-5)}{(x^2+5)^2}=(10-2x^2)(x^2+5)^{-2}$$

Question Number	Scheme	Marks
8.	(a) (£) 19500	B1 (1)
	(b) $9500=17000e^{-0.25t}+2000e^{-0.5t}+500$	
	$17e^{-0.25t}+2e^{-0.5t}=9$	
	$(\times e^{0.5t})\Rightarrow 17e^{0.25t}+2=9e^{0.5t}$	
	$0=9e^{0.5t}-17e^{0.25t}-2$	M1
	$0=(9e^{0.25t}+1)(e^{0.25t}-2)$	M1
	$e^{0.25t}=2$	A1
	$t=4\ln(2)$ *oe*	A1 (4)
	(c)	
	$(\frac{dV}{dt})=-4250e^{-0.25t}-1000e^{-0.5t}$	M1A1
	When t=8 Decrease = 593 (£/year)	M1A1 (4)
		(9 marks)

(a) B1 19500. The £ sign is not important for this mark

(b) M1 Substitute V=9500, collect terms and set on 1 side of an equation =0. Indices must be correct
Accept $17000e^{-0.25t}+2000e^{-0.5t}-9000=0$ and $17000x+2000x^2-9000=0$ where $x=e^{-0.25t}$

M1 Factorise the quadratic in $e^{0.25t}$ or $e^{-0.25t}$
For your information the factorised quadratic in $e^{-0.25t}$ is $(2e^{-0.25t}-1)(e^{-0.25t}+9)=0$
Alternatively let $'x'=e^{0.25t}$ or otherwise and factorise a quadratic equation in x

A1 Correct solution of the quadratic. Either $e^{0.25t}=2$ or $e^{-0.25t}=\frac{1}{2}$ oe.

A1 Correct exact value of t. Accept variations of $4\ln(2)$, such as $\ln(16)$, $\frac{\ln(\frac{1}{2})}{-0.25}$, $\frac{\ln(2)}{0.25}$, $-4\ln(\frac{1}{2})$

.(c) M1 Differentiates $V=17000e^{-0.25t}+2000e^{-0.5t}+500$ by the chain rule.
Accept answers of the form $(\frac{dV}{dt})=\pm Ae^{-0.25t}\pm Be^{-0.5t}$ A,B are constants $\neq 0$

A1 Correct derivative $(\frac{dV}{dt})=-4250e^{-0.25t}-1000e^{-0.5t}$.
There is no need for it to be simplified so accept

$$(\frac{dV}{dt})=17000\times-0.25e^{-0.25t}+2000\times-0.5e^{-0.5t} \quad oe$$

M1 Substitute t=8 into their $\frac{dV}{dt}$.
This is not dependent upon the first M1 but there must have been some attempt to differentiate.
Do not accept t=8 in V

A1 ±593. Ignore the sign and the units. If the candidate then divides by 8, withhold this mark. **This would not be isw. Be aware that sub t=8 into V first and then differentiating can achieve 593. This is M0A0M0A0.**

Further copies of this publication are available from
Edexcel Publications, Adamsway, Mansfield, Notts, NG18 4FN

Telephone 01623 467467
Fax 01623 450481
Email publication.orders@edexcel.com

Order Code US034365 January 2013

For more information on Edexcel qualifications, please visit our website
www.edexcel.com

Pearson Education Limited. Registered company number 872828
with its registered office at Edinburgh Gate, Harlow, Essex CM20 2JE

Llywodraeth Cynulliad Cymru
Welsh Assembly Government

Rewarding Learning

GCE

Edexcel GCE

Core Mathematics C3 (6665)

Summer 2005

advancing learning, changing lives

Mark Scheme (Results)

Edexcel GCE
Core Mathematics C3 (6665)

June 2005
6665 Core C3
Mark Scheme

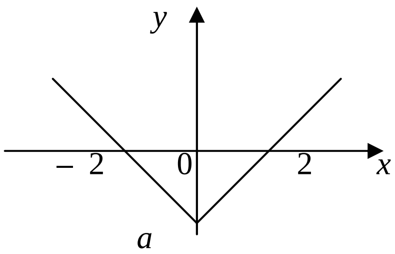
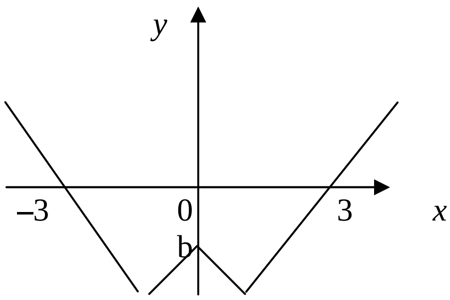
Question Number	Scheme	Marks
1. (a)	Dividing by $\cos^2\theta$: $\dfrac{\sin^2\theta}{\cos^2\theta}+\dfrac{\cos^2\theta}{\cos^2\theta}\equiv\dfrac{1}{\cos^2\theta}$	M1
	Completion **:** $1+\tan^2\theta\equiv\sec^2\theta$ (no errors seen)	A1
		(2)
(b)	Use of $1+\tan^2\theta=\sec^2\theta$: $2(\sec^2\theta-1)+\sec\theta=1$	M1
	$[\ 2\sec^2\theta+\sec\theta-3=0\ ]$	
	Factorising or solving: $(2\sec\theta+3)(\sec\theta-1)=0$	M1
	$[\ \sec\theta=-\frac{3}{2}$ or $\sec\theta=1\ ]$	
	$\theta=0$	B1
	$\cos\theta=-\frac{2}{3}$; $\theta_1=131.8°$	M1 A1
	$\theta_2=228.2°$	A1√
		(6)
	[A1ft for $\theta_2=360°-\theta_1$]	**[8]**

Question Number	Scheme	Marks
2. (a)	(i) $6\sin x\cos x + 2\sec 2x\tan 2x$ **or** $3\sin 2x + 2\sec 2x\tan 2x$ [M1 for 6 sinx]	M1A1A1 (3)
	(ii) $3(x+\ln 2x)^2(1+\frac{1}{x})$ [B1 for $3(x+\ln 2x)^2$]	B1M1A1 (3)
(b)	Differentiating numerator to obtain $10x - 10$	B1
	Differentiating denominator to obtain $2(x\text{-}1)$	B1
	Using quotient rule formula correctly:	M1
	To obtain $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(x-1)^2(10x-10)-(5x^2-10x+9)2(x-1)}{(x-1)^4}$	A1
	Simplifying to form $\frac{2(x-1)[5(x-1)^2-(5x^2-10x+9)}{(x-1)^4}$	M1
	$= -\frac{8}{(x-1)^3}$ * (c.s.o.)	A1 (6)
		[12]
	Alternatives for (b)	
	Either Using product rule formula correctly:	M1
	Obtaining 10x – 10	B1
	Obtaining $-2(x-1)^{-3}$	B1
	To obtain $\frac{\mathrm{d}y}{\mathrm{d}x} = (5x^2-10x+9)\{-2(x-1)^{-3}\}+(10x-10)(x-1)^{-2}$	A1 cao
	Simplifying to form $\frac{10(x-1)^2 - 2(5x^2-10x+9)}{(x-1)^3}$	M1
	$= -\frac{8}{(x-1)^3}$ * (c.s.o.)	A1 (6)
	Or Splitting fraction to give $5+\frac{4}{(x-1)^2}$	M1 B1 B1
	Then differentiating to give answer	M1 A1 A1 (6)

Question Number	Scheme	Marks
3(a)	$\dfrac{5x+1}{(x+2)(x-1)} - \dfrac{3}{x+2}$	B1
	$= \dfrac{5x+1-3(x-1)}{(x+2)(x-1)}$	M1
	M1 for combining fractions even if the denominator is not lowest common	
	$= \dfrac{2x+4}{(x+2)(x-1)} = \dfrac{2(x+2)}{(x+2)(x-1)} = \dfrac{2}{x-1}$ *	M1 A1 cso
	M1 must have linear numerator	(4)
(b)	$y = \dfrac{2}{x-1} \Rightarrow xy - y = 2 \Rightarrow xy = 2 + y$	M1A1
	$f^{-1}(x) = \dfrac{2+x}{x}$ o.e.	A1 (3)
(c)	$fg(x) = \dfrac{2}{x^2+4}$ (attempt) $\left[\dfrac{2}{"g"-1}\right]$	M1
	Setting $\dfrac{2}{x^2+4} = \dfrac{1}{4}$ and finding $x^2 = \ldots$; $x = \pm\ 2$	M1; A1 (3)
		[10]

Question Number	Scheme	Marks
4 (a)	$f'(x) = 3e^x - \dfrac{1}{2x}$	M1A1A1 (3)
(b)	$3e^x - \dfrac{1}{2x} = 0$	M1
	$\Rightarrow 6\alpha e^{\alpha} = 1 \quad \Rightarrow \alpha = \frac{1}{6}e^{-\alpha} \quad (*)$	A1 cso (2)
(c)	$x_1 = 0.0613...,\ x_2 = 0.1568..,\ x_3 = 0.1425...,\ x_4 = 0.1445....$	M1 A1 (2)
	[M1 at least x_1 correct, A1 all correct to 4 d.p.]	
	(d) Using $f'(x) = 3e^x - \dfrac{1}{2x}$ with suitable interval	
	e.g. $f'(0.14425) = -0.0007$ $f'(0.14435) = +0.002(1)$	M1
	Accuracy (change of sign and correct values)	A1 (2)
		[9]

Question Number	Scheme	Marks
5. (a)	$\cos 2A = \cos^2 A - \sin^2 A$ (+ use of $\cos^2 A + \sin^2 A \equiv 1$)	M1
	$= (1 - \sin^2 A); - \sin^2 A = 1 - 2\sin^2 A$ (*)	A1 (2)
(b)	$2\sin 2\theta - 3\cos 2\theta - 3\sin\theta + 3 \equiv 4\sin\theta\cos\theta; -3(1 - 2\sin^2\theta) - 3\sin\theta + 3$	B1; M1
	$\equiv 4\sin\theta\cos\theta + 6\sin^2\theta - 3\sin\theta$	M1
	$\equiv \sin\theta(4\cos\theta + 6\sin\theta - 3)$ (*)	A1 (4)
(c)	$4\cos\theta + 6\sin\theta \equiv R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$	
	Complete method for R (may be implied by correct answer)	
	[$R^2 = 4^2 + 6^2$, $R\sin\alpha = 4$, $R\cos\alpha = 6$]	M1
	$R = \sqrt{52}$ or 7.21	A1
	Complete method for α; $\alpha = 0.588$ (allow 33.7º)	M1 A1
		(4)
(d)	$\sin\theta\ (4\cos\theta + 6\sin\theta - 3) = 0$	M1
	$\theta = 0$	B1
	$\sin(\theta + 0.588) = \dfrac{3}{\sqrt{52}} = 0.4160..$ (24.6º)	M1
	$\theta + 0.588 = (0.4291),\ 2.7125$ [or $\theta + 33.7° = (24.6º),\ 155.4°$]	dM1
	$\theta = 2.12$ cao	A1
		(5)
		[15]

Question Number	Scheme	Marks
6. (a)	 Translation ← by 1	M1
	Intercepts correct	A1 (2)
(b)	 $x \geq 0$, correct “shape” provided graph is not original graph	B1
	Reflection in y-axis	B1√
	Intercepts correct	B1 (3)
(c)	$a = -2, \quad b = -1$	B1B1 (2)
(d)	Intersection of $y = 5x$ with $y = -x - 1$	M1A1
	Solving to give $x = -\frac{1}{6}$	M1A1 (4)
	[Notes: (i) If both values found for $5x = -x - 1$ and $5x = x - 3$, or solved algebraically, can score 3 out of 4 for $x = -\frac{1}{6}$ and $x = -¾$; required to eliminate $x = -¾$ for final mark. (ii) Squaring approach: M1 correct method, $24x^2 + 22x + 3 = 0$ (correct 3 term quadratic, any form) A1 Solving M1, Final correct answer A1.]	**[11]**

7. (a)	Setting $p = 300$ at $t = 0 \Rightarrow \; 300 = \dfrac{2800a}{1 + a}$	M1
	(300 = 2500a); a = 0.12 (c.s.o) *	dM1A1 (3)
(b)	$1850 = \dfrac{2800(0.12)e^{0.2t}}{1 + 0.12e^{0.2t}}$; $e^{0.2t} = 16.2...$	M1A1
	Correctly taking logs to $0.2\,t = \ln k$	M1
	$t = 14$ (13.9..)	A1 (4)
(c)	Correct derivation: (Showing division of num. and den. by $e^{0.2t}$; using a)	B1 (1)
(d)	Using $t \to \infty$, $e^{-0.2t} \to 0$,	M1
	$p \to \dfrac{336}{0.12} = 2800$	A1 (2)
		[10]

GCE

Edexcel GCE

Mathematics

Core Mathematics C3 (6665)

June 2006

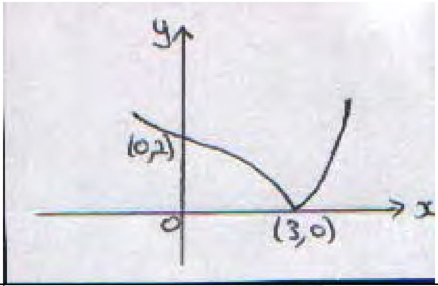
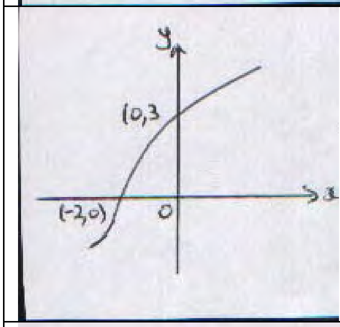
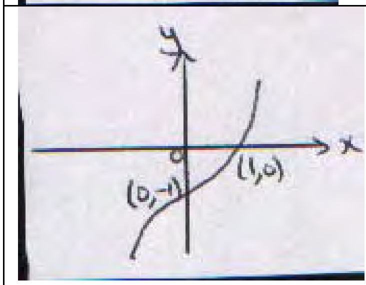
advancing learning, changing lives

Mark Scheme (Results)

Edexcel GCE

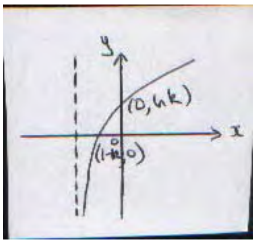
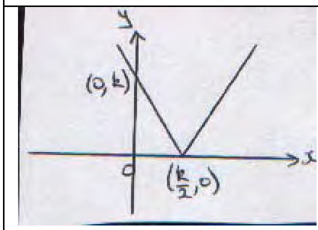
Mathematics

Question Number	Scheme	Marks
1. (*a*)	$\dfrac{(3x+2)(x-1)}{(x+1)(x-1)}, \quad = \quad \dfrac{3x+2}{x+1}$ **Notes** M1 attempt to factorise numerator, *usual rules* B1 factorising denominator seen anywhere in (a), A1 given answer If factorisation of denom. not seen, correct answer implies B1	M1B1, A1 (3)
(*b*)	Expressing over common denominator $\dfrac{3x+2}{x+1} - \dfrac{1}{x(x+1)} = \dfrac{x(3x+2)-1}{x(x+1)}$ [Or "Otherwise" : $\dfrac{(3x^2 - x - 2)x - (x-1)}{x(x^2 - 1)}$] Multiplying out numerator and attempt to factorise $\left[3x^2 + 2x - 1 \equiv (3x-1)(x+1)\right]$ Answer: $\dfrac{3x-1}{x}$	M1 M1 A1 (3) **(6 marks)**
2. (*a*)	$\dfrac{dy}{dx} = 3e^{3x} + \dfrac{1}{x}$ **Notes** B1 $3e^{3x}$ M1 : $\dfrac{a}{bx}$ A1: $3e^{3x} + \dfrac{1}{x}$	B1M1A1(3)
(b)	$(5+x^2)^{\frac{1}{2}}$ $\dfrac{3}{2}(5+x^2)^{\frac{1}{2}} \; . \; 2x \quad = 3x(5+x^2)^{\frac{1}{2}}$ M1 for $kx(5+x^2)^m$	B1 M1 A1 (3) **(6 marks)**

Question Number		Scheme		Marks
3. (*a*)		Mod graph, reflect for $y < 0$	M1	
		(0, 2), (3, 0) or marked on axes	A1	
		Correct shape, including cusp	A1	(3)
(*b*)		Attempt at reflection in $y = x$	M1	
		Curvature correct	A1	
		(–2, 0), (0, 3) or equiv.	B1	(3)
(*c*)		Attempt at 'stretches'	M1	
		(0, –1) or equiv.	B1	
		(1, 0)	B1	(3)
				(9 marks)
4. (*a*)	425 ºC		B1	(1)
(*b*)	$300 = 400\,e^{-0.05t} + 25 \quad \Rightarrow 400\,e^{-0.05t} = 275$	sub. $T = 300$ and attempt to rearrange to $e^{-0.05t} = a$, where $a \in \mathbb{Q}$	M1	
	$e^{-0.05t} = \dfrac{275}{400}$		A1	
	M1 correct application of logs		M1	
	$t =$ 7.49		A1	(4)
(*c*)	$\dfrac{dT}{dt} = -20\ e^{-0.05\ t}$	(M1 for $k\,e^{-0.05\ t}$)	M1 A1	
	At $t = 50$, rate of decrease = (±) 1.64 ºC/min		A1	(3)
(*d*)	$T > 25$, (since $e^{-0.05\ t} \to 0$ as $t \to \infty$)		B1	(1)
			(9 marks)	

Question Number	Scheme	Marks
5. (*a*)	Using product rule: $\frac{dy}{dx} = 2\tan 2x + 2(2x-1)\sec^2 2x$	M1 A1 A1
	Use of "$\tan 2x = \frac{\sin 2x}{\cos 2x}$" and "$\sec 2x = \frac{1}{\cos 2x}$"	M1
	$\left[= 2\frac{\sin 2x}{\cos 2x} + 2(2x-1)\frac{1}{\cos^2 2x}\right]$	
	Setting $\frac{dy}{dx} = 0$ and multiplying through to eliminate fractions	M1
	$[\Rightarrow \quad 2\sin 2x\cos 2x + 2(2x-1) = 0]$	
	Completion: producing $4k + \sin 4k - 2 = 0$ with no wrong working seen and at least previous line seen. AG	A1* (6)
(*b*)	$x_1 = 0.2670$, $x_2 = 0.2809$, $x_3 = 0.2746$, $x_4 = 0.2774$,	M1 A1 A1 (3)
	Note: M1 for first correct application, first A1 for two correct, second A1 for all four correct Max –1 deduction, if ALL correct to > 4 d.p. M1 A0 A1 SC: degree mode: M1 $x_1 = 0.4948$, A1 for $x_2 = 0$ .4914, then A0; max 2	
(*c*)	Choose suitable interval for k: e.g. [0.2765, 0.2775] and evaluate f(x) at these values	M1
	Show that $4k + \sin 4k - 2$ changes sign and deduction	A1 (2)
	[f(0.2765) = –0.000087.., f(0.2775) = +0.0057]	
	Note: Continued iteration: (no marks in degree mode) Some evidence of further iterations leading to 0.2765 or better M1; Deduction A1	
		(11 marks)

Question Number	Scheme	Marks
6. (*a*)	Dividing $\sin^2\theta + \cos^2\theta \equiv 1$ by $\sin^2\theta$ to give $\dfrac{\sin^2\theta}{\sin^2\theta} + \dfrac{\cos^2\theta}{\sin^2\theta} \equiv \dfrac{1}{\sin^2\theta}$	M1
	Completion: $1 + \cot^2\theta \equiv \operatorname{cosec}^2\theta \Rightarrow \operatorname{cosec}^2\theta - \cot^2\theta \equiv 1$ AG	A1* (2)
(*b*)	$\operatorname{cosec}^4\theta - \cot^4\theta \equiv (\operatorname{cosec}^2\theta - \cot^2\theta)(\operatorname{cosec}^2\theta + \cot^2\theta)$	M1
	$\equiv (\operatorname{cosec}^2\theta + \cot^2\theta)$ using (a) AG	A1* (2)
	Notes: (i) Using LHS = $(1+\cot^2\theta)^2 - \cot^4\theta$, using (a) & elim. $\cot^4\theta$ M1, conclusion {using (a) again} A1* (ii) Conversion to sines and cosines: needs $\dfrac{(1-\cos^2\theta)(1+\cos^2\theta)}{\sin^4\theta}$ for M1	
(*c*)	Using (b) to form $\operatorname{cosec}^2\theta + \cot^2\theta \equiv 2 - \cot\theta$	M1
	Forming quadratic in $\cot\theta$	M1
	$\Rightarrow 1 + \cot^2\theta + \cot^2\theta \equiv 2 - \cot\theta$ {using (a)}	
	$2\cot^2\theta + \cot\theta - 1 = 0$	A1
	Solving: $(2\cot\theta - 1)(\cot\theta + 1) = 0$ to $\cot\theta =$	M1
	$\left(\cot\theta = \frac{1}{2}\right)$ or $\cot\theta = -1$	A1
	$\theta = 135°$ (or correct value(s) for candidate dep. on 3Ms)	A1√ (6)
	Note: Ignore solutions outside range Extra "solutions" in range loses A1√, but candidate may possibly have more than one "correct" solution.	**(10 marks)**

Question Number	Scheme	Marks
7. (*a*)	 Log graph: Shape	B1
	Intersection with –ve x-axis	dB1
	$(0, \ln k)$, $(1 - k, 0)$	B1
	 Mod graph :V shape, vertex on +ve x-axis	B1
	$(0, k)$ and $\left(\frac{k}{2}, 0\right)$	B1 (5)
(*b*)	$f(x) \in \mathbb{R}$, $-\infty < f(x) < \infty$, $-\infty < y < \infty$	B1 (1)
(*c*)	$fg\left(\frac{k}{4}\right) = \ln\{k + \lvert\frac{2k}{4} - k\rvert\}$ or $f\left(\lvert-\frac{k}{2}\rvert\right)$	M1
	$= \ln\left(\frac{3k}{2}\right)$	A1 (2)
(*d*)	$\frac{dy}{dx} = \frac{1}{x + k}$	B1
	Equating (with $x = 3$) to grad. of line; $\frac{1}{3 + k} = \frac{2}{9}$	M1; A1
	$k = 1\frac{1}{2}$	A1√ (4)
		(12 marks)

Question Number	Scheme	Marks
8. (*a*)	Method for finding sin A	M1
	$\sin A = -\dfrac{\sqrt{7}}{4}$	A1 A1
	Note: First A1 for $\dfrac{\sqrt{7}}{4}$, exact. Second A1 for sign (even if dec. answer given)	
	Use of $\sin 2A \equiv 2\sin A\cos A$	M1
	$\sin 2A = -\dfrac{3\sqrt{7}}{8}$ or equivalent exact	A1√ (5)
	Note: ± f.t. Requires exact value, dependent on 2nd M	
(*b*)(i)	$\cos\left(2x+\frac{\pi}{3}\right)+\cos\left(2x-\frac{\pi}{3}\right) \equiv \cos 2x\cos\frac{\pi}{3} - \sin 2x\sin\frac{\pi}{3} + \cos 2x\cos\frac{\pi}{3} + \sin 2x\sin\frac{\pi}{3}$	M1
	$\equiv 2\cos 2x\cos\dfrac{\pi}{3}$	A1
	[This can be just written down (using factor formulae) for M1A1]	
	$\equiv \cos 2x$ AG	A1* (3)
	Note: M1A1 earned, if $\equiv 2\cos 2x\cos\dfrac{\pi}{3}$ just written down, using factor theorem Final A1* requires some working after first result.	
(*b*)(ii)	$\dfrac{dy}{dx} = 6\sin x\cos x - 2\sin 2x$	B1 B1
	or $6\sin x\cos x - 2\sin\left(2x+\dfrac{\pi}{3}\right) - 2\sin\left(2x-\dfrac{\pi}{3}\right)$	
	$= 3\sin 2x - 2\sin 2x$	M1
	$= \sin 2x$ AG	A1* (4)
	Note: First B1 for $6\sin x\cos x$; second B1 for remaining term(s)	
		(12 marks)

Mark Scheme (Results)

Summer 2007

GCE

GCE Mathematics

Core Mathematics C3 (6665)

Edexcel Limited. Registered in England and Wales No. 4496750
Registered Office: One90 High Holborn, London WC1V 7BH

A PEARSON COMPANY

June 2007
6665 Core Mathematics C3
Mark Scheme

Question Number	Scheme	Marks
1. (*a*)	$\ln 3x = \ln 6$ or $\ln x = \ln\left(\frac{6}{3}\right)$ [implied by 0.69…] or $\ln\left(\frac{3x}{6}\right) = 0$	M1
	$x = 2$ (only this answer)	A1 (cso) (2)
(*b*)	$(e^x)^2 - 4e^x + 3 = 0$ (any 3 term form)	M1
	$(e^x - 3)(e^x - 1) = 0$	
	$e^x = 3$ or $e^x = 1$ Solving quadratic	M1 dep
	$x = \ln 3$, $x = 0$ (or ln 1)	M1 A1 (4)
		(6 marks)

Notes: (a) Answer $x = 2$ with no working or no incorrect working seen: M1A1

Beware $x = 2$ from $\ln x = \frac{\ln 6}{\ln 3} = \ln 2$ M0A0

$\ln x = \ln 6 - \ln 3 \Rightarrow x = e^{(\ln 6\, -\, \ln 3)}$ allow M1, $x = 2$ (no wrong working) A1

(b) 1st M1 for attempting to multiply through by e^x : Allow y, X , even x, for e^x

Be generous for M1 e.g $e^{2x} + 3 = 4$, $e^{x^2} + 3 = 4e^x$,

$3y^2 + 1 = 12y$ (from $3e^{-x} = \frac{1}{3e^x}$), $e^x + 3 = 4e^x$

2nd M1 is for solving quadratic (may be by formula or completing the square) as far as getting two values for e^x or y or X etc

3rd M1 is for converting their answer(s) of the form $e^x = k$ to $x = \ln k$ (must be exact)

A1 is for ln3 **and** ln1 or 0 (Both required and no further solutions)

2.	(*a*)	$2x^2 + 3x - 2 = (2x-1)(x+2)$ at any stage $f(x) = \dfrac{(2x+3)(2x-1)-(9+2x)}{(2x-1)(x+2)}$ f.t. on error in denominator factors (need not be single fraction) Simplifying numerator to quadratic form [$= \dfrac{4x^2+4x-3-9-2x}{(2x-1)(x+2)}$] Correct **numerator** $= \dfrac{4x^2+2x-12}{[(2x-1)(x+2)]}$ Factorising numerator, **with** a denominator $= \dfrac{2(2x-3)(x+2)}{(2x-1)(x+2)}$ o.e. [$= \dfrac{2(2x-3)}{2x-1}$] $= \dfrac{4x-6}{2x-1}$ (✱)	B1 M1, A1√ M1 A1 M1 A1 cso (7)
	Alt.(*a*)	$2x^2 + 3x - 2 = (2x-1)(x+2)$ at any stage B1 $f(x) = \dfrac{(2x+3)(2x^2+3x-2)-(9+2x)(x+2)}{(x+2)(2x^2+3x-2)}$ M1A1 f.t. $= \dfrac{4x^3+10x^2-8x-24}{(x+2)(2x^2+3x-2)}$ $= \dfrac{2(x+2)(2x^2+x-6)}{(x+2)(2x^2+3x-2)}$ or $\dfrac{2(2x-3)(x^2+4x+4)}{(x+2)(2x^2+3x+2)}$ o.e. Any one linear factor × quadratic factor in **numerator** M1, A1 $= \dfrac{2(x+2)(x+2)(2x-3)}{(x+2)(2x^2+3x-2)}$ o.e. M1 $= \dfrac{2(2x-3)}{2x-1}$ $\dfrac{4x-6}{2x-1}$ (✱) A1	
	(*b*)	Complete method for $f'(x)$; e.g $f'(x) = \dfrac{(2x-1)\times 4-(4x-6)\times 2}{(2x-1)^2}$ o.e $= \dfrac{8}{(2x-1)^2}$ or $8(2x-1)^{-2}$ Not treating f^{-1} (for f') as misread	M1 A1 A1 (3) **(10 marks)**

Notes: (a) 1st M1 in either version is for correct method

1st A1 Allow $\dfrac{2x+3(2x-1)-(9+2x)}{(2x-1)(x+2)}$ or $\dfrac{(2x+3)(2x-1)-9+2x}{(2x-1)(x+2)}$ or $\dfrac{2x+3(2x-1)-9+2x}{(2x-1)(x+2)}$ (fractions)

2nd M1 in (main a) is for forming 3 term quadratic in **numerator**

3rd M1 is for factorising their quadratic (usual rules) ; factor of 2 need not be extracted

(✱) A1 is given answer so is cso

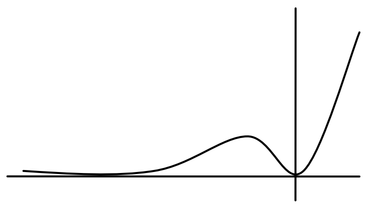
Alt (a) 3rd M1 is for factorising resulting quadratic

Notice that B1 likely to be scored very late but on ePen scored first

(b) SC: For M allow ± given expression or one error in product rule

Alt: Attempt at $f(x) = 2 - 4(2x-1)^{-1}$ and diff. M1; $k(2x-1)^{-2}$ A1; A1 as above

Accept $8(4x^2-4x+1)^{-2}$. Differentiating original function – mark as scheme.

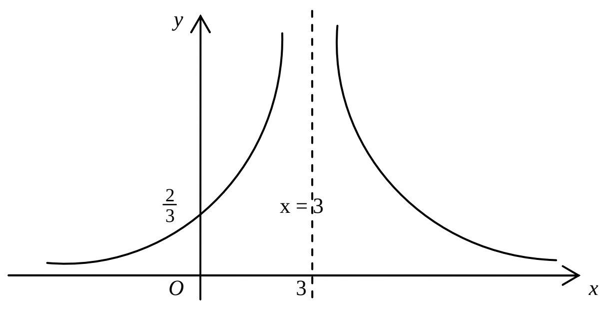
Question Number	Scheme	Marks
3. (*a*)	$\dfrac{dy}{dx} = x^2e^x + 2xe^x$	M1,A1,A1 (3)
(*b*)	If $\dfrac{dy}{dx} = 0$, $e^x(x^2 + 2x) = 0$ setting (*a*) = 0 $[e^x \neq 0]$ $x(x + 2) = 0$ ($x = 0$) or $x = -2$ $x = 0, y = 0$ **and** $x = -2, y = 4e^{-2}$ (= 0.54…)	M1 A1 A1 √ (3)
(*c*)	$\dfrac{d^2y}{dx^2} = x^2e^x + 2xe^x + 2xe^x + 2e^x$ $\left[= (x^2 + 4x + 2)e^x\right]$	M1, A1 (2)
(*d*) Alt.(*d*)	$x = 0$, $\dfrac{d^2y}{dx^2} > 0$ (=2) $x = -2$, $\dfrac{d^2y}{dx^2} < 0$ [= $-2e^{-2}$ (= –0.270…)] M1: Evaluate, or state sign of, candidate's (c) for at least one of candidate's x value(s) from (b) $\therefore$ minimum $\therefore$ maximum For M1: Evaluate, or state sign of, $\dfrac{dy}{dx}$ at two appropriate values – on either side of at least one of their answers from (b) or Evaluate y at two appropriate values – on either side of at least one of their answers from (b) or Sketch curve 	M1 A1 (cso) (2) **(10 marks)**

Notes: (a) Generous M for attempt at $f(x)g'(x) + f'(x)g(x)$
1st A1 for one correct, 2nd A1 for the other correct.
Note that x^2e^x on its own scores no marks

(b) 1st A1 ($x = 0$) may be omitted, but for
2nd A1 both sets of coordinates needed ; f.t only on candidate's $x = -2$

(c) M1 requires complete method for candidate's (a), result may be unsimplified for A1

(d) A1 is cso; $x = 0$, min, and $x = -2$, max and no incorrect working seen.,
or (in alternative) sign of $\frac{dy}{dx}$ either side correct, or values of y appropriate to t.p.
Need only consider the quadratic, as may assume $e^x > 0$.
If all marks gained in (a) and (c), and correct x values, give M1A1 for correct statements with no working

Question Number	Scheme	Marks
4. (*a*)	$x^2(3-x)-1=0$ o.e. (e.g. $x^2(-x+3)=1$) $x=\sqrt{\dfrac{1}{3-x}}$ (✱) Note(✱), answer is given: need to see appropriate working and A1 is cso [Reverse process: Squaring and non-fractional equation M1, form f(x) A1]	M1 A1 (cso) (2)
(*b*)	$x_2=0.6455$, $x_3=0.6517$, $x_4=0.6526$ 1st B1 is for one correct, 2nd B1 for other two correct If all three are to greater accuracy, award B0 B1	B1; B1 (2)
(*c*) Alt (i) Alt (ii)	Choose values in interval (0.6525, 0.6535) or tighter and evaluate both f(0.6525) = –0.0005 (372… f(0.6535) = 0.002 (101… At least one correct "up to bracket", i.e. -0.0005 or 0.002 **Change of sign,** $\therefore x=0.653$ is a root (correct) to 3 d.p. Requires both correct "up to bracket" and conclusion as above Continued iterations at least as far as x_6 M1 $x_5=0.6527$, $x_6=0.6527$, $x_7=$ … two correct to at least 4 s.f. A1 Conclusion : Two values correct to 4 d.p., so 0.653 is root to 3 d.p. A1 If use g(0.6525) = 0.6527..>0.6525 and g(0.6535) = 0.6528..<0.6535 M1A1 Conclusion : Both results correct, so 0.653 is root to 3 d.p. A1	M1 A1 A1 (3) **(7 marks)**
5. (*a*)	Finding g(4) = k and f(k) = …. or fg(x) = $\ln\left(\dfrac{4}{x-3}-1\right)$ [f(2) = ln(2x2 – 1) fg(4) = ln(4 – 1)] $=\ln 3$	M1 A1 (2)
(*b*)	$y=\ln(2x-1) \Rightarrow e^y=2x-1$ or $e^x=2y-1$ $f^{-1}(x)=\frac{1}{2}(e^x+1)$ Allow $y=\frac{1}{2}(e^x+1)$ Domain $x\in\Re$ [Allow $\Re$, all reals, $(-\infty,\infty)$] independent	M1, A1 A1 B1 (4)
(*c*)	 Shape, and x-axis should appear to be asymptote **Equation x = 3 needed,** may see in diagram (ignore others) Intercept $(0, \frac{2}{3})$ no other; accept y = ⅔ (0.67) or on graph	B1 B1 ind. B1 ind (3)
(*d*) Alt:	$\dfrac{2}{x-3}=3 \Rightarrow x=3\frac{2}{3}$ or exact equiv. $\dfrac{2}{x-3}=-3$, $\Rightarrow x=2\frac{1}{3}$ or exact equiv. Note: 2 = 3(x + 3) or 2 = 3(–x – 3) o.e. is M0A0 Squaring to quadratic ($9x^2-54x+77=0$) **and** solving M1; B1A1	B1 M1, A1 (3) **(12 marks)**

6.	(*a*)	Complete method for R: e.g. $R\cos\alpha = 3$, $R\sin\alpha = 2$, $R = \sqrt{(3^2+2^2)}$	M1
		$R = \sqrt{13}$ or 3.61 (or more accurate)	A1
		Complete method for $\tan\alpha = \frac{2}{3}$ [Allow $\tan\alpha = \frac{3}{2}$]	M1
		$\alpha = 0.588$ (Allow 33.7°)	A1 (4)
	(*b*)	Greatest value $= \left(\sqrt{13}\right)^4 = 169$	M1, A1 (2)
	(*c*)	$\sin(x+0.588) = \frac{1}{\sqrt{13}}$ (= 0.27735…) $\sin(x + \text{their } \alpha) = \frac{1}{\text{their } R}$	M1
		$(x + 0.588) = 0.281(03\ldots$ or 16.1°	A1
		$(x + 0.588) = \pi - 0.28103\ldots$ Must be π – their 0.281 or 180° – their 16.1°	M1
		or $(x + 0.588) = 2\pi + 0.28103\ldots$ Must be 2π + their 0.281 or 360° + their 16.1°	M1
		$x = 2.273$ or $x = 5.976$ (awrt) Both (radians only) If 0.281 or 16.1° not seen, correct answers imply this A mark	A1 (5) **(11 marks)**

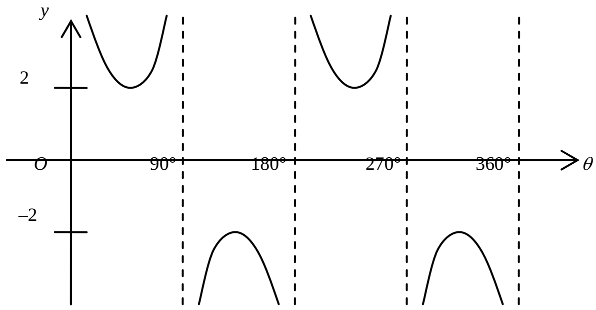
Notes: (a) 1st M1 on Epen for correct method for R, even if found second
2nd M1 for correct method for $\tan\alpha$
No working at all: M1A1 for √13, M1A1 for 0.588 or 33.7°.
N.B. $R\cos\alpha = 2$, $R\sin\alpha = 3$ used, can still score M1A1 for R, but loses the A mark for α.
$\cos\alpha = 3$, $\sin\alpha = 2$: apply the same marking.

(b) M1 for realising $\sin(x + \alpha) = \pm 1$, so finding R^4.

(c) Working in mixed degrees/rads : first two marks available
Working consistently in degrees: Possible to score first 4 marks
[Degree answers, just for reference, Only are 130.2° and 342.4°]
Third M1 can be gained for candidate's 0.281 – candidate's 0.588 + 2π or equiv. in degrees
One of the answers correct in radians or degrees implies the corresponding M mark.

Alt: (c) (i) Squaring to form quadratic in $\sin x$ or $\cos x$ M1
$[13\cos^2 x - 4\cos x - 8 = 0,\quad 13\sin^2 x - 6\sin x - 3 = 0]$
Correct values for $\cos x = 0.953\ldots, -0.646$; or $\sin x = 0.767, 2.27$ awrt A1
For any one value of $\cos x$ or $\sin x$, correct method for two values of x M1
$x = 2.273$ or $x = 5.976$ (awrt) Both seen anywhere A1
Checking other values (0.307, 4.011 or 0.869, 3.449) and discarding M1

(ii) Squaring and forming equation of form $a\cos 2x + b\sin 2x = c$
$9\sin^2 x + 4\cos^2 x + 12\sin 2x = 1 \Rightarrow 12\sin 2x + 5\cos 2x = 11$
Setting up to solve using R formula e.g. $\sqrt{13}\cos(2x - 1.176) = 11$ M1
$(2x - 1.176) = \cos^{-1}\left(\frac{11}{\sqrt{13}}\right) = 0.562(0\ldots$ (α) A1
$(2x - 1.176) = 2\pi - \alpha,\ 2\pi + \alpha, \ldots\ldots\ldots$ M1
$x = 2.273$ or $x = 5.976$ (awrt) Both seen anywhere A1
Checking other values and discarding M1

Question Number	Scheme	Marks
7. (*a*)	$\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}=\frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}$ M1 Use of common denominator to obtain single fraction	M1
	$=\frac{1}{\cos\theta\sin\theta}$ M1 Use of appropriate trig identity (in this case $\sin^2\theta+\cos^2\theta=1$)	M1
	$=\frac{1}{\frac{1}{2}\sin 2\theta}$ Use of $\sin 2\theta=2\sin\theta\cos\theta$	M1
	$=2\mathrm{cosec}2\theta$ (✱)	A1 cso (4)
Alt.(*a*)	$\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}=\tan\theta+\frac{1}{\tan\theta}=\frac{\tan^2\theta+1}{\tan\theta}$ M1 $=\frac{\sec^2\theta}{\tan\theta}$ M1 $=\frac{1}{\cos\theta\sin\theta}=\frac{1}{\frac{1}{2}\sin 2\theta}$ M1 $=2\mathrm{cosec}2\theta$ (✱) (cso) A1 If show two expressions are equal, need conclusion such as QED, tick, true.	
(*b*)	 Shape (May be translated but need to see 4"sections")	B1
	T.P.s at $y = \pm 2$, asymptotic at correct x-values (dotted lines not required)	B1 dep. (2)
(*c*)	$2\mathrm{cosec}2\theta=3$ $\sin 2\theta=\frac{2}{3}$ Allow $\frac{2}{\sin 2\theta}=3$ [M1 for equation in sin2θ]	M1, A1
	(2θ) = [41.810…°, 138.189…° ; 401.810…°, 498.189…°] 1st M1 for α, 180 − α ; 2nd M1 adding 360° to at least one of values	M1; M1
	θ = 20.9°, 69.1°, 200.9°, 249.1° (1 d.p.) awrt	
Note	1st A1 for any two correct, 2nd A1 for other two Extra solutions in range lose final A1 only **SC:** Final 4 marks:**θ = 20.9°, after M0M0 is B1; record as M0M0A1A0**	A1,A1 (6)
Alt.(*c*)	$\tan\theta+\frac{1}{\tan\theta}=3$ and form quadratic , $\tan^2\theta-3\tan\theta+1=0$ M1, A1 (M1 for attempt to multiply through by tanθ, A1 for correct equation above) Solving quadratic [$\tan\theta=\frac{3\pm\sqrt{5}}{2}$ = 2.618… or = 0.3819…] M1 θ = 69.1°, 249.1° θ= 20.9°, 200.9° (1 d.p.) M1, A1, A1 (M1 is for one use of 180° + α °, A1A1 as for main scheme)	**(12 marks)**

Question Number	Scheme	Marks
8. (*a*)	$D = 10$, $t = 5$, $x = 10e^{-\frac{1}{8}\times 5}$ $= 5.353$ awrt	M1 A1 (2)
(*b*)	$D = 10 + 10e^{-\frac{5}{8}}$, $t = 1$, $x = 15.3526\ldots \times e^{-\frac{1}{8}}$ $x = 13.549$ (✱)	M1 A1 cso (2)
Alt.(*b*)	$x = 10e^{-\frac{1}{8}\times 6} + 10e^{-\frac{1}{8}\times 1}$ M1 $x = 13.549$ (✱) A1 cso	
(*c*)	$15.3526\ldots e^{-\frac{1}{8}T} = 3$ $e^{-\frac{1}{8}T} = \dfrac{3}{15.3526\ldots} = 0.1954\ldots$ $-\dfrac{1}{8}T = \ln 0.1954\ldots$ $T = 13.06\ldots$ or 13.1 or 13	M1 M1 A1 (3) **(7 marks)**

Notes: (b) (main scheme) M1 is for $(10 + 10e^{-\frac{5}{8}})e^{-\frac{1}{8}}$, or $\{10 + \text{their(a)}\}e^{-(1/8)}$

N.B. The answer is given. There are many correct answers seen which deserve M0A0 or M1A0

(If adding two values, these should be 4.724 and 8.825)

(c) 1st M is for $(10 + 10e^{-\frac{5}{8}})\, e^{-\frac{T}{8}} = 3$

2nd M is for converting $e^{-\frac{T}{8}} = k$ ($k > 0$) to $-\dfrac{T}{8} = \ln k$. This is independent of 1st M.

Trial and improvement: M1 as scheme,
M1 correct process for their equation (two equal to 3 s.f.)
A1 as scheme

advancing learning, changing lives

Mark Scheme (Results)

Summer 2008

GCE

GCE Mathematics (6665/01)

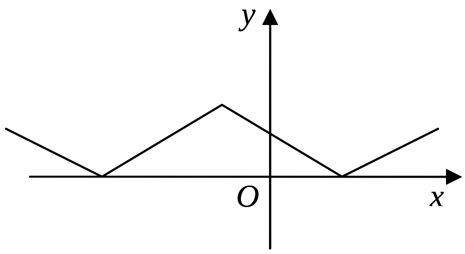
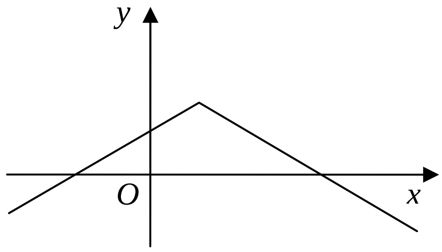
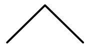
Edexcel Limited. Registered in England and Wales No. 4496750
Registered Office: One90 High Holborn, London WC1V 7BH

A PEARSON COMPANY

June 2008
6665 Core Mathematics C3
Mark Scheme

Question Number	Scheme	Marks	
1.	(a) $e^{2x+1} = 2$		
	$2x+1 = \ln 2$	M1	
	$x = \frac{1}{2}(\ln 2 - 1)$	A1	(2)
	(b) $\frac{dy}{dx} = 8e^{2x+1}$	B1	
	$x = \frac{1}{2}(\ln 2 - 1) \Rightarrow \frac{dy}{dx} = 16$	B1	
	$y - 8 = 16\left(x - \frac{1}{2}(\ln 2 - 1)\right)$	M1	
	$y = 16x + 16 - 8\ln 2$	A1	(4)
			[6]

Question Number	Scheme		Marks	
2.	(a) $R^2 = 5^2 + 12^2$		M1	
	$R = 13$		A1	
	$\tan\alpha = \dfrac{12}{5}$		M1	
	$\alpha \approx 1.176$	cao	A1	(4)
	(b) $\cos(x - \alpha) = \dfrac{6}{13}$		M1	
	$x - \alpha = \arccos\dfrac{6}{13} = 1.091\ \ldots$		A1	
	$x = 1.091\ \ldots + 1.176\ \ldots \approx 2.267\ldots$	awrt 2.3	A1	
	$x - \alpha = -1.091\ \ldots$	accept … = 5.19 … for M	M1	
	$x = -1.091\ \ldots + 1.176\ \ldots \approx 0.0849\ldots$	awrt 0.084 or 0.085	A1	(5)
	(c)(i) $R_{\max} = 13$	ft their R	B1 ft	
	(ii) At the maximum, $\cos(x - \alpha) = 1$ *or* $x - \alpha = 0$		M1	
	$x = \alpha = 1.176\ \ldots$	awrt 1.2, ft their α	A1ft	(3) **[12]**

Question Number	Scheme	Marks
3.	(a) 	
	 shape	B1
	Vertices correctly placed	B1 (2)
	(b) 	
	 shape	B1
	Vertex and intersections with axes correctly placed	B1 (2)
	(c) $P:(-1, 2)$	B1
	$Q:(0,1)$	B1
	$R:(1, 0)$	B1 (3)
	(d) $x > -1$; $\quad 2 - x - 1 = \frac{1}{2}x$	M1 A1
	Leading to $x = \frac{2}{3}$	A1
	$x < -1$; $\quad 2 + x + 1 = \frac{1}{2}x$	M1
	Leading to $x = -6$	A1 (5)
		[12]

Question Number	Scheme		Marks	
4.	(a) $x^2-2x-3=(x-3)(x+1)$		B1	
	$\mathrm{f}(x)=\dfrac{2(x-1)-(x+1)}{(x-3)(x+1)}$ $\left(or\ \dfrac{2(x-1)}{(x-3)(x+1)}-\dfrac{x+1}{(x-3)(x+1)}\right)$		M1 A1	
	$=\dfrac{x-3}{(x-3)(x+1)}=\dfrac{1}{x+1}$ $*$	cso	A1	(4)
	(b) $\left(0,\dfrac{1}{4}\right)$	Accept $0<y<\dfrac{1}{4},\ 0<\mathrm{f}(x)<\dfrac{1}{4}$ etc.	B1 B1	(2)
	(c) Let $y=\mathrm{f}(x)$ $\quad y=\dfrac{1}{x+1}$			
	$x=\dfrac{1}{y+1}$			
	$yx+x=1$			
	$y=\dfrac{1-x}{x}$	or $\dfrac{1}{x}-1$	M1 A1	
	$\mathrm{f}^{-1}(x)=\dfrac{1-x}{x}$			
	Domain of f^{-1} is $\left(0,\dfrac{1}{4}\right)$	ft their part (b)	B1 ft	(3)
	(d) $\mathrm{fg}(x)=\dfrac{1}{2x^2-3+1}$			
	$\dfrac{1}{2x^2-2}=\dfrac{1}{8}$		M1	
	$x^2=5$		A1	
	$x=\pm\sqrt{5}$	both	A1	(3)
				[12]

Question Number	Scheme	Marks
5.	(a) $\sin^2\theta + \cos^2\theta = 1$	
	$\div \sin^2\theta$ $\quad \dfrac{\sin^2\theta}{\sin^2\theta} + \dfrac{\cos^2\theta}{\sin^2\theta} = \dfrac{1}{\sin^2\theta}$	M1
	$1 + \cot^2\theta = \operatorname{cosec}^2\theta$ $\ast$ cso	A1 (2)
	Alternative for (a)	
	$1 + \cot^2\theta = 1 + \dfrac{\cos^2\theta}{\sin^2\theta} = \dfrac{\sin^2\theta + \cos^2\theta}{\sin^2\theta} = \dfrac{1}{\sin^2\theta}$	M1
	$= \operatorname{cosec}^2\theta$ $\ast$ cso	A1
	(b) $2\left(\operatorname{cosec}^2\theta - 1\right) - 9\operatorname{cosec}\theta = 3$	M1
	$2\operatorname{cosec}^2\theta - 9\operatorname{cosec}\theta - 5 = 0$ *or* $5\sin^2\theta + 9\sin\theta - 2 = 0$	M1
	$\left(2\operatorname{cosec}\theta + 1\right)\left(\operatorname{cosec}\theta - 5\right) = 0$ *or* $\left(5\sin\theta - 1\right)\left(\sin\theta + 2\right) = 0$	M1
	$\operatorname{cosec}\theta = 5$ *or* $\sin\theta = \dfrac{1}{5}$	A1
	$\theta = 11.5°, 168.5°$	A1 A1 (6)
		[8]

Question Number	Scheme	Marks
6.	(a)(i) $\frac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{e}^{3x}(\sin x+2\cos x)\right)=3\mathrm{e}^{3x}(\sin x+2\cos x)+\mathrm{e}^{3x}(\cos x-2\sin x)$	M1 A1 A1 (3)
	$\left(=\mathrm{e}^{3x}(\sin x+7\cos x)\right)$	
	(ii) $\frac{\mathrm{d}}{\mathrm{d}x}\left(x^3\ln(5x+2)\right)=3x^2\ln(5x+2)+\frac{5x^3}{5x+2}$	M1 A1 A1 (3)
	(b) $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(x+1)^2(6x+6)-2(x+1)(3x^2+6x-7)}{(x+1)^4}$	M1 $\frac{\text{A1}}{\text{A1}}$
	$=\frac{(x+1)(6x^2+12x+6-6x^2-12x+14)}{(x+1)^4}$	M1
	$=\frac{20}{(x+1)^3}$ * cso	A1 (5)
	(c) $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\frac{60}{(x+1)^4}=-\frac{15}{4}$	M1
	$(x+1)^4=16$	M1
	$x=1,-3$ both	A1 (3) **[14]**
	Note: The simplification in part (b) can be carried out as follows	
	$\frac{(x+1)^2(6x+6)-2(x+1)(3x^2+6x-7)}{(x+1)^4}$	
	$=\frac{(6x^3+18x^2+18x+6)-(6x^3+18x^2-2x-14)}{(x+1)^4}$	
	$=\frac{20x+20}{(x+1)^4}=\frac{20(x+1)}{(x+1)^4}=\frac{20}{(x+1)^3}$	M1 A1

Question Number	Scheme	Marks
7.	(a) $\mathrm{f}\left(1.4\right)=-0.568\ \ldots<0$	
	$\mathrm{f}\left(1.45\right)=0.245\ \ldots>0$	M1
	Change of sign (and continuity) $\Rightarrow\ \alpha\in\left(1.4,1.45\right)$	A1 (2)
	(b) $3x^3=2x+6$	
	$x^3=\dfrac{2x}{3}+2$	
	$x^2=\dfrac{2}{3}+\dfrac{2}{x}$	M1 A1
	$x=\sqrt{\left(\dfrac{2}{x}+\dfrac{2}{3}\right)}$ ∗ cso	A1 (3)
	(c) $x_1=1.4371$	B1
	$x_2=1.4347$	B1
	$x_3=1.4355$	B1 (3)
	(d) Choosing the interval $\left(1.4345,1.4355\right)$ or appropriate tighter interval.	M1
	$\mathrm{f}\left(1.4345\right)=-0.01$ ...	
	$\mathrm{f}\left(1.4355\right)=0.003\ \ldots$	M1
	Change of sign (and continuity) $\Rightarrow\ \alpha\in\left(1.4345,1.4355\right)$	
	$\Rightarrow\alpha=1.435$, correct to 3 decimal places ∗ cso	A1 (3) **[11]**
	Note: $\alpha=1.435\,304\,553\ \ldots$	

Mark Scheme (Results)
Summer 2009

GCE

GCE Mathematics (6665/01)

June 2009
6665 Core Mathematics C3
Mark Scheme

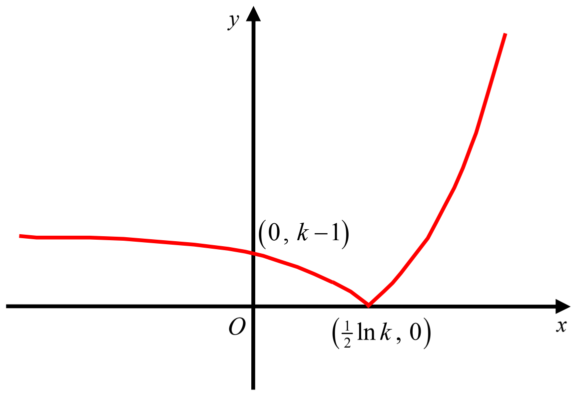
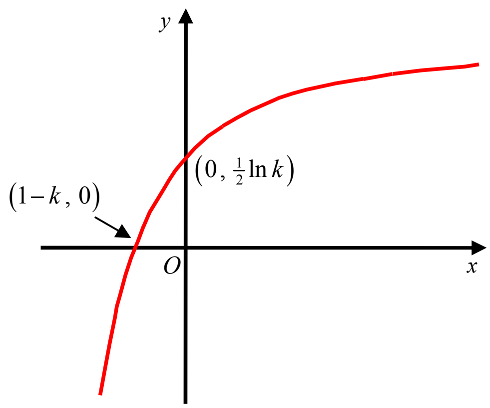
Question Number	Scheme		Marks
Q1 (a)	Iterative formula: $x_{n+1} = \dfrac{2}{(x_n)^2} + 2$, $x_0 = 2.5$		
	$x_1 = \dfrac{2}{(2.5)^2} + 2$	An attempt to substitute $x_0 = 2.5$ into the iterative formula. Can be implied by $x_1 = 2.32$ or 2.320	M1
	$x_1 = 2.32$ $x_2 = 2.371581451...$	Both $x_1 = 2.32(0)$ and $x_2 =$ awrt 2.372	A1
	$x_3 = 2.355593575...$ $x_4 = 2.360436923...$	Both $x_3 =$ awrt 2.356 and $x_4 =$ awrt 2.360 or 2.36	A1 cso
			(3)
(b)	Let $f(x) = -x^3 + 2x^2 + 2 = 0$		
	$f(2.3585) = 0.00583577...$	Choose suitable interval for x, e.g. [2.3585, 2.3595] or tighter	M1
	$f(2.3595) = -0.00142286...$	any one value awrt 1 sf or truncated 1 sf	dM1
	Sign change (and $f(x)$ is continuous) therefore a root α is such that $\alpha \in (2.3585, 2.3595) \Rightarrow \alpha = 2.359$ (3 dp)	both values correct, sign change and conclusion	A1
		At a minimum, both values must be correct to 1sf or truncated 1sf, candidate states "change of sign, hence root".	(3)
			[6]

Question Number	Scheme		Marks
Q2 (a)	$\cos^2\theta + \sin^2\theta = 1 \quad (\div \cos^2\theta)$		
	$\underline{\dfrac{\cos^2\theta}{\cos^2\theta} + \dfrac{\sin^2\theta}{\cos^2\theta} = \dfrac{1}{\cos^2\theta}}$	Dividing $\cos^2\theta + \sin^2\theta = 1$ by $\cos^2\theta$ to give underlined equation.	M1
	$1 + \tan^2\theta = \sec^2\theta$		
	$\tan^2\theta = \sec^2\theta - 1$ (as required) **AG**	Complete proof. No errors seen.	A1 cso
			(2)
(b)	$2\tan^2\theta + 4\sec\theta + \sec^2\theta = 2, \quad (\text{eqn }*) \qquad 0 \le \theta < 360^\circ$		
	$2(\sec^2\theta - 1) + 4\sec\theta + \sec^2\theta = 2$	Substituting $\tan^2\theta = \sec^2\theta - 1$ into eqn * to get a quadratic in $\sec\theta$ only	M1
	$2\sec^2\theta - 2 + 4\sec\theta + \sec^2\theta = 2$		
	$\underline{3\sec^2\theta + 4\sec\theta - 4} = 0$	Forming a three term "one sided" quadratic expression in $\sec\theta$.	M1
	$(\sec\theta + 2)(3\sec\theta - 2) = 0$	Attempt to factorise or solve a quadratic.	M1
	$\sec\theta = -2$ or $\sec\theta = \frac{2}{3}$		
	$\dfrac{1}{\cos\theta} = -2$ or $\dfrac{1}{\cos\theta} = \dfrac{2}{3}$		
	$\underline{\cos\theta = -\tfrac{1}{2}}$; or $\cos\theta = \tfrac{3}{2}$	$\underline{\cos\theta = -\tfrac{1}{2}}$	A1;
	$\alpha = 120^\circ$ or $\alpha =$ no solutions		
	$\theta_1 = \underline{120^\circ}$	$\underline{120^\circ}$	$\underline{\text{A1}}$
	$\theta_2 = 240^\circ$	$\underline{240^\circ}$ or $\theta_2 = 360^\circ - \theta_1$ when solving using $\cos\theta = \ldots$	B1 $\sqrt{}$
	$\theta = \{120^\circ, 240^\circ\}$	Note the final A1 mark has been changed to a B1 mark.	(6)
			[8]

Question Number	Scheme		Marks
Q3	$P = 80\mathrm{e}^{\frac{t}{5}}$		
(a)	$t = 0 \Rightarrow P = 80\mathrm{e}^{\frac{0}{5}} = 80(1) = \underline{80}$	$\underline{80}$	B1
			(1)
(b)	$P = 1000 \Rightarrow 1000 = 80\mathrm{e}^{\frac{t}{5}} \Rightarrow \frac{1000}{80} = \mathrm{e}^{\frac{t}{5}}$	Substitutes $P = 1000$ and rearranges equation to make $\mathrm{e}^{\frac{t}{5}}$ the subject.	M1
	$\therefore\ t = 5\ln\left(\frac{1000}{80}\right)$		
	$t = 12.6286...$	awrt 12.6 or 13 years	A1
		Note $t = 12$ or $t =$ awrt $12.6 \Rightarrow t = 12$ will score A0	(2)
(c)	$\frac{\mathrm{d}P}{\mathrm{d}t} = 16\mathrm{e}^{\frac{t}{5}}$	$k\mathrm{e}^{\frac{1}{5}t}$ and $k \neq 80$.	M1
		$16\mathrm{e}^{\frac{1}{5}t}$	A1
			(2)
(d)	$50 = 16\mathrm{e}^{\frac{t}{5}}$		
	$\therefore\ t = 5\ln\left(\frac{50}{16}\right)$ $\{= 5.69717...\}$	Using $50 = \frac{\mathrm{d}P}{\mathrm{d}t}$ and an attempt to solve to find the value of t or $\frac{t}{5}$.	M1
	$P = 80\mathrm{e}^{\frac{1}{5}\left(5\ln\left(\frac{50}{16}\right)\right)}$ or $P = 80\mathrm{e}^{\frac{1}{5}(5.69717...)}$	Substitutes their value of t back into the equation for P.	dM1
	$P = \frac{80(50)}{16} = \underline{250}$	$\underline{250}$ or awrt 250	A1
			(3)
			[8]

Question Number	Scheme		Marks
Q4 (i)(a)	$y = x^2 \cos 3x$		
	Apply product rule: $\begin{cases} u = x^2 & v = \cos 3x \\ \dfrac{du}{dx} = 2x & \dfrac{dv}{dx} = -3\sin 3x \end{cases}$		
	$\dfrac{dy}{dx} = 2x\cos 3x - 3x^2 \sin 3x$	Applies $vu' + uv'$ correctly for their u, u', v, v' AND gives an expression of the form $\alpha x \cos 3x \pm \beta x^2 \sin 3x$	M1
		Any one term correct	A1
		Both terms correct and no further simplification to terms in $\cos \alpha x^2$ or $\sin \beta x^3$.	A1
			(3)
(b)	$y = \dfrac{\ln(x^2+1)}{x^2+1}$		
	$u = \ln(x^2+1) \quad \Rightarrow \quad \dfrac{du}{dx} = \dfrac{2x}{x^2+1}$	$\ln(x^2+1) \rightarrow \dfrac{\text{something}}{x^2+1}$	M1
		$\ln(x^2+1) \rightarrow \dfrac{2x}{x^2+1}$	A1
	Apply quotient rule: $\begin{cases} u = \ln(x^2+1) & v = x^2+1 \\ \dfrac{du}{dx} = \dfrac{2x}{x^2+1} & \dfrac{dv}{dx} = 2x \end{cases}$		
	$\dfrac{dy}{dx} = \dfrac{\left(\dfrac{2x}{x^2+1}\right)\left(x^2+1\right) - 2x\ln(x^2+1)}{\left(x^2+1\right)^2}$	Applying $\dfrac{vu' - uv'}{v^2}$	M1
		Correct differentiation with correct bracketing but allow recovery.	A1
			(4)
	$\left\{\dfrac{dy}{dx} = \dfrac{2x - 2x\ln(x^2+1)}{\left(x^2+1\right)^2}\right\}$	{Ignore subsequent working.}	

Question Number	Scheme		Marks
(ii)	$y=\sqrt{4x+1},\ \ x>-\frac{1}{4}$		
	At P, $y=\sqrt{4(2)+1}\ =\ \underline{\sqrt{9}}\ =\ \underline{3}$	At P, $y=\underline{\sqrt{9}}$ or $\underline{3}$	B1
	$\dfrac{dy}{dx}=\underline{\dfrac{1}{2}\left(4x+1\right)^{-\frac{1}{2}}(4)}$	$\pm k\,(4x+1)^{-\frac{1}{2}}$ $2\,(4x+1)^{-\frac{1}{2}}$	M1* A1 aef
	$\dfrac{dy}{dx}=\dfrac{2}{(4x+1)^{\frac{1}{2}}}$		
	At P, $\dfrac{dy}{dx}=\dfrac{2}{\left(4(2)+1\right)^{\frac{1}{2}}}$	Substituting $x=2$ into an equation involving $\frac{dy}{dx}$;	M1
	Hence $m(\mathbf{T})=\dfrac{2}{3}$		
	Either $\mathbf{T}$: $y-3=\frac{2}{3}(x-2)$; or $y=\frac{2}{3}x+c$ and $3=\frac{2}{3}(2)+c\ \Rightarrow\ c=3-\frac{4}{3}\ =\ \frac{5}{3}$;	$y-y_1=m(x-2)$ or $y-y_1=m(x-\text{their stated }x)$ with 'their TANGENT gradient' and their y_1; or uses $y=mx+c$ with 'their TANGENT gradient', their x and their y_1.	dM1*;
	Either $\mathbf{T}$: $3y-9=2(x-2)$;		
	$\mathbf{T}$: $3y-9=2x-4$		
	$\mathbf{T}$: $\underline{2x-3y+5=0}$	$\underline{2x-3y+5=0}$ Tangent must be stated in the form $ax+by+c=0$, where a, b and c are integers.	A1
			(6)
	or $\mathbf{T}$: $y=\frac{2}{3}x+\frac{5}{3}$		
	$\mathbf{T}$: $3y=2x+5$		
	$\mathbf{T}$: $\underline{2x-3y+5=0}$		
			[13]

Question Number	Scheme		Marks
Q5 (a)		Curve retains shape when $x > \tfrac{1}{2}\ln k$	B1
		Curve reflects through the x-axis when $x < \tfrac{1}{2}\ln k$	B1
		$(0,\ k-1)$ and $(\tfrac{1}{2}\ln k,\ 0)$ marked in the correct positions.	B1
			(3)
(b)		Correct shape of curve. The curve should be contained in quadrants 1, 2 and 3 (Ignore asymptote)	B1
		$(1-k,\ 0)$ and $(0,\ \tfrac{1}{2}\ln k)$	B1
			(2)
(c)	Range of f: $\underline{\mathrm{f}(x) > -k}$ or $\underline{y > -k}$ or $\underline{(-k,\ \infty)}$	Either $\underline{\mathrm{f}(x) > -k}$ or $\underline{y > -k}$ or $\underline{(-k,\ \infty)}$ or $\underline{\mathrm{f} > -k}$ or $\underline{\text{Range} > -k}$.	B1
			(1)
(d)	$y = \mathrm{e}^{2x} - k \ \Rightarrow\ y + k = \mathrm{e}^{2x}$	Attempt to make x (or swapped y) the subject	M1
	$\Rightarrow \ln(y+k) = 2x$ $\Rightarrow \tfrac{1}{2}\ln(y+k) = x$	Makes e^{2x} the subject and takes ln of both sides	M1
	Hence $\mathrm{f}^{-1}(x) = \underline{\tfrac{1}{2}\ln(x+k)}$	$\underline{\tfrac{1}{2}\ln(x+k)}$ or $\underline{\ln\sqrt{(x+k)}}$	$\underline{\text{A1}}$ cao
			(3)
(e)	$\mathrm{f}^{-1}(x)$: Domain: $\underline{x > -k}$ or $\underline{(-k,\ \infty)}$	Either $\underline{x > -k}$ or $\underline{(-k,\ \infty)}$ or Domain $> -k$ or x "ft one sided inequality" their part (c) RANGE answer	B1 $\sqrt{}$
			(1)
			[10]

Question Number	Scheme		Marks
Q6 (a)	$A = B \Rightarrow \cos(A+A) = \cos 2A = \underline{\cos A\cos A - \sin A\sin A}$	Applies $A = B$ to $\cos(A+B)$ to give the <u>underlined</u> equation or $\cos 2A = \underline{\cos^2 A - \sin^2 A}$	M1
	$\cos 2A = \cos^2 A - \sin^2 A$ and $\cos^2 A + \sin^2 A = 1$ gives		
	$\underline{\cos 2A} = 1 - \sin^2 A - \sin^2 A = \underline{1 - 2\sin^2 A}$ (as required)	<u>Complete proof, with a link between LHS and RHS</u>. No errors seen.	A1 AG
			(2)
(b)	$C_1 = C_2 \Rightarrow 3\sin 2x = 4\sin^2 x - 2\cos 2x$	Eliminating y correctly.	M1
	$3\sin 2x = 4\left(\dfrac{1-\cos 2x}{2}\right) - 2\cos 2x$	Using result in part (a) to substitute for $\sin^2 x$ as $\dfrac{\pm 1 \pm \cos 2x}{2}$ or $k\sin^2 x$ as $k\left(\dfrac{\pm 1 \pm \cos 2x}{2}\right)$ to produce an equation in only double angles.	M1
	$3\sin 2x = 2(1-\cos 2x) - 2\cos 2x$		
	$3\sin 2x = 2 - 2\cos 2x - 2\cos 2x$		
	$3\sin 2x + 4\cos 2x = 2$	Rearranges to give correct result	A1 AG
			(3)
(c)	$3\sin 2x + 4\cos 2x = R\cos(2x-\alpha)$		
	$3\sin 2x + 4\cos 2x = R\cos 2x\cos\alpha + R\sin 2x\sin\alpha$		
	Equate $\sin 2x$: $3 = R\sin\alpha$ Equate $\cos 2x$: $4 = R\cos\alpha$		
	$R = \sqrt{3^2+4^2}\,; = \sqrt{25} = 5$	$R = 5$	B1
	$\tan\alpha = \frac{3}{4} \Rightarrow \alpha = 36.86989765...^\circ$	$\tan\alpha = \pm\frac{3}{4}$ or $\tan\alpha = \pm\frac{4}{3}$ or $\sin\alpha = \pm\frac{3}{\text{their } R}$ or $\cos\alpha = \pm\frac{4}{\text{their } R}$ awrt 36.87	M1 A1
	Hence, $3\sin 2x + 4\cos 2x = 5\cos(2x - 36.87)$		
			(3)

Question Number	Scheme		Marks
(d)	$3\sin 2x + 4\cos 2x = 2$		
	$5\cos(2x - 36.87) = 2$		
	$\cos(2x - 36.87) = \frac{2}{5}$	$\cos(2x \pm \text{their } \alpha) = \frac{2}{\text{their } R}$	M1
	$(2x - 36.87) = 66.42182...^\circ$	awrt 66	A1
	$(2x - 36.87) = 360 - 66.42182...^\circ$		
	Hence, $x = 51.64591...^\circ,\ 165.22409...^\circ$	One of either awrt 51.6 or awrt 51.7 or awrt 165.2 or awrt 165.3	A1
		Both awrt 51.6 AND awrt 165.2	A1 (4)
		If there are any EXTRA solutions inside the range $0 \le x < 180^\circ$ then withhold the final accuracy mark. Also ignore EXTRA solutions outside the range $0 \le x < 180^\circ$.	
			[12]

Question Number	Scheme		Marks
Q7	$\mathrm{f}(x) = 1 - \dfrac{2}{(x+4)} + \dfrac{x-8}{(x-2)(x+4)}$ $x \in \mathbb{R},\ x \neq -4,\ x \neq 2.$		
(a)	$\mathrm{f}(x) = \dfrac{(x-2)(x+4) - 2(x-2) + x - 8}{(x-2)(x+4)}$	An attempt to combine to one fraction Correct result of combining all three fractions	M1 A1
	$= \dfrac{x^2 + 2x - 8 - 2x + 4 + x - 8}{(x-2)(x+4)}$		
	$= \dfrac{x^2 + x - 12}{[(x+4)(x-2)]}$	Simplifies to give the correct numerator. Ignore omission of denominator	A1
	$= \dfrac{(x+4)(x-3)}{[(x+4)(x-2)]}$	An attempt to factorise the numerator.	dM1
	$= \dfrac{(x-3)}{(x-2)}$	Correct result	A1 cso **AG** (5)
(b)	$\mathrm{g}(x) = \dfrac{\mathrm{e}^x - 3}{\mathrm{e}^x - 2} \quad x \in \mathbb{R},\ x \neq \ln 2.$		
	Apply quotient rule: $\left\{ \begin{array}{ll} u = \mathrm{e}^x - 3 & v = \mathrm{e}^x - 2 \\ \dfrac{\mathrm{d}u}{\mathrm{d}x} = \mathrm{e}^x & \dfrac{\mathrm{d}v}{\mathrm{d}x} = \mathrm{e}^x \end{array} \right\}$		
	$\mathrm{g}'(x) = \dfrac{\mathrm{e}^x(\mathrm{e}^x - 2) - \mathrm{e}^x(\mathrm{e}^x - 3)}{(\mathrm{e}^x - 2)^2}$	Applying $\dfrac{vu' - uv'}{v^2}$ Correct differentiation	M1 A1
	$= \dfrac{\mathrm{e}^{2x} - 2\mathrm{e}^x - \mathrm{e}^{2x} + 3\mathrm{e}^x}{(\mathrm{e}^x - 2)^2}$		
	$= \dfrac{\mathrm{e}^x}{(\mathrm{e}^x - 2)^2}$	Correct result	A1 **AG** cso (3)

Question Number	Scheme		Marks
(c)	$\mathrm{g}'(x) = 1 \Rightarrow \dfrac{\mathrm{e}^x}{(\mathrm{e}^x - 2)^2} = 1$		
	$\mathrm{e}^x = (\mathrm{e}^x - 2)^2$	Puts their differentiated numerator equal to their denominator.	M1
	$\mathrm{e}^x = \mathrm{e}^{2x} - 2\mathrm{e}^x - 2\mathrm{e}^x + 4$		
	$\underline{\mathrm{e}^{2x} - 5\mathrm{e}^x + 4} = 0$	$\underline{\mathrm{e}^{2x} - 5\mathrm{e}^x + 4}$	A1
	$(\mathrm{e}^x - 4)(\mathrm{e}^x - 1) = 0$	Attempt to factorise or solve quadratic in e^x	M1
	$\mathrm{e}^x = 4$ or $\mathrm{e}^x = 1$		
	$x = \ln 4$ or $x = 0$	both $x = 0,\ \ln 4$	A1
			(4)
			[12]

Question Number	Scheme		Marks
Q8 (a)	$\sin 2x = \underline{2\sin x\cos x}$	$\underline{2\sin x\cos x}$	B1 aef (1)
(b)	$\operatorname{cosec} x - 8\cos x = 0\,,\quad 0 < x < \pi$		
	$\dfrac{1}{\sin x} - 8\cos x = 0$	Using $\operatorname{cosec} x = \dfrac{1}{\sin x}$	M1
	$\dfrac{1}{\sin x} = 8\cos x$		
	$1 = 8\sin x\cos x$		
	$1 = 4(2\sin x\cos x)$		
	$1 = 4\sin 2x$		
	$\underline{\sin 2x = \tfrac{1}{4}}$	$\sin 2x = k$, where $-1 < k < 1$ and $k \neq 0$	M1
		$\underline{\sin 2x = \tfrac{1}{4}}$	$\underline{\text{A1}}$
	Radians $2x = \{0.25268..., 2.88891...\}$ Degrees $2x = \{14.4775..., 165.5225...\}$		
	Radians $x = \{0.12634..., 1.44445...\}$ Degrees $x = \{7.23875..., 82.76124...\}$	Either arwt 7.24 or 82.76 or 0.13 or 1.44 or 1.45 or awrt 0.04π or awrt 0.46π.	A1
		Both $\underline{0.13}$ and $\underline{1.44}$	A1 cao
		Solutions for the final two A marks must be given in x only. If there are any EXTRA solutions inside the range $0 < x < \pi$ then withhold the final accuracy mark. Also ignore EXTRA solutions outside the range $0 < x < \pi$.	(5)
			[6]

Mark Scheme (Results)
Summer 2010

GCE

Core Mathematics C3 (6665)

Edexcel Limited. Registered in England and Wales No. 4496750
Registered Office: One90 High Holborn, London WC1V 7BH

A PEARSON COMPANY

Edexcel is one of the leading examining and awarding bodies in the UK and throughout the world. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers.

Through a network of UK and overseas offices, Edexcel's centres receive the support they need to help them deliver their education and training programmes to learners.

For further information, please call our GCE line on 0844 576 0025, our GCSE team on 0844 576 0027, or visit our website at www.edexcel.com.

If you have any subject specific questions about the content of this Mark Scheme that require the help of a subject specialist, you may find our Ask The Expert email service helpful.

Ask The Expert can be accessed online at the following link:

http://www.edexcel.com/Aboutus/contact-us/

Summer 2010

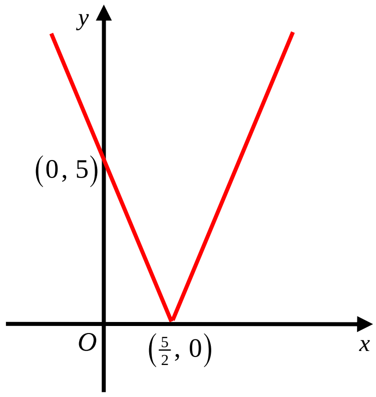
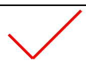
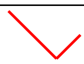
Publications Code UA023702

June 2010
6665 Core Mathematics C3
Mark Scheme

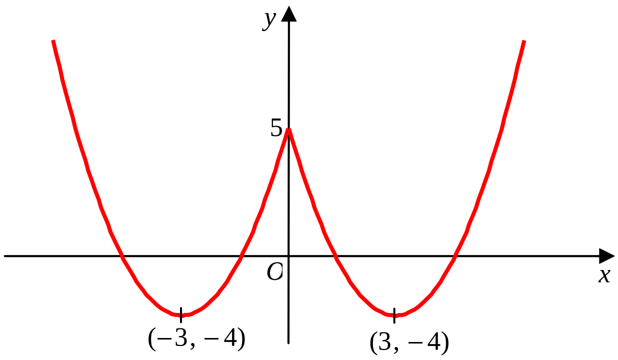
Question Number	Scheme	Marks
1. (a)	$\dfrac{2\sin\theta\cos\theta}{1+2\cos^2\theta-1}$	M1
	$\dfrac{\cancel{2}\sin\theta\cancel{\cos\theta}}{\cancel{2}\cos\theta\cancel{\cos\theta}} = \tan\theta$ (as required) **AG**	A1 **cso**
		(2)
(b)	$2\tan\theta = 1 \Rightarrow \tan\theta = \dfrac{1}{2}$	M1
	$\theta_1 = \text{awrt } 26.6^\circ$	A1
	$\theta_2 = \text{awrt } -153.4^\circ$	A1 $\sqrt{}$
		(3) **[5]**
	(a) M1: Uses **both** a correct identity for $\sin 2\theta$ **and** a correct identity for $\cos 2\theta$. Also allow a candidate writing $1+\cos 2\theta = 2\cos^2\theta$ on the denominator. Also note that angles **must be consistent** in when candidates apply these identities. A1: Correct proof. No errors seen. (b) 1st M1 for either $2\tan\theta = 1$ or $\tan\theta = \dfrac{1}{2}$, seen or implied. A1: awrt 26.6 A1 $\sqrt{}$: awrt -153.4° or $\theta_2 = -180^\circ + \theta_1$ **Special Case**: For candidate solving, $\tan\theta = k$, where $k \neq \frac{1}{2}$, to give θ_1 and $\theta_2 = -180^\circ + \theta_1$, then award M0A0B1 in part (b). **Special Case:** Note that those candidates who writes $\tan\theta = 1$, and gives ONLY two answers of 45° and -135° that are inside the range will be awarded SC M0A0B1.	

Question Number	Scheme	Marks
2.	At P, $y = \underline{3}$	B1
	$\dfrac{dy}{dx} = \underline{3(-2)(5-3x)^{-3}(-3)}$ $\left\{\text{or } \dfrac{18}{(5-3x)^3}\right\}$	M1<u>A1</u>
	$\dfrac{dy}{dx} = \dfrac{18}{(5-3(2))^3}$ $\{= -18\}$	M1
	$m(\mathbf{N}) = \dfrac{-1}{-18}$ or $\dfrac{1}{18}$	M1
	N: $y - 3 = \frac{1}{18}(x-2)$	M1
	N: $\underline{x - 18y + 52 = 0}$	A1
		[7]
	1^{st} M1: $\pm k(5-3x)^{-3}$ can be implied. See appendix for application of the quotient rule. 2^{nd} M1: Substituting $x = 2$ into an equation involving their $\frac{dy}{dx}$; 3^{rd} M1: Uses $m(\mathbf{N}) = -\dfrac{1}{\text{their } m(\mathbf{T})}$. 4^{th} M1: $y - y_1 = m(x-2)$ with 'their NORMAL gradient' or a "changed" tangent gradient and their y_1. Or uses a complete method to express the equation of the tangent in the form $y = mx + c$ with 'their NORMAL ("changed" **numerical**) gradient', their y_1 and $x = 2$. Note: To gain the final A1 mark all the previous 6 marks in this question need to be earned. Also there must be a completely correct solution given.	

Question Number	Scheme	Marks
3. (a)	$\mathrm{f}(1.2) = 0.49166551\ldots,\quad \mathrm{f}(1.3) = -0.048719817\ldots$ Sign change (and as $\mathrm{f}(x)$ is continuous) therefore a root α is such that $\alpha \in [1.2, 1.3]$	M1A1 (2)
(b)	$4\operatorname{cosec}x - 4x + 1 = 0 \Rightarrow 4x = 4\operatorname{cosec}x + 1$	M1
	$\Rightarrow x = \operatorname{cosec}x + \frac{1}{4} \Rightarrow \underline{x = \frac{1}{\sin x} + \frac{1}{4}}$	A1 *
		(2)
(c)	$x_1 = \frac{1}{\sin(1.25)} + \frac{1}{4}$	M1
	$x_1 = 1.303757858\ldots,\quad x_2 = 1.286745793\ldots$	A1
	$x_3 = 1.291744613\ldots$	A1
		(3)
(d)	$\mathrm{f}(1.2905) = 0.00044566695\ldots,\quad \mathrm{f}(1.2915) = -0.00475017278\ldots$	M1
	Sign change (and as $\mathrm{f}(x)$ is continuous) therefore a root α is such that $\alpha \in (1.2905, 1.2915) \Rightarrow \alpha = 1.291$ (3 dp)	A1
		(2) **[9]**
	(a) M1: Attempts to evaluate both $\mathrm{f}(1.2)$ and $\mathrm{f}(1.3)$ and evaluates at least one of them correctly to awrt (or truncated) 1 sf. A1: both values correct to awrt (or truncated) 1 sf, sign change and conclusion. (b) M1: Attempt to make $4x$ or x the subject of the equation. A1: Candidate must then rearrange the equation to give the required result. It must be clear that candidate has made their initial $\mathrm{f}(x) = 0$. (c) M1: An attempt to substitute $x_0 = 1.25$ into the iterative formula Eg $= \frac{1}{\sin(1.25)} + \frac{1}{4}$. Can be implied by $x_1 =$ awrt 1.3 or $x_1 =$ awrt 46°. A1: Both $x_1 =$ awrt 1.3038 and $x_2 =$ awrt 1.2867 A1: $x_3 =$ awrt 1.2917 (d) M1: Choose suitable interval for x, e.g. [1.2905, 1.2915] or tighter and at least one attempt to evaluate f(x). A1: both values correct to awrt (or truncated) 1 sf, sign change and conclusion.	

Question Number	Scheme	Marks
4. (a)		M1A1 (2)
(b)	$\underline{x = 20}$ $2x - 5 = -(15 + x)$; $\Rightarrow$ $\underline{x = -\frac{10}{3}}$	B1 M1;A1 oe. (3)
(c)	$\text{fg}(2) = \text{f}(-3) = \lvert 2(-3) - 5 \rvert; = \lvert -11 \rvert = 11$	M1;A1 (2)
(d)	$\text{g}(x) = x^2 - 4x + 1 = (x-2)^2 - 4 + 1 = (x-2)^2 - 3$. Hence $\text{g}_{\min} = -3$ Either $\text{g}_{\min} = -3$ or $\text{g}(x) \geqslant -3$ or $\text{g}(5) = 25 - 20 + 1 = 6$ $\underline{-3 \leqslant \text{g}(x) \leqslant 6}$ or $\underline{-3 \leqslant y \leqslant 6}$	M1 B1 A1 (3) **[10]**
	(a) M1: V or  or  graph with vertex on the x-axis. A1: $(\frac{5}{2}, \{0\})$ and $(\{0\}, 5)$ seen and the graph appears in both the first and second quadrants. (b) M1: Either $2x - 5 = -(15 + x)$ or $-(2x - 5) = 15 + x$ (c) M1: ***Full method*** of inserting $\text{g}(2)$ into $\text{f}(x) = \lvert 2x - 5 \rvert$ or for inserting $x = 2$ into $\lvert 2(x^2 - 4x + 1) - 5 \rvert$. There must be evidence of the modulus being applied. (d) M1: **Full method** to establish the minimum of g. Eg: $(x \pm \alpha)^2 + \beta$ leading to $\text{g}_{\min} = \beta$. Or for candidate to differentiate the quadratic, set the result equal to zero, find x and insert this value of x back into $\text{f}(x)$ in order to find the minimum. B1: For either finding the correct minimum value of g (can be implied by $\text{g}(x) \geqslant -3$ or $\text{g}(x) > -3$) or for stating that $\text{g}(5) = 6$. A1: $\underline{-3 \leqslant \text{g}(x) \leqslant 6}$ or $\underline{-3 \leqslant y \leqslant 6}$ or $\underline{-3 \leqslant \text{g} \leqslant 6}$. **Note that:** $-3 \leqslant x \leqslant 6$ is A0. **Note that:** $-3 \leqslant \text{f}(x) \leqslant 6$ is A0. **Note that**: $-3 \geqslant \text{g}(x) \geqslant 6$ is A0. **Note that:** $\text{g}(x) \geqslant -3$ or $\text{g}(x) > -3$ or $x \geqslant -3$ or $x > -3$ with no working gains M1B1A0. **Note that for the final Accuracy Mark**: If a candidate writes down $-3 < \text{g}(x) < 6$ or $-3 < y < 6$, then award M1B1A0. If, however, a candidate writes down $\text{g}(x) \geqslant -3$, $\text{g}(x) \leqslant 6$, then award A0. If a candidate writes down $\text{g}(x) \geqslant -3$ or $\text{g}(x) \leqslant 6$, then award A0.	

Question Number	Scheme	Marks
5. (a)	Either $y = 2$ or $(0,\ 2)$	B1 (1)
(b)	When $x = 2,\ y = (8 - 10 + 2)e^{-2} = 0e^{-2} = 0$ $(2x^2 - 5x + 2) = 0 \Rightarrow (x-2)(2x-1) = 0$ Either $x = 2$ (for possibly B1 above) or $\underline{x = \tfrac{1}{2}}$.	B1 M1 A1 (3)
(c)	$\dfrac{dy}{dx} = (4x-5)e^{-x} - (2x^2 - 5x + 2)e^{-x}$	M1A1A1 (3)
(d)	$(4x-5)e^{-x} - (2x^2 - 5x + 2)e^{-x} = 0$ $2x^2 - 9x + 7 = 0 \Rightarrow (2x-7)(x-1) = 0$ $x = \tfrac{7}{2},\ 1$ When $x = \tfrac{7}{2},\ y = 9e^{-\frac{7}{2}}$, when $x = 1,\ y = -e^{-1}$	M1 M1 A1 ddM1A1 (5) **[12]**
	(b) If the candidate believes that $e^{-x} = 0$ solves to $x = 0$ or gives an extra solution of $x = 0$, then withhold the final accuracy mark. (c) M1: (their u')e^{-x} + $(2x^2 - 5x + 2)$(their v') A1: Any one term correct. A1: Both terms correct. (d) 1st M1: For setting their $\frac{dy}{dx}$ found in part (c) equal to 0. 2nd M1: Factorise or eliminate out e^{-x} correctly and an attempt to factorise a 3-term quadratic or apply the formula to candidate's $ax^2 + bx + c$. See rules for solving a three term quadratic equation on page 1 of this Appendix. 3rd ddM1: An attempt to use at least one x-coordinate on $y = (2x^2 - 5x + 2)e^{-x}$. Note that this method mark is dependent on the award of the two previous method marks in this part. Some candidates write down corresponding y-coordinates without any working. It may be necessary on some occasions to use your calculator to check that at least one of the two y-coordinates found is correct to awrt 2 sf. Final A1: Both $\{x = 1\},\ y = -e^{-1}$ and $\{x = \tfrac{7}{2}\},\ y = 9e^{-\frac{7}{2}}$. **cao** Note that both exact values of y are required.	

Question Number	Scheme	Marks
6. (a) (i)	$(3, 4)$	B1 B1
(ii)	$(6, -8)$	B1 B1
		(4)
(b)		B1 B1 B1
		(3)
(c)	$\mathrm{f}(x) = (x-3)^2 - 4$ or $\mathrm{f}(x) = x^2 - 6x + 5$	M1A1
		(2)
(d)	Either: The function f is a many-one {mapping}. Or: The function f is not a one-one {mapping}.	B1
		(1) **[10]**
	(b) B1: Correct shape for $x \geqslant 0$, with the curve meeting the positive y-axis and the turning point is found below the x-axis. (providing candidate does not copy the whole of the original curve and adds nothing else to their sketch.). B1: Curve is symmetrical about the y-axis or correct shape of curve for $x < 0$. **Note:** The first two B1B1 can only be awarded if the curve has the correct shape, with a cusp on the positive y-axis and with both turning points located in the correct quadrants. Otherwise award B1B0. B1: Correct turning points of $(-3, -4)$ and $(3, -4)$. Also, $(\{0\}, 5)$ is marked where the graph cuts through the y-axis. Allow $(5, 0)$ rather than $(0, 5)$ if marked in the "correct" place on the y-axis. (c) M1: Either states $\mathrm{f}(x)$ in the form $(x \pm \alpha)^2 \pm \beta$; $\alpha, \beta \neq 0$ Or uses a complete method on $\mathrm{f}(x) = x^2 + ax + b$, with $\mathrm{f}(0) = 5$ and $\mathrm{f}(3) = -4$ to find both a and b. A1: Either $(x-3)^2 - 4$ or $x^2 - 6x + 5$ (d) B1: Or: The inverse is a one-many {mapping and not a function}. Or: Because $\mathrm{f}(0) = 5$ and also $\mathrm{f}(6) = 5$. Or: One y-coordinate has 2 corresponding x-coordinates {and therefore cannot have an inverse}.	

Question Number	Scheme	Marks
7. (a)	$R = \sqrt{6.25}$ or 2.5	B1
	$\tan\alpha = \frac{1.5}{2} = \frac{3}{4} \Rightarrow \alpha = \text{awrt } 0.6435$	M1A1
		(3)
(b) (i)	Max Value $= 2.5$	B1$\sqrt{\ }$
(ii)	$\underline{\sin(\theta - 0.6435) = 1}$ or $\underline{\theta - \text{their } \alpha = \frac{\pi}{2}}$; $\Rightarrow \theta = \text{awrt } 2.21$	$\underline{\text{M1}}$;A1 $\sqrt{\ }$
		(3)
(c)	$H_{\text{Max}} = 8.5$ (m)	B1$\sqrt{\ }$
	$\underline{\sin\left(\frac{4\pi t}{25} - 0.6435\right) = 1}$ or $\underline{\frac{4\pi t}{25} = \text{ their (b) answer}}$ $;\Rightarrow t = \text{awrt } 4.41$	M1;A1
		(3)
(d)	$\Rightarrow 6 + 2.5\sin\left(\frac{4\pi t}{25} - 0.6435\right) = 7$; $\Rightarrow \sin\left(\frac{4\pi t}{25} - 0.6435\right) = \frac{1}{2.5} = 0.4$	M1;M1
	$\left\{\frac{4\pi t}{25} - 0.6435\right\} = \sin^{-1}(0.4)$ or awrt 0.41	A1
	Either $t = \text{awrt } 2.1$ or awrt 6.7	A1
	So, $\left\{\frac{4\pi t}{25} - 0.6435\right\} = \left\{\pi - 0.411517... \text{ or } 2.730076...^c\right\}$	ddM1
	Times $= \{14:06,\ 18:43\}$	A1 (6)
		[15]
	(a) B1: $R = 2.5$ or $R = \sqrt{6.25}$. For $R = \pm 2.5$, award B0. M1: $\tan\alpha = \pm\frac{1.5}{2}$ or $\tan\alpha = \pm\frac{2}{1.5}$ A1: $\alpha = \text{awrt } 0.6435$ (b) B1$\sqrt{\ }$: 2.5 or follow through the value of R in part (a). M1: For $\sin(\theta - \text{their } \alpha) = 1$ A1$\sqrt{\ }$: awrt 2.21 or $\frac{\pi}{2}$ + their α rounding correctly to 3 sf. (c) B1$\sqrt{\ }$: 8.5 or 6 + their R found in part (a) as long as the answer is greater than 6. M1: $\sin\left(\frac{4\pi t}{25} \pm \text{their } \alpha\right) = 1$ or $\frac{4\pi t}{25} = $ their (b) answer A1: For $\sin^{-1}(0.4)$ This can be implied by awrt 4.41 or awrt 4.40. (d) M1: $6 + (\text{their } R)\sin\left(\frac{4\pi t}{25} \pm \text{their } \alpha\right) = 7$, M1: $\sin\left(\frac{4\pi t}{25} \pm \text{their } \alpha\right) = \frac{1}{\text{their } R}$ A1: For $\sin^{-1}(0.4)$. This can be implied by awrt 0.41 or awrt 2.73 or other values for different α's. Note this mark can be implied by seeing 1.055. A1: Either $t = \text{awrt } 2.1$ or $t = \text{awrt } 6.7$ ddM1: either π – their PV^c. Note that this mark is dependent upon the two M marks. This mark will usually be awarded for seeing either 2.730… or 3.373… A1: Both $t = 14:06$ and $t = 18:43$ or both 126 (min) and 403 (min) or both 2 hr 6 min and 6 hr 43 min.	

Question Number	Scheme	Marks
8. (a)	$\dfrac{(x+5)(2x-1)}{(x+5)(x-3)} = \dfrac{(2x-1)}{(x-3)}$	M1 B1 A1 **aef**
		(3)
(b)	$\ln\left(\dfrac{2x^2+9x-5}{x^2+2x-15}\right)=1$	M1
	$\dfrac{2x^2+9x-5}{x^2+2x-15}=\mathrm{e}$	dM1
	$\dfrac{2x-1}{x-3}=\mathrm{e} \Rightarrow \quad 3\mathrm{e}-1 = x(\mathrm{e}-2)$	M1
	$\Rightarrow \; x = \dfrac{3\mathrm{e}-1}{\mathrm{e}-2}$	A1 **aef cso**
		(4) **[7]**
	(a) M1: An attempt to factorise the numerator. B1: Correct factorisation of denominator to give $(x+5)(x-3)$. Can be seen anywhere. (b) M1: Uses a correct law of logarithms to combine at least two terms. This usually is achieved by the subtraction law of logarithms to give $\ln\left(\dfrac{2x^2+9x-5}{x^2+2x-15}\right)=1.$ The product law of logarithms can be used to achieve $\ln\left(2x^2+9x-5\right)=\ln\left(\mathrm{e}\left(x^2+2x-15\right)\right).$ The product and quotient law could also be used to achieve $\ln\left(\dfrac{2x^2+9x-5}{\mathrm{e}(x^2+2x-15)}\right)=0.$ dM1: Removing ln's correctly by the realisation that the anti-ln of 1 is e. Note that this mark is dependent on the previous method mark being awarded. M1: Collect x terms together and factorise. Note that this is not a dependent method mark. A1: $\dfrac{3\mathrm{e}-1}{\mathrm{e}-2}$ or $\dfrac{3\mathrm{e}^1-1}{\mathrm{e}^1-2}$ or $\dfrac{1-3\mathrm{e}}{2-\mathrm{e}}$. aef Note that the answer needs to be in terms of e. The decimal answer is 9.9610559… Note that the solution must be correct in order for you to award this final accuracy mark. **Note: See Appendix for an alternative method of long division.**	

Further copies of this publication are available from
Edexcel Publications, Adamsway, Mansfield, Notts, NG18 4FN

Telephone 01623 467467
Fax 01623 450481

Email publications@linneydirect.com

Order Code UA023702 Summer 2010

For more information on Edexcel qualifications, please visit www.edexcel.com/quals

Edexcel Limited. Registered in England and Wales no.4496750
Registered Office: One90 High Holborn, London, WC1V 7BH

Mark Scheme (Results)

June 2011

GCE Core Mathematics C3 (6665) Paper 1

June 2011

Publications Code UA027660

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

   - M marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - B marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

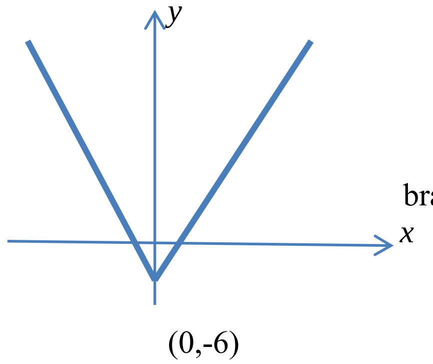
3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

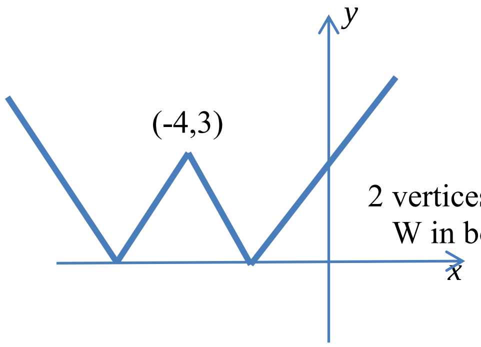
   - bod – benefit of doubt
   - ft – follow through
   - the symbol will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - * The answer is printed on the paper
   - The second mark is dependent on gaining the first mark

Question Number	Scheme	Marks
1 (a)	$\frac{1}{(x^2+3x+5)} \times \ldots,= \frac{2x+3}{(x^2+3x+5)}$	M1,A1
		(2)
(b)	Applying $\frac{vu'-uv'}{v^2}$	M1,
	$\frac{x^2\times -\sin x - \cos x \times 2x}{(x^2)^2} = \frac{-x^2\sin x - 2x\cos x}{x^4} = \frac{-x\sin x - 2\cos x}{x^3}$ oe	A2,1,0
		(3)
		5 Marks
2 (a)	f(0.75)= -0.18… f(0.85) = 0.17…..	M1
	Change of sign, hence root between x=0.75 and x=0.85	A1
		(2)
(b)	Sub x_0=0.8 into $x_{n+1} = [\arcsin(1-0.5x_n)]^{\frac{1}{2}}$ to obtain x_1	M1
	Awrt x_1=0.80219 and x_2=0.80133	A1
	Awrt x_3 = 0.80167	A1
		(3)
(c)	f(0.801565) = -2.7…..×10^{-5} f(0.801575) = +8.6….×10^{-6}	M1A1
	Change of sign and conclusion	A1
		(3)
	See Notes for continued iteration method	
		8 Marks

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>3 (a)

(b)</td><td>

V shape
vertex on y axis &both branches of graph cross x axis
„y‟ co-ordinate of R is -6



W shape
2 vertices on the negative x axis. W in both quad 1 & quad 2.
R‟=(-4,3)</td><td>B1
B1
B1
(3)

B1
B1dep
B1
(3)
6 Marks</td></tr>
<tr><td>4 (a)

(b)

(c)

(d)</td><td>$$y = 4 - \ln(x+2)$$
$$\ln(x+2) = 4 - y$$
$$x + 2 = e^{4-y}$$
$$x = e^{4-y} - 2$$
$$f^{-1}(x) = e^{4-x} - 2$$ oe

$$x \leq 4$$

$$fg(x) = 4 - \ln(e^{x^2} - 2 + 2)$$
$$fg(x) = 4 - x^2$$

$$fg(x) \leq 4$$</td><td>M1
M1A1
(3)

B1
(1)

M1
dM1A1
(3)

B1ft
(1)
8 Marks</td></tr>
</table>

Question Number	Scheme		Marks
5 (a)	p=7.5		B1 (1)
(b)	$2.5 = 7.5e^{-4k}$		M1
	$e^{-4k} = \frac{1}{3}$		M1
	$-4k = \ln(\frac{1}{3})$		dM1
	$-4k = -\ln(3)$		
	$k = \frac{1}{4}\ln(3)$		A1*
	See notes for additional correct solutions and the last A1		(4)
(c)	$\frac{dm}{\mathrm{d}t} = -kpe^{-kt}$	ft on their p and k	M1A1ft
	$-\frac{1}{4}ln3 \times 7.5e^{-\frac{1}{4}(ln3)t} = -0.6ln3$		
	$e^{-\frac{1}{4}(ln3)t} = \frac{2.4}{7.5} = (0.32)$		M1A1
	$-\frac{1}{4}(ln3)t = \ln(0.32)$		dM1
	t=4.1486….	4.15 or awrt 4.1	A1
			(6)
			11Marks

Question Number	Scheme	Marks
6 (a)	$\dfrac{1}{\sin 2\theta} - \dfrac{\cos 2\theta}{\sin 2\theta} = \dfrac{1-\cos 2\theta}{\sin 2\theta}$	M1
	$= \dfrac{2\sin^2\theta}{2\sin\theta\cos\theta}$	M1A1
	$= \dfrac{\sin\theta}{\cos\theta} = \tan\theta$ cso	A1* (4)
(b)(i)	$\tan 15° = \dfrac{1}{\sin 30°} - \dfrac{\cos 30°}{sin 30°}$	M1
	$\tan 15° = \dfrac{1}{\frac{1}{2}} - \dfrac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = 2 - \sqrt{3}$ cso	dM1 A1* (3)
(b)(ii)	$\tan 2x = 1$	M1
	$2x = 45°$	A1
	$2x = 45° + 180°$	M1
	$x = 22.5°, 112.5°, 202.5°, 292.5°$	A1(any two) A1 (5)
		12 Marks
	Alt for (b)(i) $\tan 15° = \tan(60° - 45°) \;\; or \;\; \tan(45° - 30°)$	
	$\tan 15° = \dfrac{\tan 60 - \tan 45}{1 + \tan 60 tan 45} \; or \; \dfrac{\tan 45 - \tan 30}{1 + \tan 45 \tan 30}$	M1
	$\tan 15° = \dfrac{\sqrt{3} - 1}{1 + \sqrt{3}} \; or \; \dfrac{1 - \frac{\sqrt{3}}{3}}{1 + \frac{\sqrt{3}}{3}}$	M1
	Rationalises to produce $\tan 15° = 2 - \sqrt{3}$	A1*

Question Number	Scheme	Marks
7 (a)	$x^2-9=(x+3)(x-3)$	B1
	$\frac{4x-5}{(2x+1)(x-3)}-\frac{2x}{(x+3)(x-3)}$	
	$=\frac{(4x-5)(x+3)}{(2x+1)(x-3)(x+3)}-\frac{2x(2x+1)}{(2x+1)(x+3)(x-3)}$	M1
	$=\frac{5x-15}{(2x+1)(x-3)(x+3)}$	M1A1
	$=\frac{5\cancel{(x-3)}}{(2x+1)\cancel{(x-3)}(x+3)}=\frac{5}{(2x+1)(x+3)}$	A1*
		(5)
(b)	$f\ (x)=\frac{5}{2x^2+7x+3}$	
	$f'(x)=\frac{-5(\mathbf{4x+7})}{(2x^2+7x+3)^2}$	M1**M1**A1
	$f'\ (-1)=-\frac{15}{4}$	M1A1
	Uses $m_1m_2=-1$ to give gradient of normal=$\frac{4}{15}$	M1
	$\frac{y-(-\frac{5}{2})}{(x--1)}=their\ \frac{4}{15}$	M1
	$y+\frac{5}{2}=\frac{4}{15}(x+1)$ or any equivalent form	A1
		(8)
		13 Marks

Question Number	Scheme	Marks
8 (a)	$R^2 = 2^2 + 3^2$ $R = \sqrt{13}$ or 3.61 …..	M1 A1
	$\tan\alpha = \dfrac{3}{2}$ $\alpha = 0.983 \ldots$	M1 A1
		(4)
(b)	$f'(x) = 2e^{2x}\cos3x - 3e^{2x}\sin3x$	M1A1A1
	$= e^{2x}(2\cos3x - 3\sin3x)$	M1
	$= e^{2x}(R\cos(3\mathrm{x} + \alpha)$	
	$= Re^{2x}\cos(3x + \alpha)$	A1* cso
		(5)
(c)	$f'\ (x) = 0 \qquad \cos(3\mathrm{x} + \alpha\ ) = 0$	M1
	$3x + \alpha = \dfrac{\pi}{2}$	M1
	x=0.196… awrt 0.20	A1
		(3)
		12 Marks
	Alternative to part (c)	
	$f'(x) = 0 \quad 2\cos3x - 3\sin3x = 0$	M1
	$\tan3x = \dfrac{2}{3}$	M1
	x=0.196… awrt 0.20	A1
		(3)

Further copies of this publication are available from
Edexcel Publications, Adamsway, Mansfield, Notts, NG18 4FN

Telephone 01623 467467
Fax 01623 450481
Email publication.orders@edexcel.com
Order Code UA027660 June 2011

For more information on Edexcel qualifications, please visit
www.edexcel.com/quals

Pearson Education Limited. Registered company number 872828
with its registered office at Edinburgh Gate, Harlow, Essex CM20 2JE

Llywodraeth Cynulliad Cymru
Welsh Assembly Government

Rewarding Learning

edexcel

Mark Scheme (Results)

Summer 2012

GCE Core Mathematics C3
(6665) Paper 1

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications come from Pearson, the world's leading learning company. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information, please visit our website at www.edexcel.com.

Our website subject pages hold useful resources, support material and live feeds from our subject advisors giving you access to a portal of information. If you have any subject specific questions about this specification that require the help of a subject specialist, you may find our Ask The Expert email service helpful.

www.edexcel.com/contactus

Pearson: helping people progress, everywhere

Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Summer 2012
Publications Code UA031956

Summer 2012
6665 Core Mathematics
C3 Mark Scheme

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = ...$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = ...$

2. Formula

Attempt to use correct formula (with values for *a*, *b* and *c*), leading to $x = ...$

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c, \quad q \neq 0$, leading to $x = ...$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.
Normal marking procedure is as follows:
Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.
Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required.

Question Number	Scheme	Marks
1.	$9x^2-4=(3x-2)(3x+2)$ At any stage	B1
	Eliminating the common factor of (3*x*+2) at any stage $\dfrac{2\cancel{(3x+2)}}{(3x-2)\cancel{(3x+2)}}=\dfrac{2}{3x-2}$	B1
	Use of a common denominator $\dfrac{2(3x+2)(3x+1)}{(9x^2-4)(3x+1)}-\dfrac{2(9x^2-4)}{(9x^2-4)(3x+1)}$ or $\dfrac{2(3x+1)}{(3x-2)(3x+1)}-\dfrac{2(3x-2)}{(3x+1)(3x-2)}$	M1
	$\dfrac{6}{(3x-2)(3x+1)}$ or $\dfrac{6}{9x^2-3x-2}$	A1
		(4 marks)

Notes

B1 For factorising $9x^2-4=(3x-2)(3x+2)$ using difference of two squares. It can be awarded at any stage of the answer but it must be scored on E pen as the first mark

B1 For eliminating/cancelling out a factor of (3*x*+2) at any stage of the answer.

M1 For combining two fractions to form a single fraction with a common denominator. Allow slips on the numerator but at least one must have been adapted. Condone invisible brackets. Accept two separate fractions with the same denominator as shown in the mark scheme. Amongst possible (incorrect) options scoring method marks are

$\dfrac{2(3x+2)}{(9x^2-4)(3x+1)}-\dfrac{2(9x^2-4)}{(9x^2-4)(3x+1)}$ Only one numerator adapted, separate fractions

$\dfrac{2\times3x+1-2\times3x-2}{(3x-2)(3x+1)}$ Invisible brackets, single fraction

A1 $\dfrac{6}{(3x-2)(3x+1)}$

This is not a given answer so you can allow recovery from ‘invisible’ brackets.

Alternative method

$\dfrac{2(3x+2)}{(9x^2-4)}-\dfrac{2}{(3x+1)}=\dfrac{2(3x+2)(3x+1)-2(9x^2-4)}{(9x^2-4)(3x+1)}=\dfrac{18x+12}{(9x^2-4)(3x+1)}$ has scored 0,0,1,0 so far

$=\dfrac{6\cancel{(3x+2)}}{\cancel{(3x+2)}(3x-2)(3x+1)}$ is now 1,1,1,0

$=\dfrac{6}{(3x-2)(3x+1)}$ and now 1,1,1,1

Question Number	Scheme	Marks
2.	(a) $x^3+3x^2+4x-12=0 \Rightarrow x^3+3x^2=12-4x$	
	$\Rightarrow x^2(x+3)=12-4x$	M1
	$\Rightarrow x^2=\dfrac{12-4x}{(x+3)} \Rightarrow x=\sqrt{\dfrac{4(3-x)}{(x+3)}}$	dM1A1*
		(3)
	(b) $x_1=1.41$, *awrt* $x_2=1.20$ $x_3=1.31$	M1A1,A1
		(3)
	(c) Choosing (1.2715,1.2725) or tighter containing root 1.271998323	M1
	$\text{f}(1.2725)=(+)0.00827...$ $\text{f}(1.2715)=-0.00821....$	M1
	Change of sign$\Rightarrow\alpha$=1.272	A1
		(3)
		(9 marks)

Notes

(a) M1 Moves from f(x)=0, which may be implied by subsequent working, to $x^2(x\pm3)=\pm12\pm4x$ by separating terms and factorising in either order. No need to factorise rhs for this mark.

dM1 Divides by '(x+3)' term to make x^2 the subject, then takes square root. No need for rhs to be factorised at this stage

A1* CSO. This is a given solution. Do not allow sloppy algebra or notation with root on just numerator for instance. The 12-4x needs to have been factorised.

(b) **Note that this appears B1,B1,B1 on EPEN**

M1 An attempt to substitute $x_0=1$ into the iterative formula to calculate x_1.

This can be awarded for the sight of $\sqrt{\dfrac{4(3-1)}{(3+1)}}$, $\sqrt{\dfrac{8}{4}}$, $\sqrt{2}$ and even 1.4

A1 $x_1=1.41$. The subscript is not important. Mark as the first value found, $\sqrt{2}$ is A0

A1 $x_2=\text{awrt}\,1.20$ $x_3=\text{awrt}\,1.31$. Mark as the second and third values found. Condone 1.2 for x_2

(c) **Note that this appears M1A1A1 on EPEN**

M1 Choosing the interval (1.2715,1.2725) or tighter containing the root 1.271998323.
Continued iteration is not allowed for this question and is M0

M1 Calculates f(1.2715) and f(1.2725), or **the** tighter interval with at least 1 correct to 1 sig fig rounded or truncated.
Accept f(1.2715) = -0.008 1sf rounded or truncated. Also accept f(1.2715) = -0.01 2dp
Accept f(1.2725) = (+)0.008 1sf rounded or truncated. Also accept f(1.2725) = (+)0.01 2dp

A1 Both values correct (see above),
A valid reason; Accept change of sign, or >0 <0, or f(1.2715) ×f(1.2725)<0
And a (minimal) conclusion; Accept hence root or α=1.272 or QED or □

Alternative to (a) working backwards

2(a)

	$x=\sqrt{\dfrac{4(3-x)}{(x+3)}} \Rightarrow x^2=\dfrac{4(3-x)}{(x+3)} \Rightarrow x^2(x+3)=4(3-x)$	M1
	$x^3+3x^2=12-4x \Rightarrow x^3+3x^2+4x-12=0$	dM1
	States that this is f(x)=0	A1* (3)

Alternative starting with the given result and working backwards

M1 Square (both sides) and multiply by (x+3)

dM1 Expand brackets and collect terms on one side of the equation =0

A1 A statement to the effect that this is f(x)=0

An acceptable answer to (c) with an example of a tighter interval

M1 Choosing the interval (1.2715, 1.2720). This contains the root 1.2719(98323)

M1 Calculates f(1.2715) and f(1.2720), with at least 1 correct to 1 sig fig rounded or truncated.
Accept f(1.2715) = -0.008 1sf rounded or truncated f(1.2715) = -0.01 2dp
Accept f(1.2720) = (+)0.00003 1sf rounded or f(1.2720) = (+)0.00002 truncated 1sf

A1 Both values correct (see above),
A valid reason; Accept change of sign, or >0 <0, or f(1.2715) ×f(1.2720)<0
And a (minimal) conclusion; Accept hence root or α=1.272 or QED or □

x	f(x)
1.2715	-0.00821362
1.2716	-0.00656564
1.2717	-0.00491752
1.2718	-0.00326927
1.2719	-0.00162088
1.2720	+0.00002765
1.2721	+0.00167631
1.2722	+0.00332511
1.2723	+0.00497405
1.2724	+0.00662312
1.2725	+0.00827233

An acceptable answer to (c) using g(x) where g(x)= $\sqrt{\dfrac{4(3-x)}{(x+3)}}-x$

2nd M1 Calculates g(1.2715) and g(1.2725), or **the** tighter interval with at least 1 correct to 1 sig fig rounded or truncated.
g(1.2715) = 0.0007559. Accept g(1.2715) = awrt (+)0.0008 1sf rounded or awrt 0.0007 truncated.
g(1.2725)=-0.00076105. Accept g(1.2725) = awrt -0.0008 1sf rounded or awrt -0.0007 truncated.

Question Number	Scheme	Marks
3.	(a) $\frac{dy}{dx}=\sqrt{3}e^{x\sqrt{3}}\sin 3x+3e^{x\sqrt{3}}\cos 3x$	M1A1
	$\frac{dy}{dx}=0$ $\quad e^{x\sqrt{3}}(\sqrt{3}\sin 3x+3\cos 3x)=0$	M1
	$\tan 3x=-\sqrt{3}$	A1
	$3x=\frac{2\pi}{3}\Rightarrow x=\frac{2\pi}{9}$	M1A1
		(6)
	(b) At x=0 $\quad \frac{dy}{dx}=3$	B1
	Equation of normal is $-\frac{1}{3}=\frac{y-0}{x-0}$ or any equivalent $y=-\frac{1}{3}x$	M1A1
		(3)
		(9 marks)

(a) M1 Applies the product rule vu'+uv' to $e^{x\sqrt{3}}\sin 3x$. If the rule is quoted it must be correct and there must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, ie. terms are written out u=…,u'=….,v=….,v'=….followed by their vu'+uv') only accept answers of the form $\frac{dy}{dx}=Ae^{x\sqrt{3}}\sin 3x+e^{x\sqrt{3}}\times\pm B\cos 3x$

A1 Correct expression for $\frac{dy}{dx}=\sqrt{3}e^{x\sqrt{3}}\sin 3x+3e^{x\sqrt{3}}\cos 3x$

M1 Sets **their** $\frac{dy}{dx}=0$, factorises out or divides by $e^{x\sqrt{3}}$ producing an equation in sin3x and cos3x

A1 Achieves either $\tan 3x=-\sqrt{3}$ or $\tan 3x=-\frac{3}{\sqrt{3}}$

M1 Correct order of arctan, followed by ÷3.

Accept $3x=\frac{5\pi}{3}\Rightarrow x=\frac{5\pi}{9}$ or $3x=\frac{-\pi}{3}\Rightarrow x=\frac{-\pi}{9}$ but not $x=\arctan(\frac{-\sqrt{3}}{3})$

A1 CS0 $x=\frac{2\pi}{9}$ Ignore extra solutions outside the range. Withhold mark for extra inside the range.

(b) B1 Sight of **3** for the gradient

M1 A full method for finding an equation of the normal.

Their tangent gradient m must be modified to $-\frac{1}{m}$ and used together with (0, 0).

Eg $-\frac{1}{their\ 'm'}=\frac{y-0}{x-0}$ or equivalent is acceptable

A1 $y=-\frac{1}{3}x$ or any correct equivalent including $-\frac{1}{3}=\frac{y-0}{x-0}$.

Alternative in part (a) using the form $R\sin(3x+\alpha)$ JUST LAST 3 MARKS

Question Number	Scheme	Marks
3.	(a) $\dfrac{dy}{dx} = \sqrt{3}e^{x\sqrt{3}}\sin 3x + 3e^{x\sqrt{3}}\cos 3x$	M1A1
	$\dfrac{dy}{dx} = 0 \qquad e^{x\sqrt{3}}(\sqrt{3}\sin 3x + 3\cos 3x) = 0$	M1
	$(\sqrt{12})\sin(3x + \dfrac{\pi}{3}) = 0$	A1
	$3x = \dfrac{2\pi}{3} \Rightarrow x = \dfrac{2\pi}{9}$	M1A1
		(6)

A1 Achieves either $(\sqrt{12})\sin(3x + \dfrac{\pi}{3}) = 0$ or $(\sqrt{12})\cos(3x - \dfrac{\pi}{6}) = 0$

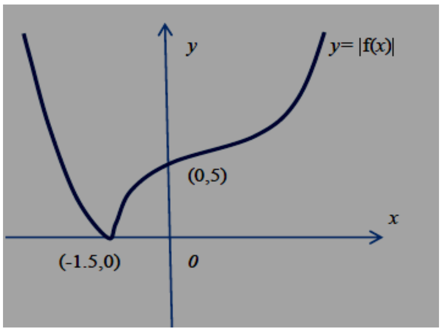
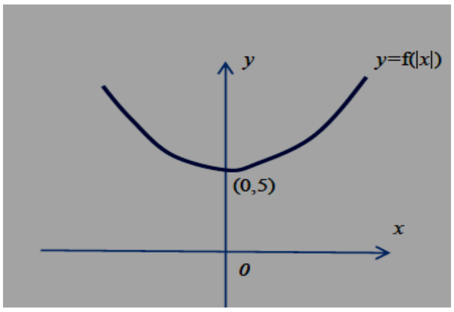
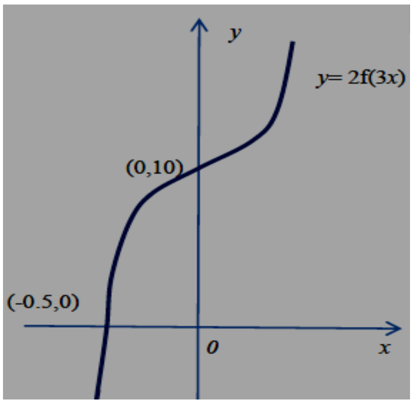
M1 Correct order of arcsin or arcos, etc to produce a value of x

Eg accept $3x + \dfrac{\pi}{3} = 0$ or π or $2\pi \Rightarrow x = ...$

A1 Cao $x = \dfrac{2\pi}{9}$ Ignore extra solutions outside the range. Withhold mark for extra inside the range.

Alternative to part (a) squaring both sides JUST LAST 3 MARKS

Question Number	Scheme	Marks
3.	(a) $\dfrac{dy}{dx} = \sqrt{3}e^{x\sqrt{3}}\sin 3x + 3e^{x\sqrt{3}}\cos 3x$	M1A1
	$\dfrac{dy}{dx} = 0 \qquad e^{x\sqrt{3}}(\sqrt{3}\sin 3x + 3\cos 3x) = 0$	M1
	$\sqrt{3}\sin 3x = -3\cos 3x \Rightarrow \cos^2(3x) = \dfrac{1}{4}$ or $\sin^2(3x) = \dfrac{3}{4}$	A1
	$x = \dfrac{1}{3}\arccos(\pm\sqrt{\dfrac{1}{4}})$ oe	M1
	$x = \dfrac{2\pi}{9}$	A1

Question Number	Scheme		Marks
4.(a)		Shape including cusp	B1
		(-1.5, 0) **and** (0, 5)	B1
			(2)
(b)		Shape	B1
		(0,5)	B1
			(2)
(c)		Shape	B1
		(0,10)	B1
		(-0.5, 0)	B1
			(3)
			(7 marks)

(a) **Note that this appears as M1A1 on EPEN**

B1 Shape (inc cusp) with graph in just quadrants 1 and 2. Do not be overly concerned about relative gradients, but the left hand section of the curve should not bend back beyond the cusp

B1 This is independent, and for the curve touching the *x*-axis at (-1.5, 0) **and** crossing the *y*-axis at (0,5)

(b) **Note that this appears as M1A1 on EPEN**

B1 For a U shaped curve symmetrical about the *y*- axis

B1 (0,5) lies on the curve

(c) **Note that this appears as M1B1B1 on EPEN**

B1 Correct shape- do not be overly concerned about relative gradients. Look for a similar shape to f(*x*)

B1 Curve **crosses** the *y* axis at (0, 10). The curve must appear in both quadrants 1 and 2

B1 Curve **crosses** the *x* axis at (-0.5, 0). The curve must appear in quadrants 3 and 2.

In all parts accept the following for any co-ordinate. Using (0,3) as an example, accept both (3,0) or 3 written on the *y* axis (as long as the curve passes through the point)

Special case with (a) and (b) completely correct but the wrong way around mark - SC(a) 0,1 SC(b) 0,1
Otherwise follow scheme

Question Number	Scheme	Marks
5.	(a) $4\operatorname{cosec}^2 2\theta - \operatorname{cosec}^2\theta = \dfrac{4}{\sin^2 2\theta} - \dfrac{1}{\sin^2\theta}$	
	$= \dfrac{4}{(2\sin\theta\cos\theta)^2} - \dfrac{1}{\sin^2\theta}$	B1 B1
		(2)
	(b) $\dfrac{4}{(2\sin\theta\cos\theta)^2} - \dfrac{1}{\sin^2\theta} = \dfrac{\cancel{4}}{\cancel{4}\sin^2\theta\cos^2\theta} - \dfrac{1}{\sin^2\theta}$	
	$= \dfrac{1}{\sin^2\theta\cos^2\theta} - \dfrac{\cos^2\theta}{\sin^2\theta\cos^2\theta}$	M1
	Using $1-\cos^2\theta = \sin^2\theta$ $\quad = \dfrac{\cancel{\sin^2\theta}}{\cancel{\sin^2\theta}\cos^2\theta}$	M1
	$= \dfrac{1}{\cos^2\theta} = \sec^2\theta$	M1A1*
		(4)
	(c) $\sec^2\theta = 4 \Rightarrow \sec\theta = \pm 2 \Rightarrow \cos\theta = \pm\dfrac{1}{2}$	M1
	$\theta = \dfrac{\pi}{3}, \dfrac{2\pi}{3}$	A1,A1
		(3)
		(9 marks)

Note (a) and (b) can be scored together

(a) B1 One term correct. Eg. writes $4\operatorname{cosec}^2 2\theta$ as $\dfrac{4}{(2\sin\theta\cos\theta)^2}$ **or** $\operatorname{cosec}^2\theta$ as $\dfrac{1}{\sin^2\theta}$. Accept terms like $\operatorname{cosec}^2\theta = 1+\cot^2\theta = 1+\dfrac{\cos^2\theta}{\sin^2\theta}$. The question merely asks for an expression in sin θ and cos θ

B1 A fully correct expression in sin θ and cos θ. Eg. $\dfrac{4}{(2\sin\theta\cos\theta)^2} - \dfrac{1}{\sin^2\theta}$ **Accept equivalents**

Allow a different variable say x's instead of θ's but do not allow mixed units.

b) M1 Attempts to combine their expression in sinθ and cosθ using a common denominator. The terms can be separate but the denominator must be correct and one of the numerators must have been adapted

M1 Attempts to form a 'single' term on the numerator by using the identity $1-\cos^2\theta = \sin^2\theta$

M1 Cancels correctly by $\sin^2\theta$ terms and replaces $\dfrac{1}{\cos^2\theta}$ with $\sec^2\theta$

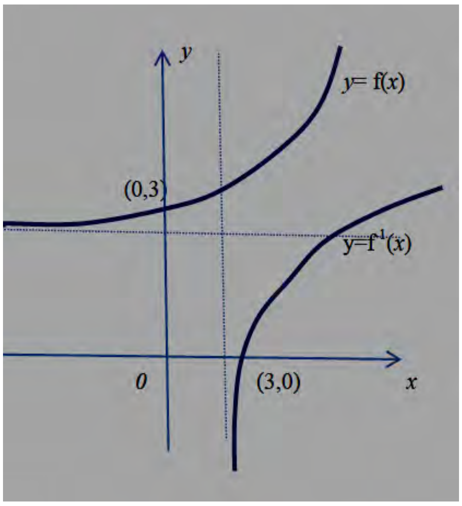
A1* Cso. This is a given answer. All aspects must be correct

IF IN ANY DOUBT SEND TO REVIEW OR CONSULT YOUR TEAM LEADER

c) M1 For $\sec^2\theta = 4$ leading to a solution of $\cos\theta$ by taking the root and inverting in either order .
Similarly accept $\tan^2\theta = 3, \sin^2\theta = \dfrac{3}{4}$ leading to solutions of $\tan\theta, \sin\theta$. Also accept $\cos 2\theta = -\dfrac{1}{2}$

A1 Obtains one correct answer usually $\theta = \dfrac{\pi}{3}$ Do not accept decimal answers or degrees

A1 Obtains both correct answers. $\theta = \dfrac{\pi}{3}, \dfrac{2\pi}{3}$ Do not award if there are extra solutions inside the range.
Ignore solutions outside the range.

Question Number	Scheme	Marks
6.	(a) f(x)>2	B1 (1)
	(b) $\mathrm{fg}(x)=e^{\ln x}+2, =x+2$	M1,A1 (2)
	(c) $e^{2x+3}+2=6 \Rightarrow e^{2x+3}=4$	M1A1
	$\Rightarrow 2x+3=\ln 4$	
	$\Rightarrow x=\dfrac{\ln 4-3}{2}$ or $\ln 2-\dfrac{3}{2}$	M1A1 (4)
	(d) Let $y=e^x+2 \Rightarrow y-2=e^x \Rightarrow \ln(y-2)=x$	M1
	$\mathrm{f}^{-1}(x)=\ln(x-2), \quad x>2.$	A1 , **B1ft** (3)
	(e) 	
	Shape for f(x)	B1
	(0, 3)	B1
	Shape for $\mathrm{f}^{-1}(x)$	B1
	(3, 0)	B1 (4)
		(14 marks)

(a) B1 Range of f(x)>2. Accept y>2, (2,∞), f>2, as well as 'range is the set of numbers bigger than 2' but **don't accept** x>2

(b) M1 For applying the correct order of operations. Look for $e^{\ln x}+2$. Note that $\ln e^x+2$ is M0

A1 Simplifies $e^{\ln x}+2$ to $x+2$. Just the answer is acceptable for both marks

(c) M1 Starts with $e^{2x+3}+2=6$ and proceeds to $e^{2x+3}=...$

A1 $e^{2x+3}=4$

M1 Takes ln's both sides, $2x+3=\ln..$ and proceeds to x=….

A1 $x=\dfrac{\ln 4-3}{2}$ oe. eg $\ln 2-\dfrac{3}{2}$ Remember to isw any incorrect working after a correct answer

(d) **Note that this is marked M1A1A1 on EPEN**

M1 Starts with $y = e^x + 2$ or $x = e^y + 2$ and attempts to change the subject.
All ln work must be correct. The 2 must be dealt with first.
Eg. $y = e^x + 2 \Rightarrow \ln y = x + \ln 2 \Rightarrow x = \ln y - \ln 2$ is M0

A1 $f^{-1}(x) = \ln(x-2)$ or $y = \ln(x-2)$ or $y = \ln|x-2|$ There must be some form of bracket

B1ft Either $x>2$, or follow through on their answer to part (a), provided that it wasn't $y \in \Re$
Do not accept y>2 or $f^{-1}(x)>2$.

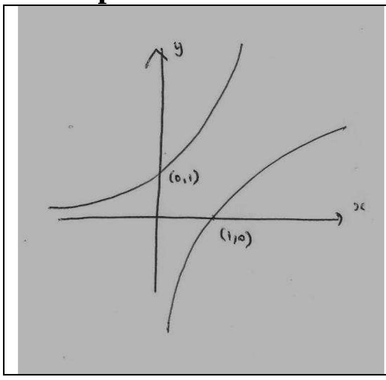
(e) B1 Shape for $y=e^x$. The graph should only lie in quadrants 1 and 2. It should start out with a gradient that is approx. 0 above the *x* axis in quadrant 2 and increase in gradient as it moves into quadrant 1. You should not see a minimum point on the graph.

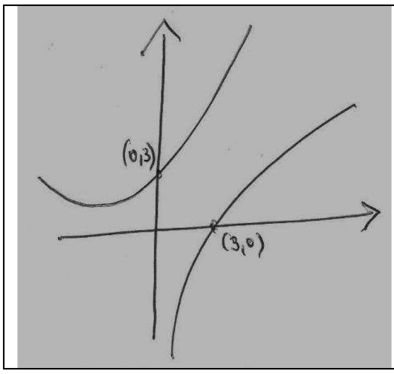
B1 (0, 3) lies on the curve. Accept 3 written on the y axis as long as the point lies on the curve

B1 Shape for y=ln*x*. The graph should only lie in quadrants 4 and 1. It should start out with gradient that is approx. infinite to the right of the *y* axis in quadrant 4 and decrease in gradient as it moves into quadrant 1. You should not see a maximum point. Also with hold this mark if it intersects $y=e^x$

B1 (3, 0) lies on the curve. Accept 3 written on the *x* axis as long as the point lies on the curve

Condone lack of labels in this part
Examples

	Scores **1,0,1,0.** Both shapes are fine, do not be concerned about asymptotes appearing at x=2, y=2. (See notes) Both co-ordinates are incorrect

	Scores **0,1,1,1** Shape for $y = e^x$ is incorrect, there is a minimum point on the graph. All other marks an be awarded

Question Number	Scheme	Marks
7.	(a)(i) $\dfrac{d}{dx}(\ln(3x)) = \dfrac{3}{3x}$	M1
	$\dfrac{d}{dx}(x^{\frac{1}{2}}\ln(3x)) = \ln(3x)\times\dfrac{1}{2}x^{-\frac{1}{2}} + x^{\frac{1}{2}}\times\dfrac{3}{3x}$	M1A1
		(3)
	(ii) $\dfrac{dy}{dx} = \dfrac{(2x-1)^5\times -10-(1-10x)\times 5(2x-1)^4\times 2}{(2x-1)^{10}}$	M1A1
	$\dfrac{dy}{dx} = \dfrac{80x}{(2x-1)^6}$	A1
		(3)
	(b) $x = 3\tan 2y \quad \Rightarrow \dfrac{dx}{dy} = 6\sec^2 2y$	M1A1
	$\Rightarrow \dfrac{dy}{dx} = \dfrac{1}{6\sec^2 2y}$	M1
	Uses $\sec^2 2y = 1+\tan^2 2y$ and uses $\tan 2y = \dfrac{x}{3}$	
	$\Rightarrow \dfrac{dy}{dx} = \dfrac{1}{6(1+(\frac{x}{3})^2)} = \left(\dfrac{3}{18+2x^2}\right)$	M1A1
		(5) **(11 marks)**

Note that this is marked B1M1A1 on EPEN

(a)(i) M1 Attempts to differentiate $\ln(3x)$ to $\dfrac{B}{x}$. Note that $\dfrac{1}{3x}$ is fine.

M1 Attempts the product rule for $x^{\frac{1}{2}}\ln(3x)$. If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms.
If the rule is not quoted nor implied from their stating of u, u', v, v' and their subsequent expression, only accept answers of the form

$$\ln(3x)\times Ax^{-\frac{1}{2}} + x^{\frac{1}{2}}\times\frac{B}{x}, \quad A, B > 0$$

A1 Any correct (un simplified) form of the answer. Remember to isw any incorrect further work

$$\frac{d}{dx}(x^{\frac{1}{2}}\ln(3x)) = \ln(3x)\times\frac{1}{2}x^{-\frac{1}{2}} + x^{\frac{1}{2}}\times\frac{3}{3x} = \left(\frac{\ln(3x)}{2\sqrt{x}} + \frac{1}{\sqrt{x}}\right) = x^{-\frac{1}{2}}\left(\frac{1}{2}\ln 3x + 1\right)$$

Note that this part does not require the answer to be in its simplest form

(ii) M1 Applies the quotient rule, a version of which appears in the formula booklet. If the formula is quoted it must be correct. There must have been an attempt to differentiate both terms. If the formula is not quoted nor implied from their stating of u, u', v, v' and their subsequent expression, only accept answers of the form

$$\frac{(2x-1)^5\times\pm 10-(1-10x)\times C(2x-1)^4}{(2x-1)^{10\text{ or }7\text{ or }25}}$$

A1 Any un simplified form of the answer. Eg $\frac{dy}{dx}=\frac{(2x-1)^5\times -10-(1-10x)\times 5(2x-1)^4\times 2}{((2x-1)^5)^2}$

A1 Cao. It must be simplified as required in the question $\frac{dy}{dx}=\frac{80x}{(2x-1)^6}$

(b) M1 Knows that $3\tan 2y$ differentiates to $C\sec^2 2y$. The lhs can be ignored for this mark. If they write $3\tan 2y$ as $\frac{3\sin 2y}{\cos 2y}$ this mark is awarded for a correct attempt of the quotient rule.

A1 Writes down $\frac{dx}{dy}=6\sec^2 2y$ or implicitly to get $1=6\sec^2 2y\frac{dy}{dx}$

Accept from the quotient rule $\frac{6}{\cos^2 2y}$ or even $\frac{\cos 2y\times 6\cos 2y-3\sin 2y\times -2\sin 2y}{\cos^2 2y}$

M1 An attempt to invert 'their' $\frac{dx}{dy}$ to reach $\frac{dy}{dx}=f(y)$, or changes the subject of their implicit

. differential to achieve a similar result $\frac{dy}{dx}=f(y)$

M1 Replaces an expression for $\sec^2 2y$ in their $\frac{dx}{dy}$ or $\frac{dy}{dx}$ with x by attempting to use $\sec^2 2y=1+\tan^2 2y$. Alternatively, replaces an expression for y in $\frac{dx}{dy}$ or $\frac{dy}{dx}$ with $\frac{1}{2}\arctan(\frac{x}{3})$

A1 Any correct form of $\frac{dy}{dx}$ in terms of x. $\frac{dy}{dx}=\frac{1}{6(1+(\frac{x}{3})^2)}$ $\frac{dy}{dx}=\frac{3}{18+2x^2}$ *or* $\frac{1}{6\sec^2(\arctan(\frac{x}{3}))}$

Question Number	Scheme	Marks
7.	**(a)(ii) Alt using the product rule**	
	Writes $\frac{1-10x}{(2x-1)^5}$ as $(1-10x)(2x-1)^{-5}$ and applies vu'+uv'.	
	See (a)(i) for rules on how to apply	
	$(2x-1)^{-5}\times -10+(1-10x)\times -5(2x-1)^{-6}\times 2$	M1A1
	Simplifies as main scheme to $80x(2x-1)^{-6}$ or equivalent	A1
		(3)
	(b) Alternative using arctan. They must attempt to differentiate to score any marks. Technically this is M1A1M1A2	
	Rearrange $x=3\tan 2y$ to $y=\frac{1}{2}\arctan(\frac{x}{3})$ **and attempt to differentiate**	M1A1
	Differentiates to a form $\frac{A}{1+(\frac{x}{3})^2}$, $=\frac{1}{2}\times\frac{1}{(1+(\frac{x}{3})^2)}\times\frac{1}{3}$ or $\frac{1}{6(1+(\frac{x}{3})^2)}$ oe	M1, A2
		(5)

Question Number	Scheme	Marks
8.	(a) $R=25$	B1
	$\tan\alpha = \frac{24}{7} \Rightarrow \alpha = \text{(awrt)}73.7^0$	M1A1
		(3)
	(b) $\cos(2x + \textit{their}\,\alpha) = \frac{12.5}{\textit{their } R}$	M1
	$2x + \textit{their}\,'\alpha' = 60^0$	A1
	$2x + \textit{their}\,'\alpha' = \text{their } 300^0 \text{ or their } 420^0 \Rightarrow x = ..$	M1
	$x = \textit{awrt}\ 113.1^0, 173.1^0$	A1A1
		(5)
	(c) Attempts to use $\cos 2x = 2\cos^2 x - 1$ **AND** $\sin 2x = 2\sin x \cos x$ in the expression	M1
	$14\cos^2 x - 48\sin x\cos x = 7(\cos 2x + 1) - 24\sin 2x$ $= 7\cos 2x - 24\sin 2x + 7$	A1
		(2)
	(d) $14\cos^2 x - 48\sin x\cos x = R\cos(2x+\alpha)+7$	
	Maximum value ='R'+'c'	M1
	= 32 cao	A1
		(2)
		(12 marks)

(a) B1 Accept 25, awrt 25.0, √625. Condone ± 25

M1 For $\tan\alpha = \pm\frac{24}{7}$ $\tan\alpha = \pm\frac{7}{24}$ $\sin\alpha = \pm\frac{24}{\textit{their } R}$, $\cos\alpha = \pm\frac{7}{\textit{their } R}$

A1 $\alpha = \text{(awrt)}73.7^0$. The answer 1.287 (radians) is A0

(b) M1 For using part (a) and dividing by their R to reach $\cos(2x + \textit{their}\,\alpha) = \frac{12.5}{\textit{their } R}$

A1 Achieving $2x + \textit{their}\ \alpha = 60^{(0)}$. This can be implied by $113.1^{(0)}/113.2^{(0)}$ or $173.1^{(0)}/173.2^{(0)}$ or $-6.8^{(0)}$ / $-6.85^{(0)}$/$-6.9^{(0)}$

M1 Finding a secondary value of x from their principal value. A correct answer will imply this mark

Look for $\dfrac{360 \pm \text{'their' principal value} \pm \text{'their' } \alpha}{2}$

A1 $x = \textit{awrt}\ 113.1^0 / 113.2^0$ OR $173.1^0 / 173.2^0$.

A1 $x = \textit{awrt}\ 113.1^0$ AND 173.1^0. Ignore solutions outside of range. Penalise this mark for extra solutions inside the range

(c) M1 Attempts to use $\cos 2x = 2\cos^2 x - 1$ **and** $\sin 2x = 2\sin x \cos x$ in expression.
Allow slips in sign on the $\cos 2x$ term. So accept $2\cos^2 x = \pm\cos 2x \pm 1$
A1 Cao $= 7\cos 2x - 24\sin 2x + 7$. The order of terms is not important. Also accept a=7, b=-24, c=7

(d) M1 This mark is scored for adding their *R* to their *c*
A1 cao 32

Radian solutions- they will lose the first time it occurs (usually in a with 1.287 radians) Part b will then be marked as follows

(b) M1 For using part (a) and dividing by their *R* to reach $\cos(2x + \textit{their}\,\alpha) = \dfrac{12.5}{\textit{their } R}$

A1 The correct principal value of $\dfrac{\pi}{3}$ or awrt 1.05 radians. Accept $60^{(0)}$
This can be implied by awrt – 0.12 radians or awrt or 1.97 radians or awrt 3.02 radians

M1 Finding a secondary value of *x* from their principal value. A correct answer will imply this mark
Look for $\dfrac{2\pi \pm \text{'their' principal value} \pm \text{'their' } \alpha}{2}$ Do not allow mixed units.

A1 $x = awrt\, 1.97$ OR 3.02.

A1 $x = awrt\, 1.97$ AND 3.02. Ignore solutions outside of range. Penalise this mark for extra solutions inside the range

Further copies of this publication are available from
Edexcel Publications, Adamsway, Mansfield, Notts, NG18 4FN

Telephone 01623 467467
Fax 01623 450481
Email publication.orders@edexcel.com

Order Code UA031956 Summer 2012

For more information on Edexcel qualifications, please visit our website
www.edexcel.com

Pearson Education Limited. Registered company number 872828
with its registered office at Edinburgh Gate, Harlow, Essex CM20 2JE

Llywodraeth Cynulliad Cymru
Welsh Assembly Government

Rewarding Learning

Mark Scheme (Results)

Summer 2013

GCE Core Mathematics 3 (6665/01R)

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications come from Pearson, the world's leading learning company. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk for our BTEC qualifications.
Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

If you have any subject specific questions about this specification that require the help of a subject specialist, you can speak directly to the subject team at Pearson. Their contact details can be found on this link: www.edexcel.com/teachingservices.

You can also use our online Ask the Expert service at www.edexcel.com/ask. You will need an Edexcel username and password to access this service.

Pearson: helping people progress, everywhere
Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Summer 2013
Publications Code UA035673

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes:

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper
- The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate's response may differ from the final mark scheme.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to x =

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to x =

2. Formula

Attempt to use correct formula (with values for *a*, *b* and *c*).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c, \quad q \neq 0,$ leading to x =...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $(x^n \to x^{n-1})$

2. Integration

Power of at least one term increased by 1. $(x^n \to x^{n+1})$

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.
Normal marking procedure is as follows:
Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.
Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required.

Question Number	Scheme	Marks
1.	(a) $x^2+x-12=(x+4)(x-3)$	B1
	Attempt as a single fraction $\dfrac{(3x+5)(x-3)-2(x^2+x-12)}{(x^2+x-12)(x-3)}$ or $\dfrac{3x+5-2(x+4)}{(x+4)(x-3)}$	M1
	$=\dfrac{x-3}{(x+4)(x-3)}$, $=\dfrac{1}{(x+4)}$ cao	A1, A1
		(4 marks)

Notes for Question 1

B1 For correctly factorising $x^2+x-12=(x+4)(x-3)$. It could appear anywhere in their solution

M1 For an attempt to combine two fractions. The denominator must be correct for 'their' fractions.
The terms could be separate but one term must have been modified.
Condone invisible brackets.
Examples of work scoring this mark are;

$\dfrac{(3x+5)(x-3)}{(x^2+x-12)(x-3)}-\dfrac{2(x^2+x-12)}{(x^2+x-12)(x-3)}$ Two separate terms

$\dfrac{3x+5-2x+4}{(x+4)(x-3)}$ Single term, invisible bracket

$\dfrac{(3x+5)}{(x^2+x-12)(x-3)}-\dfrac{2(x^2+x-12)}{(x^2+x-12)(x-3)}$ Separate terms, only one numerator modified

A1 Correct un simplified answer $\dfrac{x-3}{(x+4)(x-3)}$

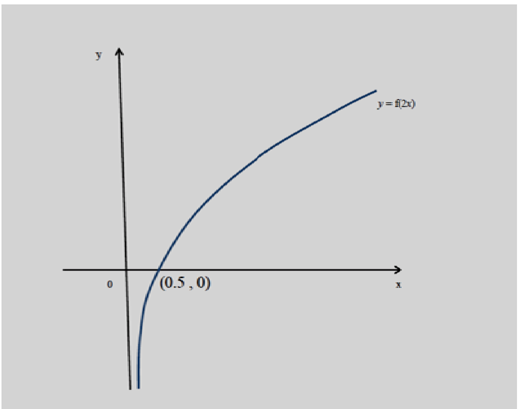
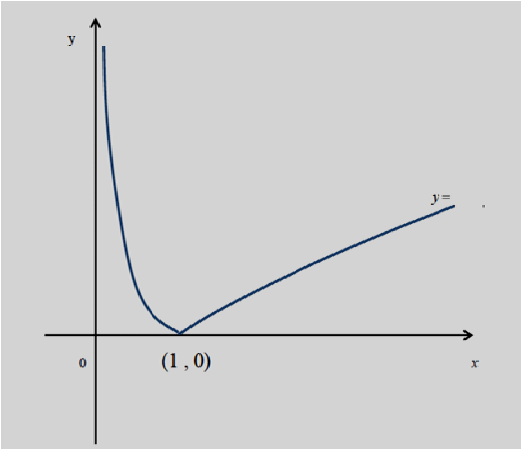
If $\dfrac{x^2-6x-9}{(x^2+x-12)(x-3)}$ scored M1 the fraction must be subsequently be reduced to a correct $\dfrac{x-3}{x^2+x-12}$ or $\dfrac{(x-3)(x-3)}{(x+4)(x-3)(x-3)}$ to score this mark.

A1 cao $\dfrac{1}{(x+4)}$

Do Not isw in this question.

The method of partial fractions is perfectly acceptable and can score full marks

$$\underbrace{\frac{3x+5}{(x+4)(x-3)}}_{B1}-\frac{2}{x-3}=\underbrace{\frac{1}{x+4}+\frac{2}{x-3}}_{M1A1}-\frac{2}{x-3}=\underbrace{\frac{1}{x+4}}_{A1}$$

Question Number	Scheme		Marks
2.(a)		Shape (0.5, 0)	B1 B1 (2)
(b)		Shape (1,0) Cusp at (1,0)	B1 B1 B1 (3) **(5 marks)**

Notes for Question 2

(a)

B1 Award for the correct shape. Look for an increasing function with decreasing gradient. Condone linear looking functions in the first quadrant. It needs to look asymptotic at the *y* axis and have no obvious maximum point. It must be wholly contained in quadrants 1 and 4

See practice and qualification items for clarification.

B1 Crosses *x* axis at $\left(\frac{1}{2}, 0\right)$. Accept $\frac{1}{2}$, 0.5 or even $\left(0, \frac{1}{2}\right)$ marked on the correct axis.

There must be a graph for this mark to be scored.

(b)

B1 Correct shape wholly contained in quadrant 1.

The shape to the rhs of the cusp must not have an obvious maximum.

Accept linear, or positive with decreasing gradient. The gradient of the curve to the lhs of the cusp/minimum should always be negative. The curve in this section should not 'bend' back past (1, 0) forming a 'C' shape or have incorrect curvature.

See practice and qualification for clarification.

B1 The curve touches or crosses the *x* axis at (1, 0). Allow for the curve passing through a point marked '1' on the *x* axis. Condone the point marked on the correct axis as (0, 1)

B1 Award for a cusp, not a minimum at (1,0)

Note that $\mathrm{f}\left(|x|\right)$ scores B0 B1 B0 under the scheme.

If the graphs are not labelled (a) and (b), then they are to be marked in the order they are presented

Question Number	Scheme	Marks
3.(a)	$7\cos x + \sin x = R\cos(x-\alpha)$	
	$R = \sqrt{(7^2+1^2)} = \sqrt{50} = (5\sqrt{2})$	B1
	$\alpha = \arctan\left(\frac{1}{7}\right) = 8.13... = \text{awrt } 8.1^0$	M1A1
		(3)
(b)	$\sqrt{50}\cos(x-8.1) = 5 \Rightarrow \cos(x-8.1) = \frac{5}{\sqrt{50}}$	M1
	$x - 8.1 = 45 \Rightarrow x = 53.1^0$	M1,A1
	AND $x - 8.1 = 315 \Rightarrow x = 323.1^0$	M1A1
		(5)
(c)	One solution if $\frac{k}{\sqrt{50}} = \pm 1, \Rightarrow k = \pm\sqrt{50}$ ft on R	M1A1ft
		(2)
		(10 marks)

Notes for Question 3

(a)

B1 $R=\sqrt{50}$. Accept $5\sqrt{2}$ Accept $R=\pm\sqrt{50}$

Do not accept $R=\sqrt{(7^2+1^2)}$ or the decimal equivalent 7.07…unless you see $\sqrt{50}$ or $5\sqrt{2}$ as well

M1 For $\tan\alpha=\pm\frac{1}{7}$ or $\tan\alpha=\pm\frac{7}{1}$. Condone if this comes from $\cos\alpha=7, \sin\alpha=1$

If *R* is used then only accept $\sin\alpha=\pm\frac{1}{R}$ or $\cos\alpha=\pm\frac{7}{R}$

A1 $\alpha=$ awrt 8.1.

Be aware that $\tan\alpha=7\Rightarrow\alpha=81.9$ can easily be mistaken for the correct answer

Note that the radian answer awrt 0.1418… is A0

(b)

M1 For using their answers to part (a) and moving from $R\cos(x\pm\alpha)=5\Rightarrow\cos(x\pm\alpha)=\frac{5}{R}$ using their numerical values of R and α

This may be implied for sight of 53.1 if *R* and α were correct

M1 For achieving $x\pm\alpha=$ awrt 45^0 or 315, leading to one value of *x* **in the range**

Note that for this to be scored *R* has to be correct (to 2sf) as awrt 45, 315 must be achieved

This may be implied for achieving an answer of either $45+\textit{their }\alpha$ or $315+\textit{their }\alpha$

A1 One correct answer, either awrt 53.1° or 323.1°

M1 For an attempt at finding a secondary value of *x* **in the range**.

Usually this is an attempt at solving $x-\textit{their }8.1^0=360^0-\textit{their }45^0\Rightarrow x=..$

A1 Both values correct awrt 53.1° and 323.1°.

Withhold this mark if there are extra values in the range.

Ignore extra values outside the range

(c)

M1 For stating that $\frac{k}{\textit{their }R}=1$ OR $\frac{k}{\textit{their }R}=-1$

This may be implied by seeing $k=(\pm)\textit{their }R$

A1ft Both values $k=\pm\sqrt{50}$ oe . Follow through on their numerical R

Answers all in radians. Lose the first time that it appears but demand an accuracy of 2dp.

Part (a) $R=\sqrt{50}$ $\alpha=awrt\ 0.14$

Part (b) $x=awrt\ 0.927,\ 5.64$. Accuracy must be to 3 sf.

With correct working this would score (a) B1M1A0 (b) M1A1A1M1A1

Mixed degrees and radians refer to the main scheme

Question Number	Scheme	Marks
4.(a)	$\mathrm{f}(x) \geqslant 3$	M1A1
		(2)
(b)	An attempt to find $2\|3-4x\|+3$ when $x=1$	M1
	Correct answer $\mathrm{fg}(1)=5$	A1
		(2)
(c)	$y=3-4x \Rightarrow 4x=3-y \Rightarrow x=\dfrac{3-y}{4}$	M1
	$\mathrm{g}^{-1}(x)=\dfrac{3-x}{4}$	A1
		(2)
(d)	$\left[g(x)\right]^2=(3-4x)^2$	B1
	$\mathrm{gg}(x)=3-4(3-4x)$	M1
	$gg(x)+\left[g(x)\right]^2=0 \Rightarrow -9+16x+9-24x+16x^2=0$	
	$16x^2-8x=0$	A1
	$8x(2x-1)=0 \Rightarrow x=0,\ 0.5$ oe	M1A1
		(5)
		(11 marks)

Notes for Question 4

(a)

M1 Attempt at calculating f at x=0. Sight of 3 is sufficient. Accept $f(x) > 3$ and $x > 3$ for M1,

A1 $f(x) \geqslant 3$. Accept $y \geqslant 3$, range$\geqslant 3$, $[3, \infty)$

Do not accept $f(x) > 3$, $x \geqslant 3$

The correct answer is sufficient for both marks.

(b)

M1 A full method of finding fg(1). The order of substituting into the expressions must be correct and $2|x|+3$ must be used as opposed to $2x+3$

Accept an attempt to calculate $2|x|+3$ when x= -1.

Accept an attempt to put x=1 into $3-4x$ and then substituting their answer to $3-4x|_{x=1}$ into $2|x|+3$

Do not accept the substitution of x=1 into $2|x|+3$, followed by their result into '3-4x'

This is evidence of incorrect order.

A1 fg(1)=5.

Watch for $1 \xrightarrow{3-4x} 1 \xrightarrow{2|x|+3} 5$ **which is M1A0**

(c)

M1 Award for an attempt to make x or a swapped y the subject of the formula. It must be a full method and cannot finish $4x = ..$

You can condone at most one 'arithmetic' error for this method mark.

$y = 3-4x \Rightarrow 4x = 3+y \Rightarrow x = \frac{3+y}{4}$ is fine for the M1 as there is only one error

$y = 3-4x \Rightarrow 4x = 3-y \Rightarrow x = \frac{3}{4} - y$ is fine for the M1 as there is only one error

$y = 3-4x \Rightarrow 4x = 3+y \Rightarrow x = \frac{3}{4} + y$ is M0 as there are two arithmetic errors

A1 Obtaining a correct expression $g^{-1}(x) = \frac{3-x}{4}$ oe such as $g^{-1}(x) = \frac{x-3}{-4}$, $g^{-1}(x) = \frac{3}{4} - \frac{x}{4}$

It must be in terms of x, but could be expressed 'y=' or $g^{-1}(x) \rightarrow$

(d)

B1 Sight of $[g(x)]^2 = (3-4x)^2$. If only the expanded version appears it must be correct

M1 A full attempt to find $gg(x) = 3-4(3-4x)$

Condone invisible brackets. Note that it may appear in an equation

A1 $16x^2 - 8x = 0$ Accept other alternatives such as $2x^2 = x$

M1 For factorising their quadratic or cancelling their $Ax^2 = Bx$ by x to get ≥ 1 value of x

If they have a 3TQ then usual methods are applicable.

A1 Both values correct $x = 0, 0.5$ oe

Question Number	Scheme	Marks
5.(a)	$\frac{\mathrm{d}}{\mathrm{d}x}(\cos 2x) = -2\sin 2x$	B1
	Applies $\frac{vu'-uv'}{v^2}$ to $\frac{\cos 2x}{\sqrt{x}}$ $= \frac{\sqrt{x}\times -2\sin 2x - \cos 2x \times \frac{1}{2}x^{-\frac{1}{2}}}{(\sqrt{x})^2}$	M1A1
	$= \frac{-2\sqrt{x}\sin 2x - \frac{1}{2}x^{-\frac{1}{2}}\cos 2x}{x}$	
		(3)
(b)	$\frac{\mathrm{d}}{\mathrm{d}x}(\sec^2 3x) = 2\sec 3x \times 3\sec 3x \tan 3x \,(= 6\sec^2 3x \tan 3x)$	M1
	$= 6(1+\tan^2 3x)\tan 3x$	dM1
	$= 6(\tan 3x + \tan^3 3x)$	A1
		(3)
(c)	$x = 2\sin\left(\frac{y}{3}\right) \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}y} = \frac{2}{3}\cos\left(\frac{y}{3}\right)$	M1A1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\frac{2}{3}\cos\left(\frac{y}{3}\right)} = \frac{1}{\frac{2}{3}\sqrt{\left(1-\sin^2\left(\frac{y}{3}\right)\right)}} = \frac{1}{\frac{2}{3}\sqrt{1-\left(\frac{x}{2}\right)^2}}$	dM1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3}{\sqrt{4-x^2}}$ cao	A1
		(4)
		(10 marks)
Alt 5(c)	$y = 3\arcsin\left(\frac{x}{2}\right) \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3}{\sqrt{1-\left(\frac{x}{2}\right)^2}} \times \frac{1}{2}$	M1dM1A1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3}{\sqrt{4-x^2}}$	A1
	M1 Rearranging to $y = A\arcsin Bx$ and differentiating to $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{A}{\sqrt{1-Bx^2}}$	
	dM1 As above, but form of the rhs must be correct $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{C}{\sqrt{1-\left(\frac{x}{2}\right)^2}}$	**(4)**
	A1 Correct but un simplified answer	

Notes for Question 5

(a)

B1 Award for the sight of $\frac{\mathrm{d}}{\mathrm{d}x}(\cos 2x) = -2\sin 2x$. This could be seen in their differential.

M1 Applies $\frac{vu'-uv'}{v^2}$ to $\frac{\cos 2x}{\sqrt{x}}$

If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, with terms written out u=…,u'=….,v=….,v'=….followed by their $\frac{vu'-uv'}{v^2}$) then only accept answers of the form

$$\frac{\sqrt{x}\times \pm A\sin 2x - \cos 2x \times Bx^{-\frac{1}{2}}}{(\sqrt{x})^2 \text{ or } x^{\frac{1}{4}}}$$

A1 Award for a correct answer. This does not need to be simplified.

Alt (a) using the product rule

B1 Award for the sight of $\frac{\mathrm{d}}{\mathrm{d}x}(\cos 2x) = -2\sin 2x$. This could be seen in their differential.

M1 Applies $vu'+uv'$ to $x^{-\frac{1}{2}}\cos 2x$. If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, with terms written out u=…,u'=….,v=….,v'=….followed by their $vu'+uv'$) then only accept answers of the form

$$\pm Ax^{-\frac{1}{2}}\sin 2x - Bx^{-\frac{3}{2}}\cos 2x$$

A1 Award for a correct answer. This does not need to be simplified.

$$-2x^{-\frac{1}{2}}\sin 2x - \frac{1}{2}x^{-\frac{3}{2}}\cos 2x$$

(b)

M1 Award for a correct application of the chain rule on $\sec^2 3x$
Sight of $C\sec 3x \sec 3x \tan 3x$ is sufficient

dM1 Replacing $\sec^2 3x = 1+\tan^2 3x$ in their derivative to create an expression in just $\tan 3x$. It is dependent upon the first M being scored.

A1 The correct answer $6(\tan 3x + \tan^3 3x)$. There is no need to write $\mu = 6$

Alt (b) using the product rule

M1 Writes $\sec^2 3x$ as $\sec 3x \times \sec 3x$ and uses the product rule with $u' = A\sec 3x \tan 3x$ and $v' = B\sec 3x \tan 3x$ to produce a derivative of the form $A\sec 3x \tan 3x \sec 3x + B\sec 3x \tan 3x \sec 3x$

dM1 Replaces $\sec^2 3x$ with $1+\tan^2 3x$ to produce an expression in just $\tan 3x$. It is dependent upon the first M being scored.

Notes for Question 5 Continued

A1 The correct answer $6(\tan 3x+\tan^3 3x)$. There is no need to write $\mu=6$

Alt (b) using $\sec 3x=\dfrac{1}{\cos 3x}$ **and proceeding by the chain or quotient rule**

M1 Writes $\sec^2 3x$ as $(\cos 3x)^{-2}$ and differentiates to $A(\cos 3x)^{-3}\sin 3x$

Alternatively writes $\sec^2 3x$ as $\dfrac{1}{(\cos 3x)^2}$ and achieves $\dfrac{(\cos 3x)^2\times 0-1\times A\cos 3x\sin 3x}{\left(\cos^2 3x\right)^2}$

dM1 Uses $\dfrac{\sin 3x}{\cos 3x}=\tan 3x$ and $\dfrac{1}{\cos^2 3x}=\sec^2 3x$ and $\sec^2 3x=1+\tan^2 3x$ in their derivative to create an expression in just $\tan 3x$. It is dependent upon the first M being scored.

A1 The correct answer $6(\tan 3x+\tan^3 3x)$. There is no need to write $\mu=6$

Alt (b) using $\sec^2 3x=1+\tan^2 3x$

M1 Writes $\sec^2 3x$ as $1+\tan^2 3x$ and
uses chain rule to produce a derivative of the form $A\tan 3x\sec^2 3x$
or the product rule to produce a derivative of the form $C\tan 3x\sec^2 3x+D\tan 3x\sec^2 3x$

dM1 Replaces $\sec^2 3x=1+\tan^2 3x$ to produce an expression in just $\tan 3x$. It is dependent upon the first M being scored.

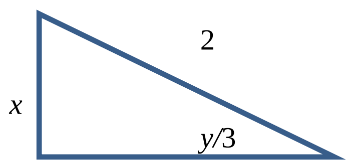
A1 The correct answer $6(\tan 3x+\tan^3 3x)$. There is no need to write $\mu=6$

(c)

M1 Award for knowing the method that $\sin\left(\dfrac{y}{3}\right)$ differentiates to $\cos\left(\dfrac{y}{3}\right)$ The lhs does not need to be correct/present. Award for $2\sin\left(\dfrac{y}{3}\right)\to A\cos\left(\dfrac{y}{3}\right)$

A1 $x=2\sin\left(\dfrac{y}{3}\right)\Rightarrow\dfrac{\mathrm{d}x}{\mathrm{d}y}=\dfrac{2}{3}\cos\left(\dfrac{y}{3}\right)$. Both sides must be correct

dM1 Award for inverting their $\dfrac{\mathrm{d}x}{\mathrm{d}y}$ and using $\sin^2\dfrac{y}{3}+\cos^2\dfrac{y}{3}=1$ to produce an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x only. It is dependent upon the first M 1 being scored.
An alternative to Pythagoras is a triangle.



$$\sin\left(\frac{y}{3}\right)=\frac{x}{2}\Rightarrow\cos\left(\frac{y}{3}\right)=\frac{\sqrt{4-x^2}}{2}$$

Notes for Question 5 Continued
Candidates who write $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3}{2\cos\left(\arcsin\left(\frac{x}{2}\right)\right)}$ do not score the mark. BUT $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3}{2\sqrt{1-\sin^2\left(\arcsin\left(\frac{x}{2}\right)\right)}}$ does score M1 as they clearly use a correct Pythagorean identity as required by the notes. A1 $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3}{\sqrt{4-x^2}}$. Expression must be in its simplest form. Do not accept $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3}{2\sqrt{1-\frac{1}{4}x^2}}$ or $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{\frac{1}{3}\sqrt{4-x^2}}$ for the final A1

Question Number	Scheme	Marks
6.(i)	$\text{cosec}2x = \dfrac{1}{\sin 2x}$	M1
	$= \dfrac{1}{2\sin x \cos x}$	M1
	$= \dfrac{1}{2}\text{cosec}x \sec x \Rightarrow \quad \lambda = \dfrac{1}{2}$	A1
		(3)
(ii)	$3\sec^2\theta + 3\sec\theta = 2\tan^2\theta \Rightarrow 3\sec^2\theta + 3\sec\theta = 2(\sec^2\theta - 1)$	M1
	$\sec^2\theta + 3\sec\theta + 2 = 0$	
	$(\sec\theta + 2)(\sec\theta + 1) = 0$	M1
	$\sec\theta = -2, -1$	A1
	$\cos\theta = -0.5, -1$	M1
	$\theta = \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \pi$	A1A1
		(6)
		(9 marks)
ALT (ii)	$3\sec^2\theta + 3\sec\theta = 2\tan^2\theta \Rightarrow 3\times\dfrac{1}{\cos^2\theta} + 3\times\dfrac{1}{\cos\theta} = 2\times\dfrac{\sin^2\theta}{\cos^2\theta}$	
	$3 + 3\cos\theta = 2\sin^2\theta$	
	$3 + 3\cos\theta = 2(1 - \cos^2\theta)$	M1
	$2\cos^2\theta + 3\cos\theta + 1 = 0$	
	$(2\cos\theta + 1)(\cos\theta + 1) = 0 \Rightarrow \cos\theta = -0.5, -1$	M1A1
	$\theta = \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \pi$	M1,A1,A1
		(6)
		(9 marks)

Notes for Question 6	
(i)	
M1	Uses the identity $\text{cosec}2x = \dfrac{1}{\sin 2x}$
M1	Uses the correct identity for $\sin 2x = 2\sin x\cos x$ in their expression. Accept $\sin 2x = \sin x\cos x + \cos x\sin x$
A1	$\lambda = \dfrac{1}{2}$ following correct working
(ii)	
M1	Replaces $\tan^2\theta$ by $\pm\sec^2\theta \pm 1$ to produce an equation in just $\sec\theta$
M1	Award for a forming a 3TQ=0 in $\sec\theta$ and applying a correct method for factorising, or using the formula, or completing the square to find two answers to $\sec\theta$ If they replace $\sec\theta = \dfrac{1}{\cos\theta}$ it is for forming a 3TQ in $\cos\theta$ and applying a correct method for finding two answers to $\cos\theta$
A1	Correct answers to $\sec\theta = -2, -1$ or $\cos\theta = -\dfrac{1}{2}, -1$
M1	Award for using the identity $\sec\theta = \dfrac{1}{\cos\theta}$ and proceeding to find at least one value for θ. If the 3TQ was in cosine then it is for finding at least one value of θ.
A1	Two correct values of θ. All method marks must have been scored. Accept two of $120°, 180°, 240°$ or two of $\dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \pi$ or two of awrt 2dp 2.09, 3.14, 4.19
A1	All three answers correct. They must be given in terms of π as stated in the question. Accept $0.\dot{6}\pi, 1.\dot{3}\pi, \pi$ Withhold this mark if further values in the range are given. All method marks must have been scored. Ignore any answers outside the range.
Alt (ii)	
M1	Award for replacing $\sec^2\theta$ with $\dfrac{1}{\cos^2\theta}$, $\sec\theta$ with $\dfrac{1}{\cos\theta}$, $\tan^2\theta$ with $\dfrac{\sin^2\theta}{\cos^2\theta}$ multiplying through by $\cos^2\theta$ (seen in at least 2 terms) and replacing $\sin^2\theta$ with $\pm1 \pm \cos^2\theta$ to produce an equation in just $\cos\theta$
M1	Award for a forming a 3TQ=0 in $\cos\theta$ and applying a correct method for factorising, or using the formula, or completing the square to find two answers to $\cos\theta$
A1	$\cos\theta = -\dfrac{1}{2}, -1$
M1	Proceeding to finding at least one value of θ from an equation in $\cos\theta$.
A1	Two correct values of θ. All method marks must have been scored Accept two of $120°, 180°, 240°$ or two of $\dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \pi$ or two of awrt 2dp 2.09, 3.14, 4.19
A1	All three answers correct. They must be given in terms of π as stated in the question.

Notes for Question 6 Continued
Accept $0.\dot{6}\pi, 1.\dot{3}\pi, \pi$ All method marks must have been scored. Withhold this mark if further values in the range are given. Ignore any answers outside the range

Question Number	Scheme	Marks
7.(a)	$f(x)=0 \Rightarrow x^2+3x+1=0$	
	$\Rightarrow x=\dfrac{-3\pm\sqrt{5}}{2}=$ awrt -0.382, -2.618	M1A1
		(2)
(b)	Uses $vu'+uv'$ $\quad \mathrm{f}'(x)=e^{x^2}(2x+3)+(x^2+3x+1)e^{x^2}\times 2x$	M1A1A1
		(3)
(c)	$e^{x^2}(2x+3)+(x^2+3x+1)e^{x^2}\times 2x=0$	
	$\Rightarrow e^{x^2}\{2x^3+6x^2+4x+3\}=0$	M1
	$\Rightarrow x(2x^2+4)=-3(2x^2+1)$	M1
	$\Rightarrow x=-\dfrac{3(2x^2+1)}{2(x^2+2)}$	A1*
		(3)
(d)	Sub $x_0=-2.4$ into $x_{n+1}=-\dfrac{3(2x_n{}^2+1)}{2(x_n{}^2+2)}$	
	$x_1=awrt-2.420,\ x_2=awrt-2.427\ \ x_3=awrt-2.430$	M1A1,A1
		(3)
(e)	Sub x= - 2.425 and -2.435 into $\mathrm{f}'(x)$ and start to compare signs	
	$\mathrm{f}'(-2.425)=+22.4,\ \mathrm{f}'(-2.435)=-15.02$	**M1**
	Change in sign, hence $\mathrm{f}'(x)=0$ in between. Therefore α = - 2.43 (2dp)	A1
		(2)
		(13 marks)
Alt 7.(c)	$x=-\dfrac{3(2x^2+1)}{2(x^2+2)} \Rightarrow 2x(x^2+2)=-3(2x^2+1) \Rightarrow 2x^3+6x^2+4x+3=0$	M1
	$\mathrm{f}'(x)=e^{x^2}\{2x^3+6x^2+4x+3\}=0$ when $2x^3+6x^2+4x+3=0$	M1
	Hence the minimum point occurs when $x=-\dfrac{3(2x^2+1)}{(2x^2+4)}$	**A1**

Question Number	Scheme	Marks
Alt 1 7(e)	Sub x= - 2.425 and -2.435 into cubic part of $\text{f}'(x) = 2x^3 + 6x^2 + 4x + 3$ and start to compare signs Adapted $\text{f}'(-2.425) = +0.06, \text{f}'(-2.435) = -0.04$ Change in sign, hence $\text{f}'(x) = 0$ in between. Therefore α = - 2.43 (2dp)	**M1** A1 **(2)**
Alt 2 7 (e)	Sub x= - 2.425, -2.43 and -2.435 into $\text{f}(x) = (x^2 + 3x + 1)e^{x^2}$ and start to compare sizes $\text{f}(-2.425) = -141.2, \text{f}(-2.435) = -141.2, \text{f}(-2.43) = -141.3$ $\text{f}(-2.43) < \text{f}(-2.425), \text{f}(-2.43) < \text{f}(-2.435)$. Therefore α = - 2.43 (2dp)	**M1** A1 **(2)**

Notes for Question 7

(a)

M1 Solves $x^2 + 3x + 1 = 0$ by completing the square or the formula, producing two 'non integer answers. **Do not accept factorisation here** . Accept awrt -0.4 and -2.6 for this mark

A1 Answers correct. Accept awrt -0.382, -2.618.

Accept just the answers for both marks. Don't withhold the marks for incorrect labelling.

(b)

M1 Applies the product rule $vu' + uv'$ to $(x^2 + 3x + 1)e^{x^2}$.

If the rule is quoted it must be correct and there must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, ie. terms are written out u=…,u'=….,v=….,v'=….followed by their vu'+uv') only accept answers of the form

$$\left(\frac{\text{d}y}{\text{d}x}\right) = \text{f}'(x) = e^{x^2}(Ax + B) + (x^2 + 3x + 1)Cxe^{x^2}$$

A1 One term of $\text{f}'(x) = e^{x^2}(2x + 3) + (x^2 + 3x + 1)e^{x^2} \times 2x$ correct.

There is no need to simplify

A1 A fully correct (un simplified) answer $\text{f}'(x) = e^{x^2}(2x + 3) + (x^2 + 3x + 1)e^{x^2} \times 2x$

(c)

M1 Sets their $\text{f}'(x) = 0$ and either factorises out, or cancels by e^{x^2} to produce a polynomial equation in x

M1 Rearranges the cubic polynomial to $Ax^3 + Bx = Cx^2 + D$ and factorises to reach $x(Ax^2 + B) = Cx^2 + D$ or equivalent

A1* Correctly proceeds to $x = -\dfrac{3(2x^2 + 1)}{2(x^2 + 2)}$. This is a given answer

Notes on Question 7 Continued

(c) Alternative to (c) working backwards

M1 Moves correctly from $x = -\frac{3(2x^2+1)}{2(x^2+2)}$ to $2x^3 + 6x^2 + 4x + 3 = 0$

M1 States or implies that $f'(x) = 0$

A1 Makes a conclusion to tie up the argument

For example, hence the minimum point occurs when $x = -\frac{3(2x^2+1)}{(2x^2+4)}$

(d)

M1 Sub $x_0 = -2.4$ into $x_{n+1} = -\frac{3(2x_n{}^2+1)}{2(x_n{}^2+2)}$

This may be implied by awrt -2.42, or $x_{n+1} = -\frac{3(2\times -2.4^2+1)}{2(-2.4^2+2)}$

A1 Awrt. $x_1 = -2.420$.

The subscript is not important. Mark as the first value given

A1 awrt $x_2 = -2.427$ awrt $x_3 = -2.430$

The subscripts are not important. Mark as the second and third values given

(e)

Note that continued iteration is not allowed

M1 Sub x= - 2.425 and -2.435 into $f'(x)$, starts to compare signs and gets at least one correct to 1 sf rounded or truncated.

A1 Both values correct (1sf rounded or truncated), a reason and a minimal conclusion

Acceptable reasons are change in sign, positive and negative and $f'(a) \times f'(b) < 0$

Minimal conclusions are hence $\alpha = -2.43$, hence shown, hence root

Alt 1 using adapted $f'(x)$

(e)

M1 Sub x= - 2.425 and -2.435 into cubic part of $f'(x)$, starts to compare signs and gets at least one correct to 1 sf rounded or truncated.

A1 Both values correct of adapted $f'(x)$ correct (1sf rounded or truncated), a reason and a minimal conclusion

Acceptable reasons are change in sign, positive and negative and $f'(a) \times f'(b) < 0$

Minimal conclusions are hence $\alpha = -2.43$, hence shown, hence root

Alt 2 using $f(x)$

(e)

M1 Sub x= - 2.425, -2.43 and -2.435 into $f(x)$, starts to compare sizes and gets at least one correct to 4sf rounded

A1 All three values correct of $f(x)$ correct (4sf rounded), a reason and a minimal conclusion

Acceptable reasons are $f(-2.43) < f(-2.425), f(-2.43) < f(-2.435)$, a sketch

Minimal conclusions are hence $\alpha = -2.43$, hence shown, hence root

Question Number	Scheme	Marks
8.(a)	$t=0 \Rightarrow P=\dfrac{8000}{1+7}=1000$ cao	M1A1
		(2)
(b)	$t \to \infty \quad P \to \dfrac{8000}{1}=8000$	B1
		(1)
(c)	$t=3,\ P=2500 \Rightarrow 2500=\dfrac{8000}{1+7e^{-3k}}$	B1
	$e^{-3k}=\dfrac{2.2}{7}=(0.31..)$ oe	M1,A1
	$k=-\dfrac{1}{3}\ln\left(\dfrac{2.2}{7}\right)=$ awrt 0.386	M1A1
		(5)
(d)	Sub t=10 into $P=\dfrac{8000}{1+7e^{-0.386t}} \Rightarrow P=6970$ cao	M1A1
		(2)
(e)	$\dfrac{dP}{dt}=-\dfrac{8000}{(1+7e^{-kt})^2}\times -7ke^{-kt}$	M1,A1
	Sub t=10 $\left.\dfrac{dP}{dt}\right\rvert_{t=10}=346$	A1
		(3)
		(13 marks)

Notes for Question 8

(a)

M1 Sets t=0, giving $e^{-k\times 0}=1$. Award if candidate attempts $\dfrac{8000}{1+7\times 1},\dfrac{8000}{8}$

A1 Correct answer only 1000. Accept 1000 for both marks as long as no incorrect working is seen.

(b)

B1 8000. Accept $P<8000$. Condone $P\leqslant 8000$ but not $P>8000$

(c)

B1 Sets both $t=3$, and $P=2500\Rightarrow 2500=\dfrac{8000}{1+7e^{-3k}}$

This may be implied by a subsequent correct line.

M1 Rearranges the equation to make $e^{\pm 3k}$ the subject. They need to multiply by the $1+7e^{-3k}$ term, and proceed to $e^{\pm 3k}=A,\quad A>0$

A1 The correct intermediate answer of $e^{-3k}=\dfrac{2.2}{7},\dfrac{11}{35}$ or equivalent. Accept awrt 0.31..

Alternatively accept $e^{3k}=\dfrac{35}{11}, 3.18..$ or equivalent.

M1 Proceeds from $e^{\pm 3k}=A,\quad A>0$ by correctly taking *ln*'s and then making *k* the subject of the formula.

Award for $e^{-3k}=A\Rightarrow -3k=\ln(A)\Rightarrow k=\dfrac{\ln(A)}{-3}$

If e^{3k} was found accept $e^{3k}=C\Rightarrow 3k=\ln C\Rightarrow k=\dfrac{\ln C}{3}$ As with method 1, C >0

A1 Awrt $k=0.386$ 3dp

(d)

M1 Substitutes t=10 into $P=\dfrac{8000}{1+7e^{-kt}}$ with their numerical value of *k* to find *P*

A1 $(P=)\,6970$ or other exact equivalents like 6.97×10^3

(e)

M1 Differentiates using the chain rule to a form $\dfrac{dP}{dt}=\dfrac{C}{(1+7e^{-kt})^2}\times e^{-kt}$

Accept an application of the quotient rule to achieve $\dfrac{(1+7e^{-kt})\times 0-C\times -e^{-kt}}{(1+7e^{-kt})^2}$

A1 A correct un simplified $\dfrac{dP}{dt}=-\dfrac{8000}{(1+7e^{-kt})^2}\times -7ke^{-kt}$.

The derivative can be given in terms of *k*. If a numerical value is used you may follow through on incorrect values.

A1 Awrt 346. Note that M1 must have been achieved. Just the answer scores 0

Further copies of this publication are available from
Edexcel Publications, Adamsway, Mansfield, Notts, NG18 4FN

Telephone 01623 467467
Fax 01623 450481
Email publication.orders@edexcel.com

Order Code UA035673 Summer 2013

For more information on Edexcel qualifications, please visit our website www.edexcel.com

Pearson Education Limited. Registered company number 872828
with its registered office at Edinburgh Gate, Harlow, Essex CM20 2JE

Ofqual

Llywodraeth Cynulliad Cymru
Welsh Assembly Government

cea
Rewarding Learning

Mark Scheme (Results)

Summer 2013

GCE Core Mathematics 3 (6665/01)

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications come from Pearson, the world's leading learning company. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk for our BTEC qualifications.

Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

If you have any subject specific questions about this specification that require the help of a subject specialist, you can speak directly to the subject team at Pearson.

Their contact details can be found on this link: www.edexcel.com/teachingservices.

You can also use our online Ask the Expert service at www.edexcel.com/ask. You will need an Edexcel username and password to access this service.

Pearson: helping people progress, everywhere

Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Summer 2013
Publications Code UA035670

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:
- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes:

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate's response may differ from the final mark scheme.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$$(x^2 + bx + c) = (x + p)(x + q), \text{ where } |pq| = |c|, \text{ leading to x = }$$

$$(ax^2 + bx + c) = (mx + p)(nx + q), \text{ where } |pq| = |c| \text{ and } |mn| = |a|, \text{ leading to x = }$$

2. Formula

Attempt to use correct formula (with values for *a*, *b* and *c*).

3. Completing the square

$$\text{Solving } x^2 + bx + c = 0: \quad \left(x \pm \frac{b}{2}\right)^2 \pm q \pm c, \quad q \neq 0, \quad \text{leading to x =...}$$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required.

Question Number	Scheme	Marks
1	$$\begin{array}{r} 3x^2-2x+7 \\ x^2(+0x)-4\overline{)3x^4-2x^3-5x^2+(0x)-4} \\ \underline{3x^4+0x^3-12x^2} \\ -2x^3+7x^2+0x \\ \underline{-2x^3+0x^2+8x} \\ 7x^2-8x-4 \\ \underline{7x^2+0x-28} \\ -8x+24 \end{array}$$	
By Division		
	$a=3$	**B1**
	Long division as far as $$\begin{array}{r} 3x^2-2x....... \\ x^2(+0x)-4\overline{)3x^4-2x^3-5x^2+(0x)-4} \\ \underline{3x^4+0x^3-12x^2} \\ -2x^3+............... \\ \underline{-2x^3+................} \end{array}$$	**M1**
	Two of $b=-2\quad c=7\quad d=-8\quad e=24$	**A1**
	All four of $b=-2\quad c=7\quad d=-8\quad e=24$	**A1**
		(4 marks)

Notes for Question 1

B1 Stating $a=3$. This can also be scored by the coefficient of x^2 in $3x^2-2x+7$

M1 Using long division by x^2-4 and getting as far as the 'x' term. The coefficients need not be correct.
Award if you see the whole number part as $...x^2+...x$ following some working. You may also see this in a table/ grid.
Long division by $(x+2)$ will not score anything until $(x-2)$ has been divided into the new quotient. It is very unlikely to score full marks and the mark scheme can be applied.

A1 Achieving two of $b=-2$ $c=7$ $d=-8$ $e=24$.
The answers may be embedded within the division sum and can be implied.

A1 Achieving all of $b=-2$ $c=7$ $d=-8$ and $e=24$

Accept a correct long division for 3 out of the 4 marks scoring B1M1A1A0

Need to see a=…, b=…, or the values embedded in the rhs for all 4 marks

Question Number	Scheme	Marks
Alt 1 **By Multiplication**	* $3x^4-2x^3-5x^2-4\equiv(ax^2+bx+c)(x^2-4)+dx+e$	
	Compares the x^4 terms $a=3$	B1
	Compares coefficients to obtain a numerical value of one **furthe**r constant $-2=b,\quad -5=-4a+c\Rightarrow c=..,$	M1
	Two of $b=-2\quad c=7\quad d=-8\quad e=24$	A1
	All four of $b=-2\quad c=7\quad d=-8\quad e=24$	A1
		(4 marks)

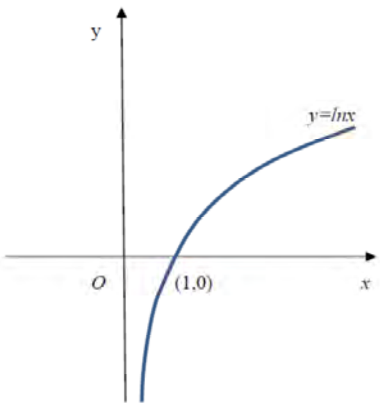
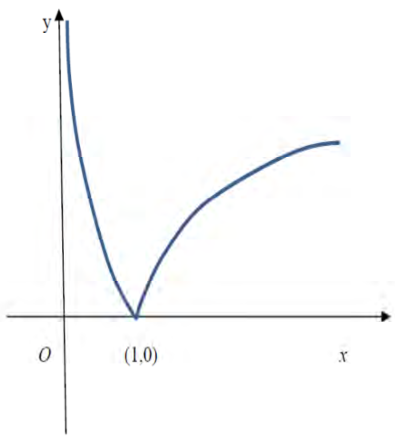
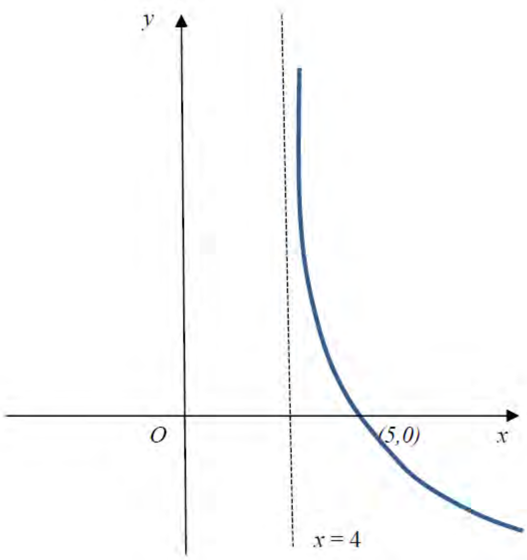
Notes for Question 2

B1 Stating $a=3$. This can also be scored for writing $3x^4=ax^4$

M1 Multiply out expression given to get *. Condone slips only on signs of either expression.
Then compare the coefficient of any term (other than x^4) to obtain a numerical value of one further constant. In reality this means a valid attempt at either b or c
The method may be implied by a correct additional constant to a.

A1 Achieving two of $b=-2$ $c=7$ $d=-8$ $e=24$

A1 Achieving all of $b=-2$ $c=7$ $d=-8$ and $e=24$

Question Number	Scheme	Marks
2(i)	 *ln* graph crossing *x* axis at (1,0) and asymptote at *x*=0	B1
2(ii)	 Shape including cusp Touches or crosses the *x* axis at (1,0) Asymptote given as *x*=0	B1ft B1ft B1
2(iii)	 Shape Crosses at (5, 0) Asymptote given as *x*=4	B1 B1ft B1 **(7 marks)**

Notes for Question 2

(i) B1 Correct shape, correct position and passing through (1, 0).

Graph must 'start' to the rhs of the y - axis in quadrant 4 with a gradient that is large. The gradient then decreases as it moves through (1, 0) into quadrant 1. There must not be an obvious maximum point but condone 'slips'. Condone the point marked (0,1) on the correct axis. See practice and qualification for clarification. **Do not with hold this mark if (x=0) the asymptote is incorrect or not given**.

(ii) B1ft Correct shape **including the cusp** wholly contained in quadrant 1.

The shape to the rhs of the cusp should have a decreasing gradient and must not have an obvious maximum.. The shape to the lhs of the cusp should not bend backwards past (1,0)
Tolerate a 'linear' looking section here but not one with incorrect curvature (See examples sheet (ii) number 3. For further clarification see practice and qualification items.
Follow through on an incorrect sketch in part (i) as long as it was above and below the x axis.

B1ft The curve touches or crosses the x axis at (1, 0). Allow for the curve passing through a point marked '1' on the x axis. Condone the point marked on the correct axis as (0, 1)
Follow through on an incorrect intersection in part (i).

B1 Award for the asymptote to the curve given/ marked as x = 0. Do not allow for it given/ marked as 'the y axis'. There must be a graph for this mark to be awarded, and there must be an asymptote on the graph at x = 0. Accept if x=0 is drawn separately to the y axis.

(iii)

B1 Correct shape.

The gradient should always be negative and becoming less steep. It must be approximately infinite at the *lh* end and not have an obvious minimum. The lh end must not bend 'forwards' to make a C shape.
The position is not important for this mark. See practice and qualification for clarification.

B1ft The graph crosses (or touches) the x axis at (5, 0). Allow for the curve passing through a point marked '5' on the x axis. Condone the point marked on the correct axis as (0, 5)
Follow through on an incorrect intersection in part (i). Allow for $\left((i)+4,0\right)$

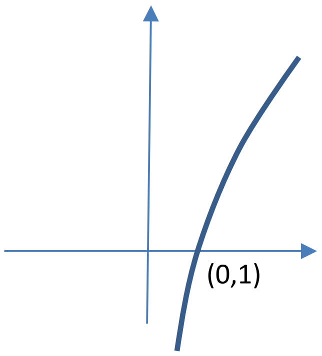
B1 The asymptote is given/ marked as x = 4. There must be a graph for this to be awarded and there must be an asymptote on the graph (in the correct place to the rhs of the y axis).

If the graphs are not labelled as (i), (ii) and (iii) mark them in the order that they are given.

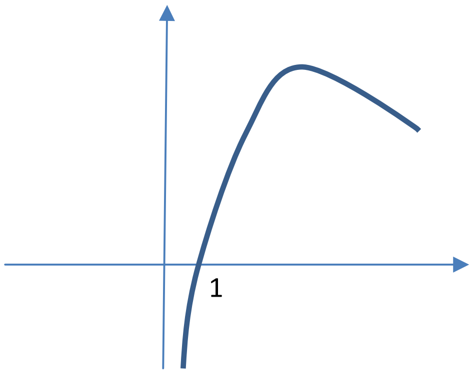
Examples of graphs in number 2

Part (i)

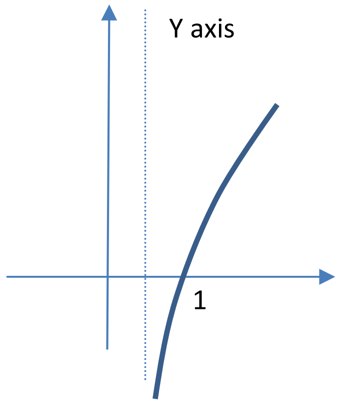
Condoned



Not condoned

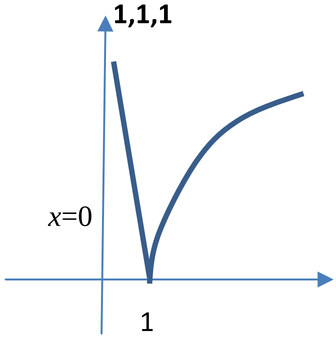


Condone

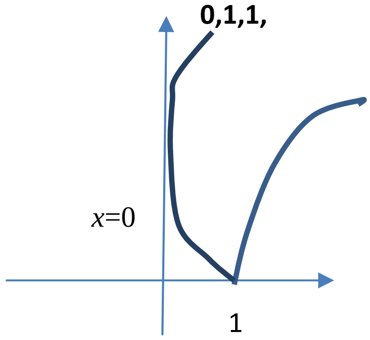


Part (ii)

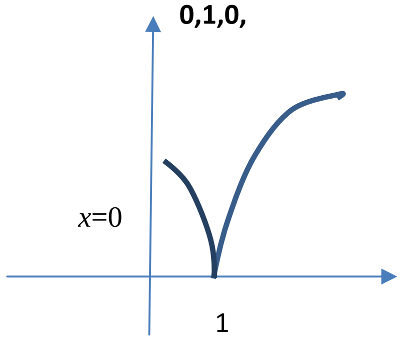
1,1,1



0,1,1,

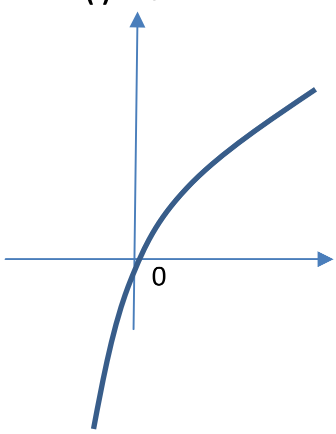


0,1,0,

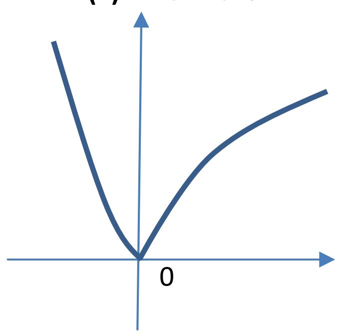


Example of follow through in part (ii) and (iii)

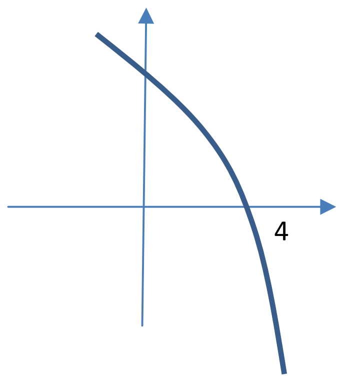
(i) B0



(ii) B1ftB1ftB0



(iii) B0B1ftB0



Question Number	Scheme	Marks
3(a)	$2\cos x\cos 50 - 2\sin x\sin 50 = \sin x\cos 40 + \cos x\sin 40$	M1
	$\sin x(\cos 40 + 2\sin 50) = \cos x(2\cos 50 - \sin 40)$	
	$\div\cos x \Rightarrow \tan x(\cos 40 + 2\sin 50) = 2\cos 50 - \sin 40$	M1
	$\tan x = \dfrac{2\cos 50 - \sin 40}{\cos 40 + 2\sin 50}$, (or numerical answer awrt 0.28)	A1
	States or uses cos50 = sin40 and cos40 =sin50 and so $\tan x^{\circ} = \frac{1}{3}\tan 40^{\circ}$ * cao	A1 *
		(4)
(b)	Deduces $\tan 2\theta = \frac{1}{3}\tan 40$	M1
	$2\theta =$ 15.6 so $\theta =$ awrt 7.8(1) One answer	A1
	Also $2\theta =$ 195.6, 375.6 , 555.6 $\Rightarrow \theta = ..$	M1
	$\theta =$ awrt 7.8 , 97.8, 187.8, 277.8 All 4 answers	A1
		(4)
		[8 marks]
Alt 1 3(a)	$2\cos x\cos 50 - 2\sin x\sin 50 = \sin x\cos 40 + \cos x\sin 40$	M1
	$2\cos x\sin 40 - 2\sin x\cos 40 = \sin x\cos 40 + \cos x\sin 40$	
	$\div\cos x \Rightarrow 2\sin 40 - 2\tan x\cos 40 = \tan x\cos 40 + \sin 40$	M1
	$\tan x = \dfrac{\sin 40}{3\cos 40}$(or numerical answer awrt 0.28), $\Rightarrow \tan x = \dfrac{1}{3}\tan 40$	A1,A1
Alt 2 3(a)	$2\cos(x+50) = \sin(x+40) \Rightarrow 2\sin(40-x) = \sin(x+40)$	
	$2\cos x\sin 40 - 2\sin x\cos 40 = \sin x\cos 40 + \cos x\sin 40$	M1
	$\div\cos x \Rightarrow 2\sin 40 - 2\tan x\cos 40 = \tan x\cos 40 + \sin 40$	M1
	$\tan x = \dfrac{\sin 40}{3\cos 40}$(or numerical answer awrt 0.28), $\Rightarrow \tan x = \dfrac{1}{3}\tan 40$	A1,A1

Notes for Question 3
(a) M1 Expand both expressions using $\cos(x+50)=\cos x\cos 50-\sin x\sin 50$ and $\sin(x+40)=\sin x\cos 40+\cos x\sin 40$. Condone a missing bracket on the lhs. The terms of the expansions must be correct as these are given identities. You may condone a sign error on one of the expressions. Allow if written separately and not in a connected equation. M1 Divide by $\cos x$ to reach an equation in $\tan x$. Below is an example of M1M1 with incorrect sign on left hand side $2\cos x\cos 50+2\sin x\sin 50=\sin x\cos 40+\cos x\sin 40$ $\Rightarrow 2\cos 50+2\tan x\sin 50=\tan x\cos 40+\sin 40$ This is independent of the first mark. A1 $\tan x=\dfrac{2\cos 50-\sin 40}{\cos 40+2\sin 50}$ Accept for this mark $\tan x=$ awrt 0.28... as long as M1M1 has been achieved. A1* States or uses $\cos 50=\sin 40$ and $\cos 40=\sin 50$ leading to showing $\tan x=\dfrac{2\cos 50-\sin 40}{\cos 40+2\sin 50}=\dfrac{\sin 40}{3\cos 40}=\dfrac{1}{3}\tan 40$ This is a given answer and all steps above must be shown. The line above is acceptable. Do not allow from $\tan x=$ awrt 0.28... (b) M1 For linking part (a) with (b). Award for writing $\tan 2\theta=\frac{1}{3}\tan 40$ A1 Solves to find one solution of θ which is usually (awrt) 7.8 M1 Uses the correct method to find at least another value of θ. It must be a full method but can be implied by any correct answer. Accept $\theta=\dfrac{180+their\alpha}{2},(or)\dfrac{360+their\alpha}{2},(or)\dfrac{540+their\alpha}{2}$ A1 Obtains all four answers awrt 1dp. $\theta=7.8, 97.8, 187.8, 277.8$. Ignore any extra solutions outside the range. Withhold this mark for extras inside the range. Condone a different variable. Accept x= 7.8, 97.8, 187.8, 277.8 Answers fully given in radians, loses the first A mark. Acceptable answers in rads are awrt 0.136, 1.71, 3.28, 4.85 Mixed units can only score the first M 1

Question Number	Scheme	Marks
4(a)	$\mathrm{f}'(x) = 50x^2\mathrm{e}^{2x} + 50x\mathrm{e}^{2x}$ oe.	M1A1
	Puts $\mathrm{f}'(x) = 0$ to give x = -1 and x = 0 or one coordinate	dM1A1
	Obtains $(0,\text{-}16)$ and $(\text{-}1,\ 25\mathrm{e}^{-2}\text{-}16)$ CSO	A1
		(5)
(b)	Puts $25x^2\mathrm{e}^{2x} - 16 = 0 \Rightarrow x^2 = \frac{16}{25}\mathrm{e}^{-2x} \Rightarrow x = \pm\frac{4}{5}\mathrm{e}^{-x}$	B1*
		(1)
(c)	Subs $x_0 = 0.5$ into $x = \frac{4}{5}\mathrm{e}^{-x} \Rightarrow x_1 = $ awrt 0.485	M1A1
	$\Rightarrow x_2 = $ awrt 0.492, $x_3 = $ awrt 0.489	A1
		(3)
(d)	$\alpha = 0.49$	B1
	$\mathrm{f}(0.485) = -0.487,\ \mathrm{f}(0.495) = (+)0.485$, sign change and deduction	B1
		(2)
		(11 marks)

Notes for Question 4

No marks can be scored in part (a) unless you see differentiation as required by the question.

(a)

M1 Uses $vu' + uv'$. If the rule is quoted it must be correct.
It can be implied by their $u = ..,v = ...,u' = ...,v' = ...$ followed by their $vu' + uv'$
If the rule is not quoted nor implied only accept answers of the form $Ax^2\mathrm{e}^{2x} + Bx\mathrm{e}^{2x}$

A1 $\mathrm{f}'(x) = 50x^2\mathrm{e}^{2x} + 50x\mathrm{e}^{2x}$.
Allow un simplified forms such as $\mathrm{f}'(x) = 25x^2 \times 2\mathrm{e}^{2x} + 50x \times \mathrm{e}^{2x}$

dM1 Sets $\mathrm{f}'(x) = 0$, factorises out/ or cancels the e^{2x} leading to at least one solution of x
This is dependent upon the first M1 being scored.

A1 Both x = -1 and x = 0 or one complete coordinate . Accept $(0,\text{-}16)$ and $(\text{-}1,\ 25\mathrm{e}^{-2}\text{-}16)$ or $(-1, awrt -12.6)$

A1 CSO. Obtains both solutions from differentiation. Coordinates can be given in any way.
x= - 1, 0 $\quad y = \frac{25}{\mathrm{e}^2} - 16, -16$ or linked together by coordinate pairs $(0,\text{-}16)$ and $(\text{-}1,\ 25\mathrm{e}^{-2}\text{-}16)$ but the ‘pairs’ must be correct and exact.

Notes for Question 4 Continued
(b) B1 This is a show that question and all elements must be seen Candidates must 1) State that f(x)=0 or writes $25x^2e^{2x}-16=0$ or $25x^2e^{2x}=16$ 2) Show at least one intermediate (correct) line with either x^2 or x the subject. Eg $x^2=\frac{16}{25}e^{-2x}$, $x=\sqrt{\frac{16}{25}e^{-2x}}$ oe or square rooting $25x^2e^{2x}=16\Rightarrow 5xe^x=\pm4$ or factorising by DOTS to give $(5xe^x+4)(5xe^x-4)=0$ 3) Show the given answer $x=\pm\frac{4}{5}e^{-x}$. Condone the minus sign just appearing on the final line. A 'reverse' proof is acceptable as long as there is a statement that f(x)=0 (c) M1 Substitutes $x_0=0.5$ into $x=\frac{4}{5}e^{-x}\Rightarrow x_1=....$ This can be implied by $x_1=\frac{4}{5}e^{-0.5}$, or awrt 0.49 A1 $x_1=$ awrt 0.485 3dp. Mark as the first value given. Don't be concerned by the subscript. A1 $x_2=$ awrt 0.492, $x_3=$ awrt 0.489 3dp. Mark as the second and third values given. (d) B1 States $\alpha=0.49$ B1 Justifies **by** **either** calculating correctly $f(0.485)$ and $f(0.495)$ to awrt 1sf or 1dp, $f(0.485)=-0.5, f(0.495)=(+)0.5$ rounded $f(0.485)=-0.4, f(0.495)=(+)0.4$ truncated giving a reason – accept change of sign, >0 <0 or $f(0.485)\times f(0.495)<0$ and giving a minimal conclusion. Eg. Accept hence root or $\alpha=0.49$ A smaller interval containing the root may be used, eg f (0.49) and f (0.495). Root = 0.49007 **or by** stating that the iteration is oscillating **or by** calculating by continued iteration to at least the value of x_4= awrt 0.491 and stating (or seeing each value round to) 0.49

Question Number	Scheme	Marks
5(a)	$\frac{dx}{dy} = 2\times 3\sec 3y \sec 3y \tan 3y = \left(6\sec^2 3y \tan 3y\right)$ $\left(\text{oe } \frac{6\sin 3y}{\cos^3 3y}\right)$	M1A1 **(2)**
(b)	Uses $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$ to obtain $\frac{dy}{dx} = \frac{1}{6\sec^2 3y \tan 3y}$	M1
	$\tan^2 3y = \sec^2 3y - 1 = x - 1$	B1
	Uses $\sec^2 3y = x$ and $\tan^2 3y = \sec^2 3y - 1 = x - 1$ to get $\frac{dy}{dx}$ or $\frac{dx}{dy}$ in just x.	M1
	$\Rightarrow \frac{dy}{dx} = \frac{1}{6x(x-1)^{\frac{1}{2}}}$ CSO	A1* **(4)**
(c)	$\frac{d^2y}{dx^2} = \frac{0-[6(x-1)^{\frac{1}{2}} + 3x(x-1)^{-\frac{1}{2}}]}{36x^2(x-1)}$	M1A1
	$\frac{d^2y}{dx^2} = \frac{6-9x}{36x^2(x-1)^{\frac{3}{2}}} = \frac{2-3x}{12x^2(x-1)^{\frac{3}{2}}}$	dM1A1
		(4)
		(10 marks)
Alt 1 to 5(a)	$x = (\cos 3y)^{-2} \Rightarrow \frac{dx}{dy} = -2(\cos 3y)^{-3} \times -3\sin 3y$	M1A1
Alt 2 to 5 (a)	$x = \sec 3y \times \sec 3y \Rightarrow \frac{dx}{dy} = \sec 3y \times 3\sec 3y \tan 3y + \sec 3y \times 3\sec 3y \tan 3y$	M1A1
Alt 1 To 5 (c)	$\frac{d^2y}{dx^2} = \frac{1}{6}[x^{-1}(-\frac{1}{2})(x-1)^{-\frac{3}{2}} + (-1)x^{-2}(x-1)^{-\frac{1}{2}}]$	M1A1
	$= \frac{1}{6}x^{-2}(x-1)^{-\frac{3}{2}}[x(-\frac{1}{2}) + (-1)(x-1)]$	dM1
	$= \frac{1}{12}x^{-2}(x-1)^{-\frac{3}{2}}[2-3x]$ oe	A1
		(4)

Notes for Question 5

(a)

M1 Uses the chain rule to get $A\sec 3y \sec 3y \tan 3y = \left(A\sec^2 3y \tan 3y\right)$.

There is no need to get the lhs of the expression. Alternatively could use the chain rule on $(\cos 3y)^{-2} \Rightarrow A(\cos 3y)^{-3}\sin 3y$

or the quotient rule on $\dfrac{1}{(\cos 3y)^2} \Rightarrow \dfrac{\pm A\cos 3y \sin 3y}{(\cos 3y)^4}$

A1 $\dfrac{dx}{dy} = 2\times 3\sec 3y \sec 3y \tan 3y$ or equivalent. There is no need to simplify the rhs but both sides must be correct.

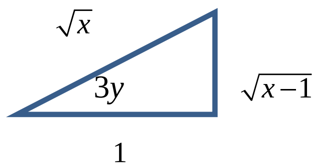
(b)

M1 Uses $\dfrac{dy}{dx} = \dfrac{1}{\frac{dx}{dy}}$ to get an expression for $\dfrac{dy}{dx}$. Follow through on their $\dfrac{dx}{dy}$

Allow slips on the coefficient but not trig expression.

B1 Writes $\tan^2 3y = \sec^2 3y - 1$ or an equivalent such as $\tan 3y = \sqrt{\sec^2 3y - 1}$ and uses $x = \sec^2 3y$ to obtain either $\tan^2 3y = x - 1$ or $\tan 3y = (x-1)^{\frac{1}{2}}$

All elements **must be present.**

Accept 

$\cos 3y = \dfrac{1}{\sqrt{x}} \Rightarrow \tan 3y = \sqrt{x-1}$

If the differential was in terms of $\sin 3y, \cos 3y$ it is awarded for $\sin 3y = \dfrac{\sqrt{x-1}}{\sqrt{x}}$

M1 Uses $\sec^2 3y = x$ and $\tan^2 3y = \sec^2 3y - 1 = x - 1$ or equivalent to get $\dfrac{dy}{dx}$ in just x. Allow slips on the signs in $\tan^2 3y = \sec^2 3y - 1$.

It may be implied- see below

A1* CSO. This is a given solution and you must be convinced that all steps are shown.

Note that the two method marks may occur the other way around

Eg. $\dfrac{dx}{dy} = 6\sec^2 3y \tan 3y = 6x(x-1)^{\frac{1}{2}} \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{6x(x-1)^{\frac{1}{2}}}$

Scores the 2nd method

Scores the 1st method

The above solution will score M1, B0, M1, A0

Notes for Question 5 Continued

Example 1- Scores 0 marks in part (b)

$$\frac{\mathrm{d}x}{\mathrm{d}y}=6\sec^2 3y\tan 3y \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{6\sec^2 3x\tan 3x}=\frac{1}{6\sec^2 3x\sqrt{\sec^2 3x-1}}=\frac{1}{6x(x-1)^{\frac{1}{2}}}$$

Example 2- Scores M1B1M1A0

$$\frac{\mathrm{d}x}{\mathrm{d}y}=2\sec^2 3y\tan 3y \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2\sec^2 3y\tan 3y}=\frac{1}{2\sec^2 3y\sqrt{\sec^2 3y-1}}=\frac{1}{2x(x-1)^{\frac{1}{2}}}$$

(c) **Using Quotient and Product Rules**

M1 Uses the quotient rule $\frac{vu'-uv'}{v^2}$ with $u=1$ and $v=6x(x-1)^{\frac{1}{2}}$ **and** achieving

$u'=0$ and $v'=A(x-1)^{\frac{1}{2}}+Bx(x-1)^{-\frac{1}{2}}$.

If the formulae are quoted, **both** must be correct. If they are not quoted nor implied by their working allow expressions of the form

$$\cancel{\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)}=\frac{0-[A(x-1)^{\frac{1}{2}}+Bx(x-1)^{-\frac{1}{2}}]}{\left(6x(x-1)^{\frac{1}{2}}\right)^2} \quad \text{or} \quad \cancel{\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)}=\frac{0-A(x-1)^{\frac{1}{2}}\pm Bx(x-1)^{-\frac{1}{2}}}{Cx^2(x-1)}$$

A1 Correct un simplified expression $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{0-[6(x-1)^{\frac{1}{2}}+3x(x-1)^{-\frac{1}{2}}]}{36x^2(x-1)}$ oe

dM1 Multiply numerator and denominator by $(x-1)^{\frac{1}{2}}$ producing a linear numerator which is then simplified by collecting like terms.

Alternatively take out a common factor of $(x-1)^{-\frac{1}{2}}$ from the numerator and collect like terms from the linear expression

This is dependent upon the 1st M1 being scored.

A1 Correct simplified expression $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{2-3x}{12x^2(x-1)^{\frac{3}{2}}}$ oe

Notes for Question 5 Continued

(c) Using Product and Chain Rules

M1 Writes $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{6x(x-1)^{\frac{1}{2}}} = Ax^{-1}(x-1)^{-\frac{1}{2}}$ and uses the product rule with u or $v = Ax^{-1}$ and v or $u = (x-1)^{-\frac{1}{2}}$. If any rule is quoted it must be correct.

If the rules are not quoted nor implied then award if you see an expression of the form $(x-1)^{-\frac{3}{2}} \times Bx^{-1} \pm C(x-1)^{-\frac{1}{2}} \times x^{-2}$

A1 $\cancel{\frac{\mathrm{d}^2y}{\mathrm{d}x^2}} = \frac{1}{6}[x^{-1}(-\frac{1}{2})(x-1)^{-\frac{3}{2}} + (-1)x^{-2}(x-1)^{-\frac{1}{2}}]$

dM1 Factorises out / uses a common denominator of $x^{-2}(x-1)^{-\frac{3}{2}}$ producing a linear factor/numerator which must be simplified by collecting like terms. Need a single fraction.

A1 Correct simplified expression $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} == \frac{1}{12}x^{-2}(x-1)^{-\frac{3}{2}}[2-3x]$ *oe*

(c) **Using Quotient and Chain rules Rules**

M1 Uses the quotient rule $\frac{vu'-uv'}{v^2}$ with $u = (x-1)^{-\frac{1}{2}}$ and $v = 6x$ **and** achieving $u' = A(x-1)^{-\frac{3}{2}}$ and $v' = B$.

If the formulae is quoted, it must be correct. If it is not quoted nor implied by their working allow an expression of the form

$$\cancel{\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)} = \frac{Cx(x-1)^{-\frac{3}{2}} - D(x-1)^{-\frac{1}{2}}}{Ex^2}$$

A1 Correct un simplified expression $\cancel{\frac{\mathrm{d}^2y}{\mathrm{d}x^2}} = \frac{6x \times -\frac{1}{2}(x-1)^{-\frac{3}{2}} - (x-1)^{-\frac{1}{2}} \times 6}{(6x)^2}$

dM1 Multiply numerator and denominator by $(x-1)^{\frac{3}{2}}$ producing a linear numerator which is then simplified by collecting like terms.

Alternatively take out a common factor of $(x-1)^{-\frac{3}{2}}$ from the numerator and collect like terms from the linear expression

This is dependent upon the 1st M1 being scored.

A1 Correct simplified expression $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{2-3x}{12x^2(x-1)^{\frac{3}{2}}}$ oe $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{(2-3x)x^{-2}(x-1)^{-\frac{3}{2}}}{12}$

Notes for Question 5 Continued	
(c)	**Using just the chain rule**
M1	Writes $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{6x(x-1)^{\frac{1}{2}}}=\frac{1}{(36x^3-36x^2)^{\frac{1}{2}}}=(36x^3-36x^2)^{-\frac{1}{2}}$ and proceeds by the chain rule to $A(36x^3-36x^2)^{-\frac{3}{2}}(Bx^2-Cx)$.
M1	Would automatically follow under this method if the first M has been scored

Question Number	Scheme	Marks
6(a)	$\ln(4-2x)(9-3x), = \ln(x+1)^2$	M1, M1
	So $36-30x+6x^2 = x^2+2x+1$ and $5x^2-32x+35=0$	A1
	Solve $5x^2-32x+35=0$ to give $x=\dfrac{7}{5}$ oe (Ignore the solution $x=5$)	M1A1
		(5)
(b)	Take $\log_e$'s to give $\ln 2^x + \ln e^{3x+1} = \ln 10$	M1
	$x\ln 2 + (3x+1)\ln e = \ln 10$	M1
	$x(\ln 2 + 3\ln e) = \ln 10 - \ln e \Rightarrow x = ..$	dM1
	and uses lne = 1	M1
	$x = \dfrac{-1+\ln 10}{3+\ln 2}$	A1
		(5)
	Note that the 4th M mark may occur on line 2	**(10 marks)**

Notes for Question 6

(a)

M1 Uses addition law on lhs of equation. Accept slips on the signs. If one of the terms is taken over to the rhs it would be for the subtraction law.

M1 Uses power rule for logs write the $2\ln(x+1)$ term as $\ln(x+1)^2$. Condone invisible brackets

A1 Undoes the logs to obtain the 3TQ =0. $5x^2-32x+35=0$. Accept equivalences. The equals zero may be implied by a subsequent solution of the equation.

M1 Solves a quadratic by any allowable method.
The quadratic cannot be a version of $(4-2x)(9-3x)=0$ however.

A1 Deduces x = 1.4 or equivalent. Accept both x=1.4 and x=5. Candidates do not have to eliminate $x=5$.
You may ignore any other solution as long as it is not in the range $-1 < x < 2$. Extra solutions in the range scores A0.

Notes for Question 6 Continued
(b) M1 Takes logs of both sides **and** splits LHS using addition law. If one of the terms is taken to the other side it can be awarded for taking logs of both sides **and** using the subtraction law. M1 Taking both powers down using power rule. It is not wholly dependent upon the first M1 but logs of both sides must have been taken. Below is an example of M0M1 $\ln 2^x \times \ln e^{3x+1} = \ln 10 \Rightarrow x \ln 2 \times (3x+1)\ln e = \ln 10$ dM1 This is dependent upon both previous two M's being scored. It can be awarded for a full method to solve their linear equation in *x*. The terms in *x* must be collected on one side of the equation and factorised. You may condone slips in signs for this mark but the process must be correct and leading to *x* = … M1 Uses ln e = 1. This could appear in line 2, but it must be part of their equation and not just a statement. Another example where it could be awarded is $e^{3x+1} = \frac{10}{2^x} \Rightarrow 3x+1 = ...$ A1 Obtains answer $x = \frac{-1+\ln 10}{3+\ln 2} = \left(\frac{\ln 10 - 1}{3+\ln 2}\right) = \left(\frac{\log_e 10 - 1}{3+\log_e 2}\right) oe$. **DO NOT ISW HERE** Note 1: If the candidate takes $\log_{10}$'s of both sides can score M1M1dM1M0A0 for 3 out of 5. Answer = $x = \frac{-\log e + \log 10}{3\log e + \log 2} = \left(\frac{-\log e + 1}{3\log e + \log 2}\right)$ Note 2: If the candidate writes $x = \frac{-1+\log 10}{3+\log 2}$ without reference to natural logs then award M4 but with hold the last A1 mark, scoring 4 out of 5.

Question Number	Scheme	Marks
Alt 1 to 6(b)	Writes lhs in e's $2^x e^{3x+1} = 10 \Rightarrow e^{x\ln 2}e^{3x+1} = 10$	1st M1
	$\Rightarrow e^{x\ln 2+3x+1} = 10, \quad x\ln 2 + 3x + 1 = \ln 10$	2nd M1, 4th M1
	$x(\ln 2+3) = \ln 10 - 1 \Rightarrow x = ..$	dM1
	$x = \dfrac{-1+\ln 10}{3+\ln 2}$	A1 **(5)**

Notes for Question 6 Alt 1

M1 Writes the lhs of the expression in e's. Seeing $2^x = e^{x\ln 2}$ in their equation is sufficient

M1 Uses the addition law on the lhs to produce a single exponential

dM1 Takes ln's of both sides to produce and attempt to solve a linear equation in *x*
You may condone slips in signs for this mark but the process must be correct leading to *x*= ..

M1 Uses ln e = 1. This could appear in line 2

Question Number	Scheme	Marks
7(a)	$0 \leqslant f(x) \leqslant 10$	B1
		(1)
(b)	ff(0) = f(5), = 3	B1,B1
		(2)
(c)	$y = \dfrac{4+3x}{5-x} \Rightarrow y(5-x) = 4+3x$	
	$\Rightarrow 5y-4 = xy+3x$	M1
	$\Rightarrow 5y-4 = x(y+3) \Rightarrow x = \dfrac{5y-4}{y+3}$	dM1
	$g^{-1}(x) = \dfrac{5x-4}{3+x}$	A1
		(3)
(d)	$gf(x) = 16 \Rightarrow f(x) = g^{-1}(16) = 4$ oe	M1A1
	$f(x) = 4 \Rightarrow x = 6$	B1
	$f(x) = 4 \Rightarrow 5-2.5x = 4 \Rightarrow x = 0.4$ oe	M1A1
		(5)
		(11 marks)
Alt 1 to 7(d)	$gf(x) = 16 \Rightarrow \dfrac{4+3(ax+b)}{5-(ax+b)} = 16$	M1
	$ax+b = x-2$ *or* $5-2.5x$	A1
	$\Rightarrow x = 6$	B1
	$\dfrac{4+3(5-2.5x)}{5-(5-2.5x)} = 16 \Rightarrow x = \ldots$	M1
	$\Rightarrow x = 0.4$ oe	A1 **(5)**

Notes for Question 7

(a)

B1 Correct range. Allow $0\leqslant f(x)\leqslant 10$, $0\leqslant f\leqslant 10$, $0\leqslant y\leqslant 10$, $0\leqslant \text{range}\leqslant 10$, $[0,10]$

Allow $f(x)\geqslant 0$ and $f(x)\leqslant 10$ but not $f(x)\geqslant 0$ or $f(x)\leqslant 10$

Do Not Allow $0\leqslant x\leqslant 10$. The inequality must include BOTH ends

(b)

B1 For correct one application of the function at x=0

Possible ways to score this mark are f(0)=5, f(5) $0\to 5\to ...$

B1: 3 ('3' can score both marks as long as no incorrect working is seen.)

(c)

M1 For an attempt to make x or a replaced y the subject of the formula. This can be scored for putting y = g(x), multiplying across, expanding and collecting x terms on one side of the equation. Condone slips on the signs

dM1 Take out a common factor of x (or a replaced y) and divide, to make x subject of formula. Only allow **one sign error** for this mark

A1 Correct answer. No need to state the domain. Allow $g^{-1}(x)=\dfrac{5x-4}{3+x}$ $y=\dfrac{5x-4}{3+x}$

Accept alternatives such as $y=\dfrac{4-5x}{-3-x}$ and $y=\dfrac{5-\frac{4}{x}}{1+\frac{3}{x}}$

(d)

M1 Stating or implying that $f(x)=g^{-1}(16)$. For example accept $\dfrac{4+3f(x)}{5-f(x)}=16\Rightarrow f(x)=..$

A1 Stating $f(x)=4$ or implying that solutions are where $f(x)=4$

B1 x = 6 and may be given if there is no working

M1 Full method to obtain other value from line $y=5-2.5x$

$5-2.5x=4\Rightarrow x=...$.

Alternatively this could be done by similar triangles. Look for $\dfrac{2}{5}=\dfrac{2-x}{4}\,(oe)\Rightarrow x=..$

A1 0.4 or 2/5

Alt 1 to (d)

M1 Writes $gf(x)=16$ with a linear $f(x)$. The order of gf(x) must be correct

Condone invisible brackets. Even accept if there is a modulus sign.

A1 Uses $f(x)=x-2$ or $f(x)=5-2.5x$ in the equation $gf(x)=16$

B1 x = 6 and may be given if there is no working

M1 Attempt at solving $\dfrac{4+3(5-2.5x)}{5-(5-2.5x)}=16\Rightarrow x=....$. The bracketing must be correct and there must be no more than one error in their calculation

A1 $x=0.4,\ \dfrac{2}{5}$ or equivalent

Question Number	Scheme	Marks
8(a)	$R=\sqrt{\left(7^2+24^2\right)}=25$	B1
	$\tan\alpha=\frac{24}{7}, \quad \Rightarrow \quad \alpha=\text{awrt}\,73.74°$	M1A1
		(3)
(b)	maximum value of 24sin$x$ + 7cos$x$ = 25 so $V_{\min}=\frac{21}{25}=(0.84)$	M1A1
		(2)
(c)	Distance $AB=\frac{7}{\sin\theta}$, with $\theta=\alpha$	M1, B1
	So distance = 7.29m $=\frac{175}{24}$m	A1
		(3)
(d)	$R\cos(\theta-\alpha)=\frac{21}{1.68}\Rightarrow\cos(\theta-\alpha)=0.5$	M1, A1
	$\theta-\alpha=60\Rightarrow\theta=..,\theta-\alpha=-60\Rightarrow\theta=..$	dM1, dM1
	$\theta=\text{awrt}\,133.7, 13.7$	A1, A1
		(6) **(14 marks)**

Notes for Question 8

(a)

B1 25. Accept 25.0 but not $\sqrt{625}$ or answers that are not exactly 25. Eg 25.0001

M1 For $\tan\alpha=\pm\frac{24}{7}, \tan\alpha=\pm\frac{7}{24}$.

If the value of R is used only accept $\sin\alpha=\pm\frac{24}{R}, \cos\alpha=\pm\frac{7}{R}$

A1 Accept answers which round to 73.74 – must be in degrees for this mark

(b)

M1 Calculates $V=\frac{21}{their\,'R'}$ NOT - R

A1 Obtains correct answer. $V=\frac{21}{25}$ Accept 0.84

Do not accept if you see incorrect working- ie from $\cos(\theta-\alpha)=-1$ or the minus just disappearing from a previous line.

Questions involving differentiation are acceptable. To score M1 the candidate would have to differentiate *V* by the quotient rule (or similar) , set *V*'=0 to find θ and then sub this back into V to find its value.

Notes for Question 8 Continued

(c)

M1 Uses the trig equation $\sin\theta = \dfrac{7}{AB}$ with a numerical θ to find $AB = ...$

B1 Uses $\theta =$ their value of α in a trig calculation involving sin. ($\sin\alpha = \dfrac{AB}{7}$ is condoned)

A1 Obtains answer $\dfrac{175}{24}$ or awrt 7.29

(d)

M1 Substitutes $V = 1.68$ and their answer to part (a) in $V = \dfrac{21}{24\sin\theta + 7\cos\theta}$ to get an equation of the form $R\cos(\theta \pm \alpha) = \dfrac{21}{1.68}$ or $1.68R\cos(\theta \pm \alpha) = 21$ or $\cos(\theta \pm \alpha) = \dfrac{21}{1.68R}$.

Follow through on their R and α

A1 Obtains $\cos(\theta \pm \alpha) = 0.5$ oe. Follow through on their α. It may be implied by later working.

dM1 Obtains one value of θ **in the range** $0 < \theta < 150$ from inverse cos +their α
It is dependent upon the first M being scored.

dM1 Obtains second angle of θ **in the range** $0 < \theta < 150$ from inverse cos +their α
It is dependent upon the first M being scored.

A1 one correct answer awrt $\theta = 133.7$ *or* 13.7 1dp

A1 both correct answers awrt $\theta = 133.7$ *and* 13.7 1dp.

Extra solutions in the range loses the last A1.
Answers in radians, lose the first time it occurs. Answers must be to 3dp
For your info $\alpha = 1.287, \theta_1 = 2.334, \theta_2 = 0.240$

Further copies of this publication are available from
Edexcel Publications, Adamsway, Mansfield, Notts, NG18 4FN

Telephone 01623 467467
Fax 01623 450481
Email publication.orders@edexcel.com

Order Code UA035670 Summer 2013

For more information on Edexcel qualifications, please visit our website
www.edexcel.com

Pearson Education Limited. Registered company number 872828
with its registered office at Edinburgh Gate, Harlow, Essex CM20 2JE

Llywodraeth Cynulliad Cymru
Welsh Assembly Government

Rewarding Learning