OCR Maths C3

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1	**(i)**	State $f(x) \le 10$	B1	**1** [Any equiv but must be or imply $\le$]
	(ii)	Attempt correct process for composition of functions	M1	[whether algebraic or numerical]
		Obtain 6 or correct expression for $ff(x)$	A1	
		Obtain -71	A1	**3**
2		<u>Either</u> Obtain $x = 0$	B1	[ignoring errors in working]
		Form linear equation with signs of $6x$ and x different	M1	[ignoring other sign errors]
		State $6x - 1 = -x + 1$	A1	[or correct equiv with or without brackets]
		Obtain $\frac{2}{7}$ and no other non-zero value	A1	**4** [or exact equiv]
	<u>Or</u>	Obtain $36x^2 - 12x + 1 = x^2 - 2x + 1$	B1	[or equiv]
		Attempt to solve quadratic equation	M1	[as far as factorisation or subn into formula]
		Obtain $\frac{2}{7}$ and no other non-zero value	A1	[or exact equiv]
		Obtain 0	B1	**(4)** [ignoring errors in working]
3	**(i)**	Attempt solution involving (natural) logarithm	M1	
		Obtain $-0.017t = \ln\frac{25}{180}$	A1	[or equiv]
		Obtain 116	A1	**3** [or greater accuracy rounding to 116]
	(ii)	Differentiate to obtain $k\,e^{-0.017t}$	M1	[any constant k different from 180; solution must involve differentiation]
		Obtain correct $-3.06e^{-0.017t}$	A1	[or unsimplified equiv; accept + or –]
		Obtain 1.2	A1	**3** [or greater accuracy; accept + or – answer]
4	**(a)**	State or imply $\int \pi y^2\, dx$	B1	
		Integrate to obtain $k \ln x$	M1	[any constant k, involving π or not; or equiv such as $k \ln 4x$]
		Obtain $4\pi \ln x$ or $4 \ln x$	A1	[or equiv]
		Obtain $4\pi \ln 5$	A1	**4** [or similarly simplified equiv]

	(b)	Attempt calculation involving attempts at y values	**M1**	[with each of 1, 4, 2 present at least once as coefficients]
		Attempt $\frac{1}{3}\times 1(y_0 + 4y_1 + 2y_2 + 4y_3 + y_4)$	**M1**	[with attempts at five y values]
		Obtain $\frac{1}{3}(\sqrt{2} + 4\sqrt{5} + 2\sqrt{10} + 4\sqrt{17} + \sqrt{26})$	**A1**	[or exact equiv or decimal equivs]
		Obtain 12.758	**A1**	**4** [or greater accuracy]
5	**(i)**	Obtain $R = \sqrt{13}$, or 3.6 or 3.61 or greater accuracy	**B1**	
		Attempt recognisable process for finding α	**M1**	[allow sine/cosine muddles]
		Obtain $\alpha = 33.7$	**A1**	**3** [or greater accuracy]
	(ii)	Attempt to find at least one value of $\theta + \alpha$	***M1**	
		Obtain value rounding to 76 or 104	**A1√**	[following their R]
		Subtract their α from at least one value	**M1**	[dependent on ***M**]
		Obtain one value rounding to 42 or 43, or to 70	**A1**	
		Obtain other value 42.4 or 70.2	**A1**	**5** [or greater accuracy; no other answers between 0 and 360; ignore answers outside 0 to 360]
6	**(a)**	Attempt use of product rule	***M1**	
		Obtain $\ln x + 1$	**A1**	[or unsimplified equiv]
		Equate attempt at first derivative to zero and obtain value involving e	**M1**	[dependent on ***M**]
		Obtain e^{-1}	**A1**	**4** [or exact equiv]
	(b)	Attempt use of quotient rule	**M1**	[or equiv using product rule or …]
		Obtain $\dfrac{(4x-c)4 - 4(4x+c)}{(4x-c)^2}$	**A1**	[or equiv]
		Show that first derivative cannot be zero	**A1**	**3** [**AG**; derivative must be correct]
7	**(i)**	State $2\cos^2 x - 1$	**B1**	**1**
	(ii)	Attempt to express left hand side in terms of $\cos x$	**M1**	[using expression of form $a\cos^2 x + b$]
		Identify $\dfrac{1}{\cos x}$ as $\sec x$	**M1**	[maybe implied]

		Confirm result	A1	**3** [**AG**; necessary detail required]
	(iii)	Use identity $\sec^2 x = 1 + \tan^2 x$ Attempt solution of quadratic equation in tan x Obtain $2\tan^2 x + 3\tan x - 9 = 0$ and hence tan x = –3, $\frac{3}{2}$ Obtain at least two of 0.983, 4.12, 1.89, 5.03 (or of 0.313π, 1.31π, 0.602π, 1.60π) Obtain all four solutions	B1 M1 A1 A1 A1	[or equiv] [allow answers with only 2 s.f.; allow greater accuracy; allow 0.983 + π, 1.89 + π allow degrees: 56, 236, 108, 288] **5** [now with at least 3 s.f.; must be radians; no other solutions in the range 0 - 2π; ignore solutions outside range 0 - 2π]
8	**(i)**	Attempt relevant calculations with 5.2 and 5.3 Obtain correct values Conclude appropriately	M1 A1 A1	x y_1 y_2 $y_1 - y_2$ 5.2 2.83 2.87 – 0.04 5.3 2.89 2.88 0.006 **3** [**AG**; comparing y values or noting sign change in difference in y values or equiv]
	(ii)	Equate expressions and attempt rearrangement to x = Obtain $x = \frac{5}{3}\ln(3x+8)$	M1 A1	**2** [**AG**; necessary detail required]
	(iii)	Obtain correct first iterate Carry out correct process to find at least two iterates in all Obtain 5.29	B1 M1 A1	**3** [must be exactly 2 decimal places; 5.2→5.2687→5.2832→5.2863→5.2869; 5.25→5.2793→5.2855→5.2868→5.2870; 5.3→5.2898→5.2877→5.2872→5.2871]
	(iv)	Obtain integral of form $k(3x+8)^{\frac{4}{3}}$ Obtain integral of form $k\,e^{\frac{1}{5}x}$	M1 M1	

		Obtain $\frac{1}{4}(3x+8)^{\frac{4}{3}} - 5e^{\frac{1}{5}x}$	A1	[or equiv]
		Apply limits 0 and their answer to **(iii)**	M1	[applied to difference of two integrals]
		Obtain 3.78	A1	**5** [or greater accuracy]
9	**(i)**	Indicate stretch and (at least one) translation	M1	[… in general terms]
		State translation by 7 units in negative x direction	A1	[or equiv; using correct terminology]
		State stretch in x direction with factor $1/m$	A1	[must follow the translation by 7; or equiv; using correct terminology]
		Indicate translation by 4 units in negative y direction	B1	**4** [or equiv; at any stage; the two translations may be combined]
	(ii)	Refer to each y value being image of unique x value	B1	[or equiv]
		Attempt correct process for finding inverse	M1	
		Obtain expression involving $(x+4)^2$ or $(y+4)^2$	M1	
		Obtain $\dfrac{(x+4)^2 - 7}{m}$	A1	**4** [or equiv]
	(iii)	Refer to fact that curves are reflections of each other in line $y = x$	B1	[or equiv]
		Attempt arrangement of either $f(x) = x$ or $f^{-1}(x) = x$	M1	
		Apply discriminant to resulting quadratic equati on	M1	
		Obtain $(m-2)(m-14) < 0$	A1	[or equiv]
		Obtain $2 < m < 14$	A1	**5**

1	Obtain integral of form $k \ln x$	**M1**		[any non-zero constant *k;* or equiv such as $k \ln 3x$]
	Obtain $3 \ln 8 - 3 \ln 2$	**A1**		[or exact equiv]
	Attempt use of at least one relevant log property	**M1**		[would be earned by initial $\ln x^3$]
	Obtain $3 \ln 4$ or $\ln 8^3 - \ln 2^3$ and hence ln 64	**A1**	**4**	[**AG**; with no errors]
2	Attempt use of identity linking $\sec^2 \theta$, $\tan^2 \theta$ and 1	**M1**		[to write eqn in terms of tan θ]
	Obtain $\tan^2 \theta - 4 \tan \theta + 3 = 0$	**A1**		[or correct unsimplified equiv]
	Attempt solution of quadratic eqn to find two values of tan θ	**M1**		[any 3 term quadratic eqn in tan θ]
	Obtain at least two correct answers	**A1**		[after correct solution of eqn]
	Obtain all four of 45, 225, 71.6, 251.6	**A1**	**5**	[allow greater accuracy or angles to nearest degree – and no other answers between 0 and 360]
3 (a)	Attempt use of product rule	**M1**		[involving … + …]
	Obtain $2x(x+1)^6$ …	**A1**		
	Obtain … + $6x^2(x+1)^5$	**A1**	**3**	[or equivs; ignore subsequent attempt at simplification]
(b)	Attempt use of quotient rule	**M1**		[or, with adjustment, product rule; allow *u* / *v* confusion]
	Obtain $\dfrac{(x^2-3)\,2x-(x^2+3)\,2x}{(x^2-3)^2}$	**A1**		[or equiv]
	Obtain –3	**A1**	**3**	[from correct derivative only]
4 (i)	State $y \le 2$	**B1**	**1**	[or equiv; allow <; allow any letter or none]
(ii)	Show correct process for composition of functions	**M1**		[numerical or algebraic]
	Obtain 0 and hence 2	**A1**	**2**	[and no other value]
(iii)	State a range of values with 2 as one end-point	**M1**		[continuous set, not just integers]
	State $0 < k \le 2$	**A1**	**2**	[with correct < and ≤ now]
5	Obtain integral of form $k(1-2x)^6$	**M1**		[any non-zero constant *k*]
	Obtain correct $-\frac{1}{12}(1-2x)^6$	**A1**		[or unsimplified equiv; allow + *c*]
	Use limits to obtain $\frac{1}{12}$	**A1**		[or exact (unsimplified) equiv]
	Obtain integral of form $k\,e^{2x-1}$	**M1**		[or equiv; any non-zero constant *k*]
	Obtain correct $\frac{1}{2}e^{2x-1} - x$	**A1**		[or equiv; allow + *c*]
	Use limits to obtain $-\frac{1}{2}e^{-1}$	**A1**		[or exact (unsimplified) equiv]
	Show correct process for finding required area	**M1**		[at any stage of solution; if process involves two definite integrals, second must be negative]
	Obtain $\frac{1}{12} + \frac{1}{2}e^{-1}$	**A1**	**8**	[or exact equiv; no + *c*]

6 (a)	Either: State proportion $\frac{440}{275}$	**B1**		
	Attempt calculation involving proportion	**M1**		[involving multn and X value]
	Obtain 704	**A1**	**3**	
	Or: Use formula of form $275\mathrm{e}^{kt}$ or $275a^t$	**M1**		[or equiv]
	Obtain $k = 0.047$ or $a = \sqrt[10]{1.6}$	**A1**		[or equiv]
	Obtain 704	**A1**	**(3)**	[allow ±0.5]
(b)(i)	Attempt correct process involving logarithm	**M1**		[or equiv including systematic trial and improvement attempt]
	Obtain $\ln\frac{20}{80} = -0.02t$	**A1**		[or equiv]
	Obtain 69	**A1**	**3**	[or greater accuracy; scheme for T&I: M1A2]
(ii)	Differentiate to obtain $k\,\mathrm{e}^{-0.02t}$	**M1**		[any constant k different from 80]
	Obtain $-1.6\,\mathrm{e}^{-0.02t}$ (or $1.6\,\mathrm{e}^{-0.02t}$)	**A1**		[or unsimplified equiv]
	Obtain 0.88	**A1**	**3**	[or greater accuracy; allow –0.88]

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7 (i)	Sketch curve showing (at least) translation in x direction	**M1**		[either positive or negative]
	Show correct sketch with one of 2 and 3π indicated	**A1**		
	… and with other one of 2 and 3π indicated	**A1**	**3**	
(ii)	Draw straight line through O with positive gradient	**B1**	**1**	[label and explanation not required]
(iii)	Attempt calculations using 1.8 and 1.9	**M1**		[allow here if degrees used]
	Obtain correct values and indicate change of sign	**A1**	**2**	[or equiv; x = 1.8: LHS = 1.93, diff = 0.13; x = 1.9: LHS = 1.35, diff = -0.55; radians needed now]
(iv)	Obtain correct first iterate 1.79 or 1.78	**B1**		[or greater accuracy]
	Attempt correct process to produce at least 3 iterates	**M1**		
	Obtain 1.82	**A1**		[answer required to exactly 2 d.p.; $2 \to 1.7859 \to 1.8280 \to 1.8200$; SR: answer 1.82 only - B2]
	Attempt rearrangement of $3\cos^{-1}(x-1) = x$ or of $x = 1 + \cos(\frac{1}{3}x)$	**M1**		[involving at least two steps]
	Obtain required formula or equation respectively	**A1**	**5**	

8	**(i)**	Differentiate to obtain $kx(5-x^2)^{-1}$	**M1**		[any non-zero constant]
		Obtain correct $-2x(5-x^2)^{-1}$	**A1**		[or equiv]
		Obtain –4 for value of derivative	**A1**		
		Attempt equation of straight line through (2, 0) with numerical value of gradient obtained from attempt at derivative	**M1**		[not for attempt at eqn of normal]
		Obtain $y = -4x + 8$	**A1**	**5**	[or equiv]
	(ii)	State or imply $h = \frac{1}{2}$	**B1**		
		Attempt calculation involving attempts at y values	**M1**		[addition with each of coefficients 1, 2, 4 occurring at least once]
		Obtain $k(\ln 5 + 4\ln 4.75 + 2\ln 4 + 4\ln 2.75 + \ln 1)$	**A1**		[or equiv perhaps with decimals; any constant k]
		Obtain 2.44	**A1**	**4**	[allow ±0.01]
	(iii)	Attempt difference of two areas	**M1**		[allow if area of their triangle < area A]
		Obtain 8 – 2.44 and hence 5.56	**A1**√	**2**	[following their tangent and area of A providing answer positive]
9	**(i)**	State $\sin 2\theta \cos\theta + \cos 2\theta \sin\theta$	**B1**		
		Use at least one of $\sin 2\theta = 2\sin\theta\cos\theta$ and $\cos 2\theta = 1 - 2\sin^2\theta$	**B1**		
		Attempt complete process to express in terms of $\sin\theta$	**M1**		[using correct identities]
		Obtain $3\sin\theta - 4\sin^3\theta$	**A1**	**4**	[**AG**; all correctly obtained]
	(ii)	State 3	**B1**		
		Obtain expression involving $\sin 10\alpha$	**M1**		[allow θ/α confusion]
		Obtain 9	**A1**	**3**	[and no other value]
	(iii)	Recognise cosec 2β as $\frac{1}{\sin 2\beta}$	**B1**		[allow θ/β confusion]
		Attempt to express equation in terms of $\sin 2\beta$ only	**M1**		[or equiv involving $\cos 2\beta$]
		Attempt to find non-zero value of $\sin 2\beta$	**M1**		[or of $\cos 2\beta$]
		Obtain at least $\sin 2\beta = \sqrt{\frac{5}{12}}$	**A1**		[or equiv, exact or approx]
		Attempt correct process to find two values of β	**M1**		[provided equation is $\sin 2\beta = k$; or equiv with $\cos 2\beta$]
		Obtain 20.1, 69.9	**A1**	**6**	[and no others between 0 and 90]

1		Differentiate to obtain $k(4x+1)^{-\frac{1}{2}}$	**M1**		any non-zero constant k
		Obtain $2(4x+1)^{-\frac{1}{2}}$	**A1**		or equiv, perhaps unsimplified
		Obtain $\frac{2}{3}$ for value of first derivative	**A1**		or unsimplified equiv
		Attempt equation of tangent through (2, 3)	**M1**		using numerical value of first derivative provided derivative is of form $k'(4x+1)^n$
		Obtain $y=\frac{2}{3}x+\frac{5}{3}$ or $2x - 3y + 5 = 0$	**A1**	**5**	or equiv involving 3 terms
2		Either: Attempt to square both sides	**M1**		producing 3 terms on each side
		Obtain $3x^2-14x+8=0$	**A1**		or inequality involving < or >
		Obtain correct values $\frac{2}{3}$ and 4	**A1**		
		Attempt valid method for solving inequality	**M1**		implied by correct answer or plausible incorrect answer
		Obtain $\frac{2}{3} < x < 4$	**A1**	**5**	or correctly expressed equiv; allow ≤ signs
		Or: Attempt solution of two linear equations or inequalities	**M1**		one eqn with signs of $2x$ and x the same, second eqn with signs different
		Obtain value $\frac{2}{3}$	**A1**		
		Obtain value 4	**B1**		
		Attempt valid method for solving inequality	**M1**		implied by correct answer or plausible incorrect answer
		Obtain $\frac{2}{3} < x < 4$	**A1**	**(5)**	or correctly expressed equiv; allow ≤ signs
3	**(i)**	Attempt evaluation of cubic expression at 2 and 3	**M1**		
		Obtain –11 and 31	**A1**		
		Conclude by noting change of sign	**A1**√	**3**	or equiv; following any calculated values provided negative then positive
	(ii)	Obtain correct first iterate	**B1**		using x_1 value such that $2 \le x_1 \le 3$
		Attempt correct process to obtain at least 3 iterates	**M1**		using any starting value now
		Obtain 2.34	**A1**	**3**	answer required to 2 d.p. exactly; 2→2.3811→2.3354→2.3410; 2.5→2.3208→2.3428→2.3401; 3→2.2572→2.3505→2.3392

4 (i)	State $\ln y = (x-1)\ln 5$	**B1**		whether following $\ln y = \ln 5^{x-1}$ or not; brackets needed
	Obtain $x = 1 + \frac{\ln y}{\ln 5}$	**B1**	**2**	**AG**; correct working needed; missing brackets maybe now implied
(ii)	Differentiate to obtain single term of form $\frac{k}{y}$	**M1**		any constant k
	Obtain $\frac{1}{y\ln 5}$	**A1**	**2**	or equiv involving y
(iii)	Substitute for y and attempt reciprocal	**M1**		or equiv method for finding derivative without using part **(ii)**
	Obtain $25\ln 5$	**A1**	**2**	or exact equiv
5 (i)	State $\sin 2\theta = 2\sin\theta\cos\theta$	**B1**	**1**	or equiv; any letter acceptable here (and in parts **(ii)** and **(iii)**)
(ii)	Attempt to find exact value of $\cos\alpha$	**M1**		using identity attempt or right-angled triangle
	Obtain $\frac{1}{4}\sqrt{15}$	**A1**		or exact equiv
	Substitute to confirm $\frac{1}{8}\sqrt{15}$	**A1**	**3**	**AG**
(iii)	State or imply $\sec\beta = \frac{1}{\cos\beta}$	**B1**		
	Use identity to produce equation involving $\sin\beta$	**M1**		
	Obtain $\sin\beta = 0.3$ and hence 17.5	**A1**	**3**	and no other values between 0 and 90; allow 17.4 or value rounding to 17.4 or 17.5
6 (i)	Either: Obtain f(–3) = –7	**B1**		maybe implied
	Show correct process for compn of functions	**M1**		
	Obtain –47	**A1**	**3**	
	Or: Show correct process for compn of functions	**M1**		using algebraic approach
	Obtain $2-(2-x^2)^2$	**A1**		or equiv
	Obtain –47	**A1**	**(3)**	
(ii)	Attempt correct process for finding inverse	**M1**		as far as $x = \ldots$ or equiv
	Obtain either one of $x = \pm\sqrt{2-y}$ or both	**A1**		or equiv perhaps involving x
	Obtain correct $-\sqrt{2-x}$	**A1**	**3**	or equiv; in terms of x now
(iii)	Draw graph showing attempt at reflection in $y = x$	**M1**		
	Draw (more or less) correct graph	**A1**		with end-point on x-axis and no minimum point in third quadrant
	Indicate coordinates 2 and $-\sqrt{2}$	**A1**	**3**	accept –1.4 in place of $-\sqrt{2}$
7 (a)	Obtain integral of form $k(4x-1)^{-1}$	**M1**		any non-zero constant k

	Obtain $-\frac{1}{2}(4x-1)^{-1}$	A1		or equiv; allow + c
	Substitute limits and attempt evaluation	M1		for any expression of form $k'(4x-1)^n$
	Obtain $\frac{2}{21}$	A1	4	or exact equiv
(b)	Integrate to obtain ln x	B1		
	Substitute limits to obtain ln 2a – ln a	B1		
	Subtract integral attempt from attempt at area of appropriate rectangle	M1		or equiv
	Obtain 1 – (ln 2a – ln a)	A1		or equiv
	Show at least one relevant logarithm property	M1		at any stage of solution
	Obtain 1 – ln 2 and hence $\ln(\frac{1}{2}\mathrm{e})$	A1	6	**AG**; full detail required

8 (i)	State R = 13	B1		or equiv
	State at least one equation of form $R\cos\alpha = k$, $R\sin\alpha = k'$, $\tan\alpha = k''$	M1		or equiv; allow sin / cos muddles; implied by correct α
	Obtain 67.4	A1	3	allow 67 or greater accuracy
(ii)	Refer to translation and stretch	M1		in either order; allow here equiv terms such as ‘move’, ‘shift’; with both transformations involving constants
	State translation in positive x direction by 67.4	A1√		or equiv; following their α; using correct terminology now
	State stretch in y direction by factor 13	A1√	3	or equiv; following their R; using correct terminology now
(iii)	Attempt value of $\cos^{-1}(2 \div R)$	M1		
	Obtain 81.15	A1√		following their R; accept 81
	Obtain 148.5 as one solution	A1		accept 148.5 or 148.6 or value rounding to either of these
	Add their α value to second value correctly attempted	M1		
	Obtain 346.2	A1	5	accept 346.2 or 346.3 or value rounding to either of these; and no other solutions

9 (i)	Attempt to express x in terms of y	*M1		obtaining two terms

	Obtain $x = e^{\frac{1}{2}y} + 1$	**A1**		or equiv
	State or imply volume involves $\int \pi x^2$	**B1**		
	Attempt to express x^2 in terms of y	***M1**		dep ***M**; expanding to produce at least 3 terms
	Obtain $k\int (e^y + 2e^{\frac{1}{2}y} + 1)\,dy$	**A1**		any constant k including 1; allow if dy absent
	Integrate to obtain $k(e^y + 4e^{\frac{1}{2}y} + y)$	**A1**		
	Use limits 0 and p	**M1**		dep ***M *M**; evidence of use of 0 needed
	Obtain $\pi(e^p + 4e^{\frac{1}{2}p} + p - 5)$	**A1**	**8**	**AG**; necessary detail required
(ii)	State or imply $\frac{dp}{dt} = 0.2$	**B1**		maybe implied by use of 0.2 in product
	Obtain $\pi(e^p + 2e^{\frac{1}{2}p} + 1)$ as derivative of V	**B1**		
	Attempt multiplication of values or expressions for $\frac{dp}{dt}$ and $\frac{dV}{dp}$	**M1**		
	Obtain $0.2\pi(e^4 + 2e^2 + 1)$	**A1**√		following their $\frac{dV}{dp}$ expression
	Obtain 44	**A1**	**5**	or greater accuracy

1		Attempt use of quotient rule to find derivative	M1		allow for numerator 'wrong way round'; or attempt use of product rule
		Obtain $\dfrac{2(3x-1)-3(2x+1)}{(3x-1)^2}$	A1		or equiv
		Obtain $-\frac{5}{4}$ for gradient	A1		or equiv
		Attempt eqn of straight line with numerical gradient	M1		obtained from their $\frac{dy}{dx}$; tangent not normal
		Obtain $5x + 4y - 11 = 0$	A1	**5**	or similar equiv
2	**(i)**	Attempt complete method for finding cot θ	M1		rt-angled triangle, identities, calculator, …
		Obtain $\frac{5}{12}$	A1	**2**	or exact equiv
	(ii)	Attempt relevant identity for cos 2θ	M1		$\pm 2\cos^2\theta \pm 1$ or $\pm 1 \pm 2\sin^2\theta$ or $\pm(\cos^2\theta - \sin^2\theta)$
		State correct identity with correct value(s) substituted	A1		
		Obtain $-\frac{119}{169}$	A1	**3**	correct answer only earns 3/3
3	**(a)**	Sketch reasonable attempt at $y = x^5$	*B1		accept non-zero gradient at O but curvature to be correct in first and third quadrants
		Sketch straight line with negative gradient	*B1		existing at least in (part of) first quadrant
		Indicate in some way single point of intersection	B1	**3**	dep *B1 *B1
	(b)	Obtain correct first iterate	B1		allow if not part of subsequent iteration
		Carry out process to find at least 3 iterates in all	M1		
		Obtain at least 1 correct iterate after the first	A1		allow for recovery after error; showing at least 3 d.p. in iterates
		Conclude 2.175	A1	**4**	answer required to precisely 3 d.p.
		[$0 \to 2.21236 \to 2.17412 \to 2.17480 \to 2.17479$; $1 \to 2.19540 \to 2.17442 \to 2.17480 \to 2.17479$; $2 \to 2.17791 \to 2.17473 \to 2.17479 \to 2.17479$; $3 \to 2.15983 \to 2.17506 \to 2.17479 \to 2.17479$]			
4	**(i)**	Obtain derivative of form $k(4t+9)^{-\frac{1}{2}}$	M1		any constant k
		Obtain correct $2(4t+9)^{-\frac{1}{2}}$	A1		or (unsimplified) equiv
		Obtain derivative of form $k\,e^{\frac{1}{2}x+1}$	M1		any constant k different from 6
		Obtain correct $3e^{\frac{1}{2}x+1}$	A1	**4**	or equiv
	(ii)	Either: Form product of two derivatives	M1		numerical or algebraic
		Substitute for t and x in product	M1		using $t = 4$ and calculated value of x
		Obtain 39.7	A1	**3**	allow ± 0.1; allow greater accuracy
		Or: Obtain $k(4t+9)^n e^{\frac{1}{2}(4t+9)^{\frac{1}{2}}+1}$	M1		differentiating $y = 6e^{\frac{1}{2}(4t+9)^{\frac{1}{2}}+1}$
		Obtain correct $6(4t+9)^{-\frac{1}{2}} e^{\frac{1}{2}(4t+9)^{\frac{1}{2}}+1}$	A1		or equiv
		Substitute $t = 4$ to obtain 39.7	A1	**(3)**	allow ± 0.1; allow greater accuracy
5	**(i)**	Obtain $R = \sqrt{17}$ or 4.12 or 4.1	B1		or greater accuracy
		Attempt recognisable process for finding α	M1		allow for sin/cos confusion
		Obtain $\alpha = 14$	A1	**3**	or greater accuracy 14.036…

(ii)	Attempt to find at least one value of $\theta + \alpha$	M1		
	Obtain or imply value 61	A1√		following R value; or value rounding to 61
	Obtain 46.9	A1		allow ±0.1; allow greater accuracy
	Show correct process for obtaining second angle	M1		
	Obtain –75	A1	**5**	allow ±0.1; allow greater accuracy; max of 4/5 if extra angles between –180 and 180

6 (i)	Obtain integral of form $k(3x+2)^{\frac{1}{2}}$	M1		any constant k
	Obtain correct $\frac{2}{3}(3x+2)^{\frac{1}{2}}$	A1		or equiv
	Substitute limits 0 and 2 and attempt evaluation	M1		for integral of form $k(3x+2)^n$
	Obtain $\frac{2}{3}(8^{\frac{1}{2}} - 2^{\frac{1}{2}})$	A1	**4**	or exact equiv suitably simplified
(ii)	State or imply $\pi\int \frac{1}{3x+2}\,dx$ or unsimplified version	B1		allow if dx absent or wrong
	Obtain integral of form $k\ln(3x+2)$	M1		any constant k involving π or not
	Obtain $\frac{1}{3}\pi\ln(3x+2)$ or $\frac{1}{3}\ln(3x+2)$	A1		
	Show correct use of ln a – ln b property	M1		
	Obtain $\frac{1}{3}\pi\ln 4$	A1	**5**	or (similarly simplified) equiv

7 (i)	State a in x-direction	B1		or clear equiv
	State factor 2 in x-direction	B1	**2**	or clear equiv
(ii)	Show (largely) increasing function crossing x-axis	M1		with correct curvature
	Show curve in first and fourth quadrants only	A1	**2**	not touching y-axis and with no maximum point; ignore intercept
(iii)	Show attempt at reflecting negative part in x-axis	M1		
	Show (more or less) correct graph	A1√	**2**	following their graph in **(ii)** and showing correct curvatures
(iv)	Identify $2a$ as asymptote or $2a + 2$ as intercept	B1		allow anywhere in question
	State $2a < x \le 2a + 2$	B1	**2**	allow < or ≤ for each inequality

8 (i)	Obtain $-2xe^{-x^2}$ as derivative of e^{-x^2}	B1		
	Attempt product rule	*M1		allow if sign errors or no chain rule
	Obtain $8x^7e^{-x^2} - 2x^9e^{-x^2}$	A1		or (unsimplified) equiv
	<u>Either</u>: Equate first derivative to zero and attempt solution	M1		dep *M; taking at least one step of solution
	Confirm 2	A1	**5**	AG
	<u>Or</u>: Substitute 2 into derivative and show attempt at evaluation	M1		
	Obtain 0	A1	**(5)**	AG; necessary correct detail required

(ii)	Attempt calculation involving attempts at *y* values	M1		with each of 1, 4, 2 present at least once as coefficients
	Attempt $k(y_0 + 4y_1 + 2y_2 + 4y_3 + y_4)$	M1		with attempts at five *y* values corresponding to correct *x* values
	Obtain $\frac{1}{6}(0 + 4 \times 0.00304 + 2 \times 0.36788$ $+ 4 \times 2.70127 + 4.68880)$	A1		or equiv with at least 3 d.p. or exact values
	Obtain 2.707	A1	**4**	or greater accuracy; allow ±0.001
(iii)	Attempt 4(*y* value) – 2(part **(ii)**)	M1		or equiv
	Obtain 13.3	A1	**2**	or greater accuracy; allow ±0.1

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9 (i)	State $-2 \le y \le 2$	B1		allow <; any notation
	State $y \le 4$	B1	**2**	allow <; any notation
(ii)	Show correct process for composition	M1		right way round
	Obtain or imply 0.959 and hence 2.16	A1		AG; necessary detail required
	Obtain g(0.5) = 3.5	B1		or (unsimplified) equiv
	Observe that 3.5 not in domain of f	B1	**4**	or equiv
(iii)	Relate quadratic expression to at least one end of range of f	M1		or equiv
	Obtain both of $4 - 2x^2 < -2$ and $4 - 2x^2 > 2$	A1		or equiv; allow any sign in each (< or ≤ or > or ≥ or =)
	Obtain at least two of the *x* values $-\sqrt{3}, -1, 1, \sqrt{3}$	A1		
	Obtain all four of the *x* values	A1		
	Attempt solution involving four *x* values	M1		to produce at least two sets of values
	Obtain $x < -\sqrt{3}$, $-1 < x < 1$, $x > \sqrt{3}$	A1	**6**	allow ≤ instead of < and/or ≥ instead of >

1	**(i)**	Attempt use of product rule	M1		
		Obtain $3x^2(x+1)^5+5x^3(x+1)^4$	A1	**2**	or equiv
		[<u>Or</u>: (following complete expansion and differentiation term by term)			
		Obtain $8x^7+35x^6+60x^5+50x^4+20x^3+3x^2$	B2		allow B1 if one term incorrect]
	(ii)	Obtain derivative of form $kx^3(3x^4+1)^n$	M1		any constants k and n
		Obtain derivative of form $kx^3(3x^4+1)^{-\frac{1}{2}}$	M1		
		Obtain correct $6x^3(3x^4+1)^{-\frac{1}{2}}$	A1	**3**	or (unsimplified) equiv
2		Identify critical value $x = 2$	B1		
		Attempt process for determining both critical values	M1		
		Obtain $\frac{1}{3}$ and 2	A1		
		Attempt process for solving inequality	M1		table, sketch …; implied by plausible answer
		Obtain $\frac{1}{3} < x < 2$	A1	**5**	
3	**(i)**	Attempt correct process for composition	M1		numerical or algebraic
		Obtain (16 and hence) 7	A1	**2**	
	(ii)	Attempt correct process for finding inverse	M1		maybe in terms of y so far
		Obtain $(x-3)^2$	A1	**2**	or equiv; in terms of x, not y
	(iii)	Sketch (more or less) correct $y = \mathrm{f}(x)$	B1		with 3 indicated or clearly implied on y-axis, correct curvature, no maximum point
		Sketch (more or less) correct $y = \mathrm{f}^{-1}(x)$	B1		right hand half of parabola only
		State reflection in line $y = x$	B1	**3**	or (explicit) equiv; independent of earlier marks
4	**(i)**	Obtain integral of form $k(2x+1)^{\frac{4}{3}}$	M1		or equiv using substitution; any constant k
		Obtain correct $\frac{3}{8}(2x+1)^{\frac{4}{3}}$	A1		or equiv
		Substitute limits in expression of form $(2x+1)^n$ and subtract the correct way round	M1		using adjusted limits if subn used
		Obtain 30	A1	**4**	
	(ii)	Attempt evaluation of $k(y_0+4y_1+y_2)$	M1		any constant k
		Identify k as $\frac{1}{3}\times 6.5$	A1		
		Obtain 29.6	A1	**3**	or greater accuracy (29.554566…)
		[SR: (using Simpson's rule with 4 strips)			
		Obtain $\frac{1}{3}\times 3.25(1+4\times\sqrt[3]{7.5}+2\times\sqrt[3]{14}+4\times\sqrt[3]{20.5}+3)$ and hence 29.9	B1		or greater accuracy (29.897…)]

5	**(i)**	State $e^{-0.04t} = 0.5$	B1		or equiv
		Attempt solution of equation of form $e^{-0.04t} = k$	M1		using sound process; maybe implied
		Obtain 17	A1	**3**	or greater accuracy (17.328…)
	(ii)	Differentiate to obtain form $k\,e^{-0.04t}$	*M1		constant k different from 240
		Obtain $(\pm)$ $9.6e^{-0.04t}$	A1		or (unsimplified) equiv
		Equate attempt at first derivative to $(\pm)$ 2.1 and attempt solution	M1		dep *M; method maybe implied
		Obtain 38	A1	**4**	or greater accuracy (37.9956…)
6	**(i)**	Obtain integral of form $k_1 e^{2x} + k_2 x^2$	M1		any non-zero constants k_1, k_2
		Obtain correct $3e^{2x} + \frac{1}{2}x^2$	A1		
		Obtain $3e^{2a} + \frac{1}{2}a^2 - 3$	A1		
		Equate definite integral to 42 and attempt rearrangement	M1		using sound processes
		Confirm $a = \frac{1}{2}\ln(15 - \frac{1}{6}a^2)$	A1	**5**	AG; necessary detail required
	(ii)	Obtain correct first iterate 1.348…	B1		
		Attempt correct process to find at least 2 iterates	M1		
		Obtain at least 3 correct iterates	A1		
		Obtain 1.344	A1	**4**	answer required to exactly 3 d.p.; allow recovery after error
		$[1 \rightarrow 1.34844 \rightarrow 1.34382 \rightarrow 1.34389]$			
7	**(i)**	Show correct general shape (alternating above and below x-axis)	M1		with no branch reaching x-axis
		Draw (more or less) correct sketch	A1	**2**	with at least one of 1 and –1 indicated or clearly implied
	(ii)	Attempt solution of $\cos x = \frac{1}{3}$	M1		maybe implied; or equiv
		Obtain 1.23 or 0.392π	A1		or greater accuracy
		Obtain 5.05 or 1.61π	A1	**3**	or greater accuracy and no others within $0 \le x \le 2\pi$; penalise answer(s) to 2sf only once
	(iii)	<u>Either</u>: Obtain equation of form $\tan\theta = k$	M1		any constant k; maybe implied
		Obtain $\tan\theta = 5$	A1		
		Obtain two values only of form $\theta,\ \theta + \pi$	M1		within $0 \le x \le 2\pi$; allow degrees at this stage
		Obtain 1.37 and 4.51 (or 0.437π and 1.44π)	A1	**4**	allow ±1 in third sig fig; or greater accuracy
		<u>Or</u>: (for methods which involve squaring,etc.) Attempt to obtain eqn in one trig ratio	M1		
		Obtain correct value	A1		$\tan^2\theta = 25, \cos^2\theta = \frac{1}{26}$, …
		Attempt solution at least to find one value in first quadrant and one value in third	M1		
		Obtain 1.37 and 4.51 (or equivs as above)	A1		ignoring values in second and fourth quadrants

8	(i)	Attempt use of quotient rule	M1		allow for numerator 'wrong way round'; or equiv
		Obtain $\dfrac{(4\ln x+3)\frac{4}{x}-(4\ln x-3)\frac{4}{x}}{(4\ln x+3)^2}$	A1		or equiv
		Confirm $\dfrac{24}{x(4\ln x+3)^2}$	A1	3	AG; necessary detail required
	(ii)	Identify $\ln x = \frac{3}{4}$	B1		or equiv
		State or imply $x = e^{\frac{3}{4}}$	B1		
		Substitute e^k completely in expression for derivative	M1		and deal with $\ln e^k$ term
		Obtain $\frac{2}{3}e^{-\frac{3}{4}}$	A1	4	or exact (single term) equiv
	(iii)	State or imply $\int \dfrac{4\pi}{x(4\ln x+3)^2}\,dx$	B1		
		Obtain integral of form $k\dfrac{4\ln x-3}{4\ln x+3}$ or $k(4\ln x+3)^{-1}$	*M1		any constant k
		Substitute both limits and subtract right way round	M1		dep *M
		Obtain $\frac{4}{21}\pi$	A1	4	or exact equiv
9	(i)	Attempt use of either of $\tan(A \pm B)$ identities	M1		
		Substitute $\tan 60° = \sqrt{3}$ or $\tan^2 60° = 3$	B1		
		Obtain $\dfrac{\tan\theta+\sqrt{3}}{1-\sqrt{3}\tan\theta} \times \dfrac{\tan\theta-\sqrt{3}}{1+\sqrt{3}\tan\theta}$	A1		or equiv (perhaps with tan 60° still involved)
		Obtain $\dfrac{\tan^2\theta-3}{1-3\tan^2\theta}$	A1	4	AG
	(ii)	Use $\sec^2\theta = 1+\tan^2\theta$	B1		
		Attempt rearrangement and simplification of equation involving $\tan^2\theta$	M1		or equiv involving $\sec\theta$
		Obtain $\tan^4\theta = \frac{1}{3}$	A1		or equiv $\sec^2\theta = 1.57735...$
		Obtain 37.2	A1		or greater accuracy
		Obtain 142.8	A1	5	or greater accuracy; and no others between 0 and 180
	(iii)	Attempt rearrangement of $\dfrac{\tan^2\theta-3}{1-3\tan^2\theta}=k^2$ to form $\tan^2\theta = \dfrac{f(k)}{g(k)}$	M1		
		Obtain $\tan^2\theta = \dfrac{k^2+3}{1+3k^2}$	A1		
		Observe that RHS is positive for all k, giving one value in each quadrant	A1	3	or convincing equiv

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1	**(i)**	Show correct process for composition of functions	**M1**	numerical or algebraic; the right way round
		Obtain (–3 and hence) –23	**A1 2**	
	(ii)	Either: State or imply $x^3+4=12$	**B1**	
		Attempt solution of equation involving x^3	**M1**	as far as $x = \ldots$
		Obtain 2	**A1 3**	and no other value
		Or: Attempt expression for f^{-1}	**M1**	involving x or y; involving cube root
		Obtain $\sqrt[3]{x-4}$ or $\sqrt[3]{y-4}$	**A1**	
		Obtain 2	**A1 (3)**	and no other value
2	**(i)**	Obtain correct first iterate 2.864	**B1**	or greater accuracy 2.864327…; condone 2 dp here and in working
		Carry out correct iteration process	**M1**	to find at least 3 iterates in all
		Obtain 2.877	**A1 3**	after at least 4 steps; answer required to exactly 3 dp
		[3 $\rightarrow$ 2.864327 $\rightarrow$ 2.878042 $\rightarrow$ 2.876661 $\rightarrow$ 2.876800]		
	(ii)	State or imply $x=\sqrt[3]{31-\frac{5}{2}x}$	**B1**	
		Attempt rearrangement of equation in x	**M1**	involving cubing and grouping non-zero terms on LHS
		Obtain equation $2x^3+5x-62=0$	**A1 3**	or equiv with integers
3	**(a)**	State correct equation involving $\cos\frac{1}{2}\alpha$	**B1**	such as $\cos\frac{1}{2}\alpha=\frac{1}{4}$ or $\frac{1}{\cos\frac{1}{2}\alpha}=4$ or …
		Attempt to find value of α	**M1**	using correct order for the steps
		Obtain 151	**A1 3**	or greater accuracy; and no other values between 0 and 180
	(b)	State or imply $\cot\beta=\frac{1}{\tan\beta}$	**B1**	
		Rearrange to the form $\tan\beta=k$	**M1**	or equiv involving $\sin\beta$ only or $\cos\beta$ only; allow missing ±
		Obtain 69.3	**A1**	
		Obtain 111	**A1 4**	or greater accuracy; and no others between 0 and 180
4	**(i)**	Obtain derivative of form $kh^5(h^6+16)^n$	**M1**	any constant k; any $n<\frac{1}{2}$; allow if – 4 term retained
		Obtain correct $3h^5(h^6+16)^{-\frac{1}{2}}$	**A1**	or (unsimplified) equiv; no –4 now
		Substitute to obtain 10.7	**A1 3**	or greater accuracy or exact equiv
	(ii)	Attempt multn or divn using 8 and answer from (i)	**M1**	
		Attempt 8 divided by answer from (i)	**M1**	
		Obtain 0.75	**A1√ 3**	or greater accuracy; allow 0.75 ± 0.01; following their answer from (i)

5	**(a)**	Obtain integral of form $k(3x+7)^{10}$	**M1**	any constant k
		Obtain (unsimplified) $\frac{1}{10}\times\frac{1}{3}(3x+7)^{10}$	**A1**	or equiv
		Obtain (simplified) $\frac{1}{30}(3x+7)^{10}+c$	**A1 3**	
	(b)	State $\int \pi(\frac{1}{2\sqrt{x}})^2\,dx$	**B1**	or equiv involving x; condone no dx
		Integrate to obtain $k\ln x$	**M1**	any constant k involving π or not; or equiv such as $k\ln 4x$ or $k\ln 2x$
		Obtain $\frac{1}{4}\pi\ln x$ or $\frac{1}{4}\ln x$ or $\frac{1}{4}\pi\ln 4x$ or $\frac{1}{4}\ln 4x$	**A1**	
		Show use of the log a – log b property	**M1**	not dependent on earlier marks
		Obtain $\frac{1}{4}\pi\ln 2$	**A1 5**	or similarly simplified equiv
6	**(i)**	Either: Refer to translation and reflection	**B1**	in either order; allow clear equivs
		State translation by 1 in negative x-direction	**B1**	or equiv but now using correct terminology
		State reflection in x-axis	**B1 3**	using correct terminology
		Or: Refer to translation and reflection	**B1**	in either order; allow clear equivs
		State reflection in y-axis	**B1**	
		State translation by 1 in positive x-direction	**B1 (3)**	with order reflection then translation clearly intended
	(ii)	Show sketch with attempt at reflection of 'negative' part in x-axis	**M1**	and curve for $0<x<1$ unchanged
		Show (more or less) correct sketch	**A1 2**	with correct curvature
	(iii)	Attempt correct process for finding at least one value	**M1**	as far as $x = \ldots$; accept decimal equivs (degrees or radians) or expressions involving $\sin(\frac{1}{3}\pi)$
		Obtain $1-\frac{1}{2}\sqrt{3}$	**A1**	or exact equiv
		Obtain $1+\frac{1}{2}\sqrt{3}$	**A1 3**	or exact equiv; give **A1A0** if extra incorrect solution(s) provided
7	**(i)**	Attempt use of product rule for $x\,e^{2x}$	**M1**	obtaining … + …
		Obtain $e^{2x}+2x\,e^{2x}$	**A1**	or equiv; maybe within QR attempt
		Attempt use of quotient rule	**M1**	with or without product rule
		Obtain unsimplified $\frac{(x+k)(e^{2x}+2xe^{2x})-xe^{2x}}{(x+k)^2}$	**A1**	
		Obtain $\frac{e^{2x}(2x^2+2kx+k)}{(x+k)^2}$	**A1 5**	**AG**; necessary detail required
	(ii)	Attempt use of discriminant	**M1**	or equiv
		Obtain $4k^2-8k=0$ or equiv and hence $k=2$	**A1**	
		Attempt solution of $2x^2+2kx+k=0$	**M1**	using their numerical value of k or solving in terms of k using correct formula
		Obtain $x=-1$	**A1**	
		Obtain $-e^{-2}$	**A1 5**	or exact equiv

8 (i)	State or imply $h = 1$	**B1**	
	Attempt calculation involving attempts at y values	**M1**	addition with each of coefficients 1, 2, 4 occurring at least once; involving at least 5 y values
	Obtain $a(1 + 4\times2 + 2\times4 + 4\times8 + 2\times16 + 4\times32 + 64)$**A1**		any constant a
	Obtain 91	**A1 4**	
(ii)	State $e^{x\ln 2}$ or $k = \ln 2$	**B1**	allow decimal equiv such as $e^{0.69x}$
	Integrate e^{kx} to obtain $\frac{1}{k}e^{kx}$	**M1**	any constant k or in terms of general k
	Obtain $\frac{1}{\ln 2}(e^{6\ln 2} - e^0)$	**A1**	or exact equiv
	Simplify to obtain $\frac{63}{\ln 2}$	**A1 4**	allow if simplification in part (iii)
(iii)	Equate answers to (i) and (ii)	**M1**	provided ln 2 involved other than in power of e
	Obtain $\frac{63}{91}$ and hence $\frac{9}{13}$	**A1 2**	**AG**; necessary correct detail required
9 (i)	State at least one of $\cos\theta\cos 60 - \sin\theta\sin 60$ and $\cos\theta\cos 30 - \sin\theta\sin 30$	**B1**	
	Attempt complete multiplication of identities of form $\pm\cos\cos\pm\sin\sin$	**M1**	with values $\frac{1}{2}\sqrt{3}$, $\frac{1}{2}$ involved
	Use $\cos^2\theta + \sin^2\theta = 1$ and $2\sin\theta\cos\theta = \sin 2\theta$	**M1**	
	Obtain $\sqrt{3} - 2\sin 2\theta$	**A1 4**	**AG**; necessary detail required
(ii)	Attempt use of 22.5 in right-hand side	**M1**	
	Obtain $\sqrt{3} - \sqrt{2}$	**A1 2**	or exact equiv
(iii)	Obtain 10.7	**B1**	or greater accuracy; allow ± 0.1
	Attempt correct process to find two angles	**M1**	from values of 2θ between 0 and 180
	Obtain 79.3	**A1 3**	or greater accuracy and no others between 0 and 90; allow ± 0.1
(iv)	Indicate or imply that critical values of $\sin 2\theta$ are –1 and 1	**M1**	
	Obtain both of $k > \sqrt{3} + 2$, $k < \sqrt{3} - 2$	**A1**	condoning decimal equivs, $\le \ge$ signs
	Obtain complete correct solution	**A1 3**	now with exact values and unambiguously stated

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1	Either:	Obtain $x = 0$	**B1**	ignoring errors in working
		Form linear equation with signs of $4x$ and $3x$ different	**M1**	ignoring other sign errors
		State $4x - 5 = -3x + 5$	**A1**	or equiv without brackets
		Obtain $\frac{10}{7}$ and no other non-zero value(s)	**A1**	or exact equiv
			$\boxed{4}$	
	Or:	Obtain $16x^2 - 40x + 25 = 9x^2 - 30x + 25$	**B1**	or equiv
		Attempt solution of quadratic equation	**M1**	at least as far as factorisation or use of formula
		Obtain $\frac{10}{7}$ and no other non-zero value(s)	**A1**	or exact equiv
		Obtain 0	**B1**	ignoring errors in working
			$\boxed{4}$	
2	**(i)**	Show graph indicating attempt at reflection in $y = x$	**M1**	with correct curvature and crossing negative y-axis and positive x-axis
		Show correct graph with x-coord 2 and y-coord -3 indicated	**A1**	
			$\boxed{2}$	
	(ii)	Show graph indicating attempt at reflection in x-axis	**M1**	with correct curvature and crossing each negative axis
		Show correct graph with x-coord -3 indicated	**A1**	
		… and y-coord -4 indicated	**A1**	
		[SC: Incorrect curve earning M0 but both correct intercepts indicated B1]		
			$\boxed{3}$	
3		Attempt use of product rule	**M1**	… + … form
		Obtain $2x\ln x + x^2 . \frac{1}{x}$	**A1**	or equiv
		Substitute e to obtain 3e for gradient	**A1**	or exact (unsimplified) equiv
		Attempt eqn of straight line with numerical gradient	**M1**	allowing approx values
		Obtain $y - \mathrm{e}^2 = 3\mathrm{e}(x - \mathrm{e})$	**A1**√	or equiv; following their gradient provided obtained by diffn attempt; allow approx values
		Obtain $y = 3\mathrm{e}x - 2\mathrm{e}^2$	**A1**	in terms of e now and in requested form
			$\boxed{6}$	
4	**(i)**	Differentiate to obtain form $kx(2x^2 + 9)^n$	**M1**	any constant k; any $n < \frac{5}{2}$
		Obtain correct $10x(2x^2 + 9)^{\frac{3}{2}}$	**A1**	or (unsimplified) equiv
		Equate to 100 and confirm $x = 10(2x^2 + 9)^{-\frac{3}{2}}$	**A1**	**AG**; necessary detail required
			$\boxed{3}$	
	(ii)	Attempt relevant calculations with 0.3 and 0.4	**M1**	
		Obtain at least one correct value	**A1**	(see table below)
		Obtain two correct values and conclude appropriately	**A1**	noting sign change or showing $0.3 < \mathrm{f}(0.3)$ and $0.4 > \mathrm{f}(0.4)$ or showing gradients either side of 100
			$\boxed{3}$	

x	$\mathrm{f}(x)$	$x - \mathrm{f}(x)$	$\mathrm{f}'(x)$
0.3	0.3595	–0.0595	83.4
0.4	0.3515	0.0485	113.8

	(iii)	Obtain correct first iterate	**B1**	
		Carry out correct process	**M1**	finding at least 3 iterates in all
		Obtain 0.3553	**A1**	answer required to exactly 4 dp
			$\boxed{3}$	

[0.3 → 0.35953 → 0.35497 → 0.35534 → 0.35531;
0.35 → 0.35575 → 0.35528 → 0.35532 (→ 0.35531);
0.4 → 0.35146 → 0.35563 → 0.35529 → 0.35532]

5	**(a)**	Obtain expression of form $\dfrac{a\tan\alpha}{b+c\tan^2\alpha}$	**M1**	any non-zero constants a, b, c
		State correct $\dfrac{2\tan\alpha}{1-\tan^2\alpha}$	**A1**	or equiv
		Attempt to produce polynomial equation in $\tan\alpha$	**M1**	using sound process
		Obtain at least one correct value of $\tan\alpha$	**A1**	$\tan\alpha = \pm\sqrt{\frac{4}{5}}$
		Obtain 41.8	**A1**	allow 42 or greater accuracy; allow 0.73
		Obtain 138.2 and no other values between 0 and 180	**A1**	allow 138 or greater accuracy
		[SC: Answers only 41.8 or … B1; 138.2 or … and no others B1]		
			$\boxed{6}$	
	(b)(i)	State $\frac{7}{6}$	**B1**	
			$\boxed{1}$	
	(ii)	Attempt use of identity linking $\cot^2\beta$ and $\operatorname{cosec}^2\beta$	**M1**	or equiv retaining exactness; condone sign errors
		Obtain $\frac{13}{36}$	**A1**	or exact equiv
			$\boxed{2}$	
6		Integrate $k_1\mathrm{e}^{nx}$ to obtain $k_2\,\mathrm{e}^{nx}$	**M1**	any constants involving π or not; any n
		Obtain correct indefinite integral of their $k_1\mathrm{e}^{nx}$	**A1**	
		Substitute limits to obtain $\frac{1}{6}\pi(\mathrm{e}^3-1)$ or $\frac{1}{6}(\mathrm{e}^3-1)$	**A1**	or exact equiv perhaps involving e^0
		Integrate $k(2x-1)^n$ to obtain $k'(2x-1)^{n+1}$	**M1**	any constants involving π or not; any n
		Obtain correct indefinite integral of their $k(2x-1)^n$	**A1**	
		Substitute limits to obtain $\frac{1}{18}\pi$ or $\frac{1}{18}$	**A1**	or exact equiv
		Apply formula $\int \pi y^2\,\mathrm{d}x$ at least once	**B1**	for $y=\mathrm{e}^{3x}$ and/or $y=(2x-1)^4$
		Subtract, correct way round, attempts at volumes	**M1**	allow with π missing but must involve y^2
		Obtain $\frac{1}{6}\pi\mathrm{e}^3-\frac{2}{9}\pi$	**A1**	or similarly simplified exact equiv
			$\boxed{9}$	
7	**(i)**	State A = 42	**B1**	
		State $k=\frac{1}{9}$	**B1**	or 0.11 or greater accuracy
		Attempt correct process for finding m	**M1**	involving logarithms or equiv
		Obtain $\frac{1}{9}\ln 2$ or 0.077	**A1**	or 0.08 or greater accuracy
			$\boxed{4}$	
	(ii)	Attempt solution for t using either formula	**M1**	using correct process (log'ms or T&I or …)
		Obtain 11.3	**A1**	or greater accuracy; allow 11.3 ± 0.1
			$\boxed{2}$	
	(iii)	Differentiate to obtain form $B\mathrm{e}^{mt}$	**M1**	where B is different from A
		Obtain $3.235\mathrm{e}^{0.077t}$	**A1**√	or equiv; following their A and m
		Obtain 47.9	**A1**	allow 48 or greater accuracy
			$\boxed{3}$	

8	**(i)**	Show at least correct cos θ cos 60 + sin θ sin 60 or cos θ cos 60 – sin θ sin 60	**B1**	
		Attempt expansion of both with exact numerical values attempted	**M1**	and with $\cos 60 \neq \sin 60$
		Obtain $\frac{1}{2}\sqrt{3}\sin\theta + \frac{5}{2}\cos\theta$	**A1**	or exact equiv
			$\boxed{3}$	
	(ii)	Attempt correct process for finding R	**M1**	whether exact or approx
		Attempt recognisable process for finding α	**M1**	allowing sin / cos muddles
		Obtain $\sqrt{7}\sin(\theta + 70.9)$	**A1**	allow 2.65 for R; allow 70.9 ± 0.1 for α
			$\boxed{3}$	
	(iii)	Attempt correct process to find any value of θ + their α	**M1**	
		Obtain any correct value for θ + 70.9	**A1**	–158, –22, 202, 338, …
		Attempt correct process to find θ + their α in 3rd quadrant	**M1**	or several values including this
		Obtain 131	**A1**	or greater accuracy and no other
		[SC for solutions with no working shown: Correct answer only B4; 131 with other answers B2]		
			$\boxed{4}$	
9	**(i)**	Attempt use of quotient rule	***M1**	or equiv; allow u / v muddles
		Obtain $\dfrac{75 - 15x^2}{(x^2+5)^2}$	**A1**	or (unsimplified) equiv; this **M1A1** available at any stage of question
		Equate attempt at first derivative to zero and rearrange to solvable form	**M1**	dep ***M**
		Obtain $x = \sqrt{5}$ or 2.24	**A1**	or greater accuracy
		Recognise range as values less than y-coord of st pt	**M1**	allowing < here
		Obtain $0 \leq y \leq \frac{3}{2}\sqrt{5}$	**A1**	any notation; with ≤ now; any exact equiv
			$\boxed{6}$	
	(ii)	State $\sqrt{5}$	**B1**√	following their x-coord of st pt; condone answer $x \geq \sqrt{5}$ but not inequality with k
			$\boxed{1}$	
	(iii)	Equate attempt at first derivative to –1 and attempt simplification	***M1**	and dependent on first **M** in part **(i)**
		Obtain $x^4 - 5x^2 + 100 = 0$	**A1**	or equiv involving 3 non-zero terms
		Attempt evaluation of discriminant or equiv	**M1**	dep ***M**
		Obtain –375 or equiv and conclude appropriately	**A1**	
			$\boxed{4}$	

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1 (i)	Obtain integral of form ke^{-2x}	M1		any constant k different from 8
	Obtain $-4e^{-2x}$	A1		or (unsimplified) equiv
(ii)	Obtain integral of form $k(4x+5)^7$	M1		any constant k
	Obtain $\frac{1}{28}(4x+5)^7$	A1		in simplified form
	Include … + c at least once	B1		in either part
			$\boxed{5}$	
2 (i)	Form expression involving attempts at y values and addition	M1		with coeffs 1, 4 and 2 present at least once
	Obtain $k(\ln 4+4\ln 6+2\ln 8+4\ln 10+\ln 12)$	A1		any constant k
	Use value of k as $\frac{1}{3}\times 2$	A1		or unsimplified equiv
	Obtain 16.27	A1	**4**	or 16.3 or greater accuracy (16.27164…)
(ii)	State 162.7 or 163	B1√	**1**	following their answer to **(i)**, maybe rounded
			$\boxed{5}$	
3 (i)	Attempt use of identity for $\tan^2\theta$	M1		using $\pm\sec^2\theta\pm 1$; or equiv
	Replace $\dfrac{1}{\cos\theta}$ by $\sec\theta$	B1		
	Obtain $2(\sec^2\theta-1)-\sec\theta$	A1	**3**	or equiv
(ii)	Attempt soln of quadratic in $\sec\theta$ or $\cos\theta$	M1		as far as factorisation or substitution in correct formula
	Relate $\sec\theta$ to $\cos\theta$ and attempt at least one value of θ	M1		may be implied
	Obtain $60°, 131.8°$	A1		allow 132 or greater accuracy
	Obtain $60°, 131.8°, 228.2°, 300°$	A1	**4**	allow 132, 228 or greater accuracy; and no others between $0°$ and $360°$
			$\boxed{7}$	
4 (i)	Obtain derivative of form $kx(4x^2+1)^4$	M1		any constant k
	Obtain $40x(4x^2+1)^4$	A1		or (unsimplified) equiv
	State $x = 0$	A1√	**3**	and no other; following their derivative of form $kx(4x^2+1)^4$
(ii)	Attempt use of quotient rule	M1		or equiv
	Obtain $\dfrac{2x\ln x - x^2.\frac{1}{x}}{(\ln x)^2}$	A1		or equiv
	Equate to zero and attempt solution	M1		as far as solution involving e
	Obtain $e^{\frac{1}{2}}$	A1	**4**	or exact equiv; and no other; allow from ± (correct numerator of derivative)
			$\boxed{7}$	

5 **(i)**	State 40	B1		
	Attempt value of k using 21 and 80	M1		or equiv
	Obtain $40\mathrm{e}^{21k} = 80$ and hence 0.033	A1		or equiv such as $\frac{1}{21}\ln 2$
	Attempt value of M for $t = 63$	M1		using established formula or using exponential property
	Obtain 320	A1	**5**	or value rounding to this
(ii)	Differentiate to obtain $c\mathrm{e}^{0.033t}$ or $40k\mathrm{e}^{kt}$	M1		any constant c different from 40
	Obtain $40 \times 0.033\mathrm{e}^{0.033t}$	A1√		following their value of k
	Obtain 2.64	A1	**3**	allow 2.6 or 2.64 ± 0.01 or greater accuracy (2.64056…)
			$\boxed{\mathbf{8}}$	
6 **(i)**	Attempt correct process for finding inverse	M1		maybe in terms of y so far
	Obtain $2x^3 - 4$	A1		or equiv; in terms of x now
	State $\sqrt[3]{2}$ or 1.26	B1	**3**	
(ii)	State reflection in $y = x$	B1		or clear equiv
	Refer to intersection of $y = x$ and $y = \mathrm{f}(x)$ and hence confirm $x = \sqrt[3]{\frac{1}{2}x + 2}$	B1	**2**	AG; or equiv
(iii)	Obtain correct first iterate	B1		
	Show correct process for iteration	M1		with at least one more step
	Obtain at least 3 correct iterates in all	A1		allowing recovery after error
	Obtain 1.39	A1	**4**	following at least 3 steps; answer required to exactly 2 d.p.

[0 → 1.259921 → 1.380330 → 1.390784 → 1.391684
1 → 1.357209 → 1.388789 → 1.391512 → 1.391747
1.26 → 1.380337 → 1.390784 → 1.391684 → 1.391761
1.5 → 1.401020 → 1.392564 → 1.391837 → 1.391775
2 → 1.442250 → 1.396099 → 1.392141 → 1.391801]

$\boxed{\mathbf{9}}$

7 **(i)**	Refer to stretch and translation	M1		in either order; allow here informal terms
	State stretch, factor $\frac{1}{k}$, in x direction	A1		or equiv; now with correct terminology
	State translation in negative y direction by a	A1	**3**	or equiv; now with correct terminology
	[SC: If M0 but one transformation completely correct – B1]			
(ii)	Show attempt to reflect negative part in x-axis	M1		ignoring curvature
	Show correct sketch	A1	**2**	with correct curvature, no pronounced 'rounding' at x-axis and no obvious maximum point
(iii)	Attempt method with $x = 0$ to find value of a	M1		... other than (or in addition to) value –12
	Obtain $a = 14$	A1		and nothing else
	Attempt to solve for k	M1		using any numerical a with sound process
	Obtain $k = 3$	A1	**4**	
			$\boxed{\mathbf{9}}$	

8	**(i)**	Attempt to express x or x^2 in terms of y	M1		
		Obtain $x^2 = \dfrac{1296}{(y+3)^4}$	A1		or (unsimplified) equiv
		Obtain integral of form $k(y+3)^{-3}$	M1		any constant k
		Obtain $-432\pi(y+3)^{-3}$ or $-432(y+3)^{-3}$	A1		or (unsimplified) equiv
		Attempt evaluation using limits 0 and p	M1		for expression of form $k(y+3)^{-n}$ obtained from integration attempt; subtraction correct way round
		Confirm $16\pi(1-\dfrac{27}{(p+3)^3})$	A1	**6**	AG; necessary detail required, including appearance of π prior to final line
	(ii)	State or obtain $\dfrac{dV}{dp} = 1296\pi(p+3)^{-4}$	B1		or equiv; perhaps involving y
		Multiply $\dfrac{dp}{dt}$ and attempt at $\dfrac{dV}{dp}$	*M1		algebraic or numerical
		Substitute p = 9 and attempt evaluation	M1		dep *M
		Obtain $\frac{1}{4}\pi$ or 0.785	A1	**4**	or greater accuracy
				$\boxed{\mathbf{10}}$	
9	**(i)**	State $\cos 2\theta \cos\theta - \sin 2\theta \sin\theta$	B1		
		Use at least one of $\cos 2\theta = 2\cos^2\theta - 1$ and $\sin 2\theta = 2\sin\theta\cos\theta$	B1		
		Attempt to express in terms of $\cos\theta$ only	M1		using correct identities for $\cos 2\theta$, $\sin 2\theta$ and $\sin^2\theta$
		Obtain $4\cos^3\theta - 3\cos\theta$	A1	**4**	AG; necessary detail required
	(ii)	<u>Either</u>: State or imply $\cos 6\theta = 2\cos^2 3\theta - 1$	B1		
		Use expression for $\cos 3\theta$ and attempt expansion	M1		for expression of form $\pm 2\cos^2 3\theta \pm 1$
		Obtain $32c^6 - 48c^4 + 18c^2 - 1$	A1	**3**	AG; necessary detail required
		<u>Or</u>: State $\cos 6\theta = 4\cos^3 2\theta - 3\cos 2\theta$	B1		maybe implied
		Express $\cos 2\theta$ in terms of $\cos\theta$ and attempt expansion	M1		for expression of form $\pm 2\cos^2\theta \pm 1$
		Obtain $32c^6 - 48c^4 + 18c^2 - 1$	A1	**(3)**	AG; necessary detail required
	(iii)	Substitute for $\cos 6\theta$	*M1		with simplification attempted
		Obtain $32c^6 - 48c^4 = 0$	A1		or equiv
		Attempt solution for c of equation	M1		dep *M
		Obtain $c^2 = \frac{3}{2}$ and observe no solutions	A1		or equiv; correct work only
		Obtain c = 0, give at least three specific angles and conclude odd multiples of 90	A1	**5**	AG; or equiv; necessary detail required; correct work only
				$\boxed{\mathbf{12}}$	

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1 (i)	State $y = \sec x$	B1		
(ii)	State $y = \cot x$	B1		
(iii)	State $y = \sin^{-1} x$	B1	3	
			$\boxed{3}$	
2	Either: State or imply $\int \pi(2x-3)^4 \, dx$	B1		or unsimplified equiv
	Obtain integral of form $k(2x-3)^5$	M1		any constant k involving π or not
	Obtain $\frac{1}{10}(2x-3)^5$ or $\frac{1}{10}\pi(2x-3)^5$	A1		
	Attempt evaluation using 0 and $\frac{3}{2}$	M1		subtraction correct way round
	Obtain $\frac{243}{10}\pi$	A1	5	or exact equiv
	Or: State or imply $\int \pi(2x-3)^4 \, dx$	B1		or unsimplified equiv
	Expand and obtain integral of order 5	M1		with at least three terms correct
	Ob'n $\frac{16}{5}x^5 - 24x^4 + 72x^3 - 108x^2 + 81x$	A1		with or without π
	Attempt evaluation using (0 and) $\frac{3}{2}$	M1		
	Obtain $\frac{243}{10}\pi$	A1	(5)	or exact equiv
			$\boxed{5}$	
3 (i)	Attempt use of identity for $\sec^2 \alpha$	M1		using $\pm\tan^2\alpha \pm 1$
	Obtain $1 + (m+2)^2 - (1+m^2)$	A1		absent brackets implied by subsequent correct working
	Obtain $4m + 4 = 16$ and hence $m = 3$	A1	3	
(ii)	Attempt subn in identity for $\tan(\alpha + \beta)$	M1		using $\dfrac{\pm\tan\alpha \pm \tan\beta}{1 \pm \tan\alpha\tan\beta}$
	Obtain $\dfrac{5+3}{1-15}$ or $\dfrac{m+2+m}{1-m(m+2)}$	A1√		following their m
	Obtain $-\frac{4}{7}$	A1	3	or exact equiv
			$\boxed{6}$	
4 (i)	Obtain $\frac{1}{3}e^{3x} + e^x$	B1		
	Substitute to obtain $\frac{1}{3}e^{9a} + e^{3a} - \frac{1}{3}e^{3a} - e^a$	B1		or equiv
	Equate definite integral to 100 and attempt rearrangement	M1		as far as $e^{9a} = \ldots$
	Introduce natural logarithm	M1		using correct process
	Obtain $a = \frac{1}{9}\ln(300 + 3e^a - 2e^{3a})$	A1	5	AG; necessary detail needed
(ii)	Obtain correct first iterate	B1		allow for 4 dp rounded or truncated
	Show correct iteration process	M1		with at least one more step
	Obtain at least three correct iterates in all	A1		allowing recovery after error
	Obtain 0.6309	A1	4	following at least three correct steps; answer required to exactly 4 dp
	[$0.6 \rightarrow 0.631269 \rightarrow 0.630884 \rightarrow 0.630889$]			
			$\boxed{9}$	

5 (i)	Either: Show correct process for comp'n	M1		correct way round and in terms of x
	Obtain $y = 3(3x+7)-2$	A1		or equiv
	Obtain $x = -\frac{19}{9}$	A1	3	or exact equiv; condone absence of $y = 0$
	Or: Use fg(x) = 0 to obtain $g(x) = \frac{2}{3}$	B1		
	Attempt solution of $g(x) = \frac{2}{3}$	M1		
	Obtain $x = -\frac{19}{9}$	A1	(3)	or exact equiv; condone absence of $y = 0$
(ii)	Attempt formation of one of the equations $3x+7 = \dfrac{x-7}{3}$ or $3x+7 = x$ or $\dfrac{x-7}{3} = x$	M1		or equiv
	Obtain $x = -\frac{7}{2}$	A1		or equiv
	Obtain $y = -\frac{7}{2}$	A1√	3	or equiv; following their value of x
(iii)	Attempt solution of modulus equation	M1		squaring both sides to obtain 3-term quadratics or forming linear equation with signs of $3x$ different on each side
	Obtain $-12x+4 = 42x+49$ or $3x-2 = -3x-7$	A1		or equiv
	Obtain $x = -\frac{5}{6}$	A1		or exact equiv; as final answer
	Obtain $y = \frac{9}{2}$	A1	4	or equiv; and no other pair of answers
			10	
6 (i)	Obtain derivative $k(37+10y-2y^2)^{-\frac{1}{2}}\mathrm{f}(y)$	M1		any constant k; any linear function for f
	Obtain $\frac{1}{2}(10-4y)(37+10y-2y^2)^{-\frac{1}{2}}$	A1	2	or equiv
(ii)	Either: Sub'te $y = 3$ in expression for $\dfrac{dx}{dy}$	*M1		
	Take reciprocal of expression/value	*M1		and without change of sign
	Obtain -7 for gradient of tangent	A1		
	Attempt equation of tangent	M1		dep *M *M
	Obtain $y = -7x+52$	A1	5	and no second equation
	Or: Sub'te $y = 3$ in expression for $\dfrac{dx}{dy}$	M1		
	Attempt formation of eq'n $x = m'y+c$	M1		where m' is attempt at $\dfrac{dx}{dy}$
	Obtain $x-7 = -\frac{1}{7}(y-3)$	A1		or equiv
	Attempt rearrangement to required form	M1		
	Obtain $y = -7x+52$	A1	(5)	and no second equation
			7	

7 (i)	State $R = 10$	B1		or equiv
	Attempt to find value of α	M1		implied by correct answer or its complement; allow sin/cos muddles
	Obtain 36.9 or $\tan^{-1}\frac{3}{4}$	A1	3	or greater accuracy 36.8699…
(ii)(a)	Show correct process for finding one angle	M1		
	Obtain (64.16 + 36.87 and hence) 101	A1		or greater accuracy 101.027…
	Show correct process for finding second angle	M1		
	Obtain (115.84 + 36.87 and hence) 153	A1√	4	following their value of α; or greater accuracy 152.711…; and no other between 0 and 360
(b)	Recognise link with part **(i)**	M1		signalled by 40 … – 20 …
	Use fact that maximum and minimum values of sine are 1 and –1	M1		may be implied; or equiv
	Obtain 60	A1	3	
			10	
8 (i)	Refer to translation and stretch	M1		in either order; allow here equiv informal terms such as 'move', …
	State translation in x direction by 6	A1		or equiv; now with correct terminology
	State stretch in y direction by 2	A1	3	or equiv; now with correct terminology
	[SC: if M0 but one transformation completely correct, give B1]			
(ii)	State $2\ln(x-6) = \ln x$	B1		or $2\ln(a-6) = \ln a$ or equiv
	Show correct use of logarithm property	*M1		
	Attempt solution of 3-term quadratic	M1		dep *M
	Obtain 9 only	A1	4	following correct solution of equation
(iii)	Attempt evaluation of form $k(y_0 + 4y_1 + y_2)$	M1		any constant k; maybe with $y_0 = 0$ implied
	Obtain $\frac{1}{3}\times 1(2\ln 1 + 8\ln 2 + 2\ln 3)$	A1		or equiv
	Obtain 2.58	A1	3	or greater accuracy 2.5808…
			10	
9 (a)	Attempt use of quotient rule	*M1		or equiv; allow numerator wrong way round and denominator errors
	Obtain $\dfrac{(kx^2+1)2kx-(kx^2-1)2kx}{(kx^2+1)^2}$	A1		or equiv; with absent brackets implied by subsequent correct working
	Obtain correct simplified numerator $4kx$	A1		
	Equate numerator of first derivative to zero	M1		dep *M
	State $x = 0$ <u>or</u> refer to $4kx$ being linear <u>or</u> observe that, with $k \neq 0$, only one sol'n	A1√	5	AG or equiv; following numerator of form $k'kx = 0$, any constant k'

(b)	Attempt use of product rule	*M1		
	Obtain $me^{mx}(x^2+mx)+e^{mx}(2x+m)$	A1		or equiv
	Equate to zero and either factorise with factor e^{mx} or divide through by e^{mx}	M1		dep *M
	Obtain $mx^2+(m^2+2)x+m=0$ or equiv and observe that e^{mx} cannot be zero	A1		
	Attempt use of discriminant	M1		using correct b^2-4ac with their a, b, c
	Simplify to obtain m^4+4	A1		or equiv
	Observe that this is positive for all *m* and hence two roots	A1	7	or equiv; AG
			$\boxed{\mathbf{12}}$	

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1		Obtain integral of form $k(2x-7)^{-1}$	M1		any constant k
		Obtain correct $-5(2x-7)^{-1}$	A1		or equiv
		Include … + c	B1	**3**	at least once; following any integral
				$\boxed{3}$	
2	**(i)**	Use $\sin 2\theta = 2\sin\theta\cos\theta$	B1		
		Attempt value of $\sin\theta$ from $k\sin\theta\cos\theta = 5\cos\theta$	M1		any constant k; or equiv
		Obtain $\frac{5}{12}$	A1	**3**	or exact equiv; ignore subsequent work
	(ii)	Use $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$ or $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$	B1		or equiv
		Attempt to produce equation involving $\cos\theta$ only	M1		using $\sin^2\theta = \pm1 \pm \cos^2\theta$ or equiv
		Obtain $3\cos^2\theta + 8\cos\theta - 3 = 0$	A1		or equiv
		Attempt solution of 3-term quadratic equation	M1		using formula or factorisation or equiv
		Obtain $\frac{1}{3}$ as only final value of $\cos\theta$	A1	**5**	or exact equiv; ignore subsequent work
				$\boxed{8}$	
3	**(i)**	Obtain or clearly imply $60\ln x$	B1		
		Obtain ($60\ln 20 - 60\ln 10$ and hence) $60\ln 2$	B1	**2**	with no error seen
	(ii)	Attempt calculation of form $k(y_0 + 4y_1 + y_2)$	M1		any constant k; using y-value attempts
		Identify k as $\frac{5}{3}$	A1		
		Obtain $\frac{5}{3}(6 + 4\times4 + 3)$ and hence $\frac{125}{3}$ or 41.7	A1	**3**	or equiv
	(iii)	Equate answers to parts **(i)** and **(ii)**	M1		provided $\ln 2$ involved
		Obtain $60\ln 2 = \frac{125}{3}$ and hence $\frac{25}{36}$	A1	**2**	AG; necessary detail required including clear use of an exact value from **(ii)**
				$\boxed{7}$	
4	**(i)**	Attempt correct process for composition	M1		numerical or algebraic
		Obtain (7 and hence) 0	A1	**2**	
	(ii)	Attempt to find x-intercept	M1		
		Obtain $x \le 7$	A1	**2**	or equiv; condone use of <
	(iii)	Attempt correct process for finding inverse	M1		
		Obtain $\pm(2-y)^3 - 1$ or $\pm(2-x)^3 - 1$	A1		
		Obtain correct $(2-x)^3 - 1$	A1	**3**	or equiv in terms of x
	(iv)	Refer to reflection in $y = x$	B1	**1**	or clear equiv
				$\boxed{8}$	

5	**(i)**	Obtain derivative of form $kx(x^2+1)^7$	M1		any constant k
		Obtain $16x(x^2+1)^7$	A1		or equiv
		Equate first derivative to 0 and confirm $x=0$ or substitute $x=0$ and verify first derivative zero	M1		AG; allow for deriv of form $kx(x^2+1)^7$
		Refer, in some way, to $x^2+1=0$ having no root	A1	**4**	or equiv
	(ii)	Attempt use of product rule	*M1		obtaining … + … form
		Obtain $16(x^2+1)^7$ + …	A1√		follow their $kx(x^2+1)^7$
		Obtain … + $224x^2(x^2+1)^6$	A1√		follow their $kx(x^2+1)^7$; or unsimplified equiv
		Substitute 0 in attempt at second derivative	M1		dep *M
		Obtain 16	A1	**5**	from second derivative which is correct at some point
				$\boxed{9}$	
6		Integrate e^{3x} to obtain $\frac{1}{3}e^{3x}$ or $e^{-\frac{1}{2}x}$ to obtain $-2e^{-\frac{1}{2}x}$	B1		or both
		Obtain indefinite integral of form $m_1e^{3x}+m_2e^{-\frac{1}{2}x}$	M1		any constants m_1 and m_2
		Obtain correct $\frac{1}{3}ke^{3x}-2(k-2)e^{-\frac{1}{2}x}$	A1		or equiv
		Obtain $e^{3\ln 4}=64$ or $e^{-\frac{1}{2}\ln 4}=\frac{1}{2}$	B1		or both
		Apply limits and equate to 185	M1		including substitution of lower limit
		Obtain $\frac{64}{3}k-(k-2)-\frac{1}{3}k+2(k-2)=185$	A1		or equiv
		Obtain $\frac{17}{2}$	A1	**7**	or equiv
				$\boxed{7}$	
7	**(a)**	<u>Either</u>: State or imply either $\frac{dA}{dr}=2\pi r$ or $\frac{dA}{dt}=250$	B1		or both
		Attempt manipulation of derivatives to find $\frac{dr}{dt}$	M1		using multiplication / division
		Obtain correct $\frac{250}{2\pi r}$	A1		or equiv
		Obtain 1.6	A1	**4**	or equiv; allow greater accuracy
		<u>Or</u>: Attempt to express r in terms of t	M1		using $A=250t$
		Obtain $r=\sqrt{\frac{250t}{\pi}}$	A1		or equiv
		Differentiate $kt^{\frac{1}{2}}$ to produce $\frac{1}{2}kt^{-\frac{1}{2}}$	M1		any constant k
		Substitute $t=7.6$ to obtain 1.6	A1	**(4)**	allow greater accuracy

(b)	State $\frac{dm}{dt} = -150ke^{-kt}$	B1	
	Equate to $(\pm)3$ and attempt value for t	M1	using valid process; condone sign confusion
	Obtain $-\frac{1}{k}\ln(\frac{1}{50k})$ or $\frac{1}{k}\ln(50k)$ or $\frac{\ln 50 + \ln k}{k}$	A1 **3**	or equiv but with correct treatment of signs
		$\boxed{7}$	

8 (i)	State scale factor is $\sqrt{2}$	B1	allow 1.4
	State translation is in negative x-direction …	B1	or clear equiv
	… by $\frac{3}{2}$ units	B1 **3**	
(ii)	Draw (more or less) correct sketch of $y = \sqrt{2x+3}$	B1	'starting' at point on negative x-axis
	Draw (more or less) correct sketch of $y = \frac{N}{x^3}$	B1	showing both branches
	Indicate one point of intersection	B1 **3**	with both sketches correct
	[SC: if neither sketch complete or correct but diagram correct for both in first quadrant B1]		
(iii) (a)	Substitute 1.9037 into $x = N^{\frac{1}{3}}(2x+3)^{-\frac{1}{6}}$	M1	or into equation $\sqrt{2x+3} = \frac{N}{x^3}$; or equiv
	Obtain 18 or value rounding to 18	A1 **2**	with no error seen
(b)	State or imply $2.6282 = N^{\frac{1}{3}}(2\times 2.6022+3)^{-\frac{1}{6}}$	B1	
	Attempt solution for N	M1	using correct process
	Obtain 52	A1 **3**	concluding with integer value
			$\boxed{11}$

9 (i)	Identify $\tan 55°$ as $\tan(45°+10°)$	B1	or equiv
	Use correct angle sum formula for $\tan(A+B)$	M1	or equiv
	Obtain $\frac{1+p}{1-p}$	A1 **3**	with $\tan 45°$ replaced by 1
(ii)	<u>Either</u>: Attempt use of identity for $\tan 2A$	*M1	linking $10°$ and $5°$
	Obtain $p = \frac{2t}{1-t^2}$	A1	
	Attempt solution for t of quadratic equation	M1	dep *M
	Obtain $\frac{-1+\sqrt{1+p^2}}{p}$	A1 **4**	or equiv; and no second expression
	<u>Or (1)</u>: Attempt expansion of $\tan(60°-55°)$	*M1	
	Obtain $\frac{\sqrt{3}-\frac{1+p}{1-p}}{1+\sqrt{3}\,\frac{1+p}{1-p}}$	A1√	follow their answer from **(i)**
	Attempt simplification to remove denominators	M1	dep *M
	Obtain $\frac{\sqrt{3}(1-p)-(1+p)}{1-p+\sqrt{3}(1+p)}$	A1 **(4)**	or equiv

Or (2):	State or imply $\tan 15° = 2-\sqrt{3}$	B1	
	Attempt expansion of $\tan(15°-10°)$	M1	with exact attempt for $\tan 15°$
	Obtain $\dfrac{2-\sqrt{3}-p}{1+p(2-\sqrt{3})}$	A2 **(4)**	
Or (3):	State or imply $\tan 15° = \frac{\sqrt{3}-1}{\sqrt{3}+1}$	B1	or exact equiv
	Attempt expansion of $\tan(15°-10°)$	M1	with exact attempt for $\tan 15°$
	Obtain $\dfrac{\sqrt{3}-1-p\sqrt{3}-p}{\sqrt{3}+1+p\sqrt{3}-p}$	A2 **(4)**	or equiv
Or (4):	Attempt expansion of $\tan(10°-5°)$	*M1	
	Obtain $t=\dfrac{p-t}{1+pt}$	A1	
	Attempt solution for t of quadratic equation	M1	dep *M
	Obtain $\dfrac{-2+\sqrt{4+4p^2}}{2p}$	A1 **(4)**	or equiv; and no second expression
(iii)	Attempt expansion of both sides	M1	
	Obtain $3\sin\theta\cos 10° + 3\cos\theta\sin 10° = 7\cos\theta\cos 10° + 7\sin\theta\sin 10°$	A1	or equiv
	Attempt division throughout by $\cos\theta\cos 10°$	M1	or by $\cos\theta$ (or $\cos 10°$) only
	Obtain $3t+3p=7+7pt$	A1	or equiv
	Obtain $\dfrac{3p-7}{7p-3}$	A1 **5**	or equiv
		$\boxed{12}$	

1	**(i)**	Attempt use of product rule	M1	producing … + … form
		Obtain $3x^2e^{2x}+2x^3e^{2x}$	A1 **2**	or equiv
	(ii)	Attempt use of chain rule to produce $\dfrac{kx}{3+2x^2}$ form	M1	any constant k
		Obtain $\dfrac{4x}{3+2x^2}$	A1 **2**	
	(iii)	Attempt use of quotient rule	M1	or equiv; condone u/v confusions
		Obtain $\dfrac{2x+1-2x}{(2x+1)^2}$ or $(2x+1)^{-1}-2x(2x+1)^{-2}$	A1 **2**	or (unsimplified) equiv

[If $...+c$ included in all three parts and all three parts otherwise correct, award M1A1, M1A1, M1A0; otherwise ignore any inclusion of $...+c$.]

$\boxed{6}$

2	**(i)**	Obtain one of $\pm\ln(\pm x\pm 4)$	M1	
		Obtain correct equation $y=-\ln(x-4)$	A1 **2**	or equiv; condone use of modulus signs instead of brackets
	(ii)	State, in any order, S, S and T	M1	or equiv such as S^2, T or 2S, T
		State T, then S, then S	A1 **2**	or equiv (note that S, S, T^9 and S, T^3, S are alternative correct answers)

$\boxed{4}$

3	**(i)**	Use $\operatorname{cosec}\theta=\dfrac{1}{\sin\theta}$	B1	
		Attempt to express equation in terms of $\sin\theta$	M1	using $\cos 2\theta=\pm1\pm2\sin^2\theta$ or equiv
		Obtain or clearly imply $6\sin^2\theta-11\sin\theta-10=0$	A1 **3**	or $-6\sin^2\theta+11\sin\theta+10=0$
	(ii)	Attempt solution to obtain at least one value of $\sin\theta$	M1	should be $s=-\frac{2}{3},\frac{5}{2}$
		Obtain –41.8	A1	allow –42 or greater accuracy
		Obtain –138	A1 **3**	or greater accuracy; and no others between –180 and 180
		[Answer(s) only: award 0 out of 3.]		

$\boxed{6}$

4	(i)	Either: Integrate to obtain $k\ln x$	B1		
		Use at least one relevant logarithm property	M1		
		Obtain $k\ln 3 = \ln 81$ and hence $k = 4$	A1	**3**	AG; accurate work required
		Or 1: (where solution involves no use of a logarithm property)			
		Integrate to obtain $k\ln x$	B1		
		Obtain correct explicit expression for k and conclude $k = 4$ with no error seen	B2	**3**	AG; e.g. $k = \frac{\ln 81}{\ln 6 - \ln 2} = 4$
		Or 2: (where solution involves verification of result by initial substitution of 4 for k)			
		Integrate to obtain $4\ln x$	B1		
		Use at least one relevant logarithm property	M1		
		Obtain $\ln 81$ legitimately with no error seen	A1	**3**	AG; accurate work required
	(ii)	State volume involves $\int \pi(\frac{4}{x})^2 \, dx$	B1		possibly implied
		Obtain integral of form $k_1 x^{-1}$	M1		any constant k_1 including π or not
		Use correct process for finding volume produced from S	M1		$\int (k_2 2^2 - k_3 y^2) \, dx$, including π or not with correct limits indicated; or equiv
		Obtain $16\pi - \frac{16}{3}\pi$ and hence $\frac{32}{3}\pi$	A1	**4**	or exact equiv
			$\boxed{7}$		

5	(i)	Attempt process for finding both critical values	M1		squaring both sides to obtain 3 terms on each side or considering 2 different linear eqns/inequalities
		Obtain -4	A1		
		Obtain $\frac{2}{3}$	A1		
		Attempt process for solving inequality	M1		table, sketch, …; needs two critical values; implied by plausible answer
		Obtain $-4 \le x \le \frac{2}{3}$	A1	**5**	with $\le$ and not $<$
	(ii)	Use correct process to find value of $\lvert x+2 \rvert$ using any value	M1		… whether part of answer to **(i)** or not
		Obtain $2\frac{2}{3}$ or $\frac{8}{3}$	A1	**2**	dependent on 5 marks awarded in part **(i)**
			$\boxed{7}$		

6	**(i)**	Attempt calculations involving 1.0 and 1.1	M1	using radians
		Obtain -0.57 and 0.76	A1	or values to 1 dp (rounded or truncated); or equivs (where eqn rearranged)
		Refer to sign change (or equiv for rearranged eqn)	A1 **3**	AG; following correct work only
	(ii)	Obtain correct first iterate	B1	using value x_1 such that $1.0 \le x_1 \le 1.1$
		Carry out iteration process	M1	obtaining at least 3 iterates in all so far
		Obtain at least 3 correct iterates	A1	showing at least 3 dp
		Obtain 1.05083	A1 **4**	answer required to exactly 5 d.p.
		[$1 \to 1.047198 \to 1.050571 \to 1.050809 \to 1.050826 \to 1.050827$; $1.05 \to 1.050769 \to 1.050823 \to 1.050827 \to 1.050827$; $1.1 \to 1.054268 \to 1.051070 \to 1.050844 \to 1.050829 \to 1.050827$]		
	(iii)	State or imply $\sec^2 2x = 1 + \tan^2 2x$	B1	
		Relate to earlier equation	M1	by halving or doubling answer to **(ii)** or carrying out equivalent iteration process
		Deduce $2x = 1.05083$ and hence 0.525	A1√ **3**	following their answer to **(ii)**; or greater accuracy
		[SC: Rearrange to obtain $x = \frac{1}{2}\cos^{-1}(2x+3)^{-\frac{1}{2}}$	B1	
		Use iterative process to obtain 0.525	B1 **2**	or greater accuracy]
			$\boxed{10}$	
7		Differentiate to obtain $k_1(3x-1)^3$	M1	any constant k_1
		Obtain correct $12(3x-1)^3$	A1	or (unsimplified) equiv
		Substitute 1 to obtain 96	A1	
		Attempt to find x-coordinate of Q	M1	using tangent with $y = 0$ or using gradient
		Obtain $\frac{5}{6}$	A1	or exact equiv
		Integrate to obtain $k_2(3x-1)^5$	M1	any constant k_2
		Obtain correct $\frac{1}{15}(3x-1)^5$	A1	or (unsimplified) equiv
		Use limits $\frac{1}{3}$ and 1 to obtain $\frac{32}{15}$	A1	
		Attempt to find shaded area by correct process	M1	integral – triangle or equiv
		Obtain ($\frac{32}{15} - \frac{1}{2} \times \frac{1}{6} \times 16$ and hence) $\frac{4}{5}$	A1	or equiv
			$\boxed{10}$	
8	**(i)**	Obtain $R = 3\sqrt{2}$ or $R = \sqrt{18}$ or $R = 4.24$	B1	or equiv
		Attempt to find value of α	M1	condone sin/cos muddles and degrees
		Obtain $\frac{1}{4}\pi$ or 0.785	A1 **3**	in radians now
	(ii) a	Equate $x - \alpha$ to $\frac{1}{2}\pi$ or attempt solution of $3\cos x + 3\sin x = 0$	M1	condone degrees here
		Obtain $\frac{3}{4}\pi$	A1 **2**	or $\ldots, -\frac{5}{4}\pi, -\frac{1}{4}\pi, \frac{7}{4}\pi, \ldots$; in radians now
	b	Attempt correct process to find value of $3x - \alpha$	*M1	with attempt at rearranging $\mathrm{T}(3x) = \frac{8}{9}\sqrt{6}$
		Obtain at least one correct exact value of $3x - \alpha$	A1	$\pm\frac{1}{6}\pi, \pm\frac{11}{6}\pi, \ldots$
		Attempt at least one positive value of x	M1	dep *M
		Obtain $\frac{1}{36}\pi$	A1 **4**	
			$\boxed{9}$	

9	**(i)**	Attempt to find x-coord of staty point or complete square	M1		
		Obtain $(\frac{3}{2}, -9)$ or $4(x-\frac{3}{2})^2 - 9$ or -9	A1		or equiv
		State $f(x) \geq -9$	A1	**3**	using any notation; with $\geq$
	(ii)	Make one correct (perhaps general) relevant statement	B1		not 1 –1 , f is many-one, … ; maybe implied if attempt is specific to this f
		Conclude with correct evidence related to this f	B1	**2**	AG; (more or less) correct sketch; correct relevant calculations, …
	(iii)	<u>Either</u>: Attempt to find expression for g^{-1}	*M1		or equiv
		Obtain $\frac{1}{a}(x-b)$	A1		or equiv
		Compare $\frac{1}{a}(x-b)$ and $ax+b$	M1		dep *M; by equating either coefficients of x or constant terms (or both); or substituting two non-zero values of x and solving eqns for a
		Obtain at least $-\frac{b}{a} = b$ and hence $a = -1$	A1	**4**	AG; necessary detail required; or equiv
		[SC1: first two steps as above, then substitute $a = -1$: max possible M1A1B1]			
		[SC2: substitute $a = -1$ at start: Attempt to find inverse M1 Obtain $-x+b$ and conclude A1 **2**]			
		<u>Or</u>: State or imply that $y = g^{-1}(x)$ is reflection of $y = g(x)$ in line $y = x$	B1		
		State that line unchanged by this reflection is perpendicular to $y = x$	M2		
		Conclude that a is –1	A1	**4**	
	(iv)	State or imply that $gf(x) = -(4x^2 - 12x) + b$	B1		
		Attempt use of discriminant or relate to range of f	M1		or equiv
		Obtain $64 + 16b < 0$ or $9 + b < 5$	A1		or equiv
		Obtain $b < -4$	A1	**4**	
				$\boxed{13}$	

1	Either: Obtain $\frac{1}{3}a$	B1		condone $\lvert x \rvert = \frac{1}{3}a$
	Attempt solution of linear eqn	M1		with signs of $3x$ and $5a$ different; allow M1 only if a given particular value and no recovery occurs; allow M1 only if a in terms of x attempted; allow M1 only if double inequality attempted but with no recovery to state actual values of x
	Obtain $-3a$	A1	**3**	as final answer
	Or: Obtain $9x^2 + 24ax + 16a^2 = 25a^2$	B1		
	Attempt solution of 3-term quad eqn	M1		as far as substitution into correct quadratic formula or correct factorisation of their quadratic; allow M1 only if a given particular value
	Obtain $-3a$ and $\frac{1}{3}a$	A1	**(3)**	or equivs; as final answers; and no others
			$\boxed{3}$	

2	Draw graph showing reflection in a horizontal axis	M1		
	Draw graph showing translation	M1		parallel to x-axis, in either direction; independent of first M1; not earned if curve still passes through O but ignore other coordinates given at this stage
	Draw (more or less) correct graph which must at least reach the negative x-axis, if not cross it, at left end of curve	A1		but ignoring no or wrong stretch in y-dir'n; condone graph existing only for $x < 0$; consider shape of curve and ignore coordinates given
	State $(-5, 24)$ and $(-3, 0)$ wherever located	B1	**4**	or clearly implied by sketch; allow for coordinates whatever sketch looks like; allow if in solution with no sketch
			$\boxed{4}$	

3	Either: State or imply $8\pi r$ as derivative	B1		or equiv
	Attempt to connect 12 and their derivative	M1		numerical or algebraic; using multiplication or division
	Obtain $8\pi \times 150 \times 12$ and hence 45000 or 14400π or 14000π	A1	**3**	or equiv; or greater accuracy (45239); condone absence of units or use of wrong units
	Or: Use $r = 12t$ to show $S = 576\pi t^2$	B1		
	Attempt $\frac{dS}{dt}$ and substitute for t	M1		
	Obtain $1152\pi \times \frac{150}{12}$ and hence 45000 or 14400π or 14000π	A1	**(3)**	or equiv; or greater accuracy (45239); condone absence of units or use of wrong units
			$\boxed{3}$	

4	**(i)**	Obtain $R = 25$	B1		allow $\sqrt{625}$ or value rounding to 25
		Attempt to find value of α	M1		implied by correct answer or its complement; allow sin/cos muddles; allow use of radians for this mark; condone $\sin\alpha = 7$, $\cos\alpha = 24$ in the working
		Obtain $16.3°$	A1	**3**	or greater accuracy 16.260…; must be degrees now; allow $16°$ here
	(ii)	Show correct process for finding one answer	M1		even if leading to answer outside 0 to 360
		Obtain (28.69 – 16.26 and hence) $12.4°$	A1		or greater accuracy 12.425… or anything rounding to 12.4
		Show correct process for finding second answer	M1		even if further incorrect answers produced
		Obtain (151.31 – 16.26 and hence) $135°$ or $135.1°$	A1	**4**	or greater accuracy 135.054…; and no other between 0 and 360
		[SC: No working shown and 2 correct angles stated - B1 only in part (ii)]			
				$\boxed{7}$	

5	Integrate to obtain form $k(3x-2)^{\frac{1}{2}}$	M1		any non-zero constant k; or equiv involving substitution
	Obtain correct $4(3x-2)^{\frac{1}{2}}$	A1		or (unsimplified) equiv such as $\dfrac{6(3x-2)^{\frac{1}{2}}}{3\times\frac{1}{2}}$
	Apply limits and attempt solution for a	M1		assuming integral of form $k(3x-2)^n$; taking solution as far as removal of root; with subtraction the right way round; if sub'n used, limits must be appropriate
	Obtain $a = 9$	A1		(this answer written down with no working scores 0/4 so far but all subsequent marks are available)
	State or imply formula $\int \dfrac{36\pi}{3x-2}\,\mathrm{d}x$	B1		or (unsimplified) equiv; condone absence of dx; allow B1 retroactively if π absent here but inserted later
	Integrate to obtain form $k\ln(3x-2)$	*M1		any constant k including π or not; condone absence of brackets
	Obtain $12\pi\ln(3x-2)$ or $12\ln(3x-2)$	A1√		following their integral of form $\int \frac{k}{3x-2}\,\mathrm{d}x$
	Apply limits the correct way round	M1		dep *M; use of limit 1 is implied by absence of second term; allow use of limit a
	Obtain $12\pi\ln 25$ (or $24\pi\ln 5$)	A1	**9**	or exact equiv but not with $\ln 1$ remaining; condone answers such as $\pi 12\ln 25$ and $12\ln 25\pi$
			$\boxed{9}$	

6	**(i)**	Attempt use of quotient rule	M1		or equiv; allow numerator wrong way round but needs minus sign in numerator; for M1 condone 'minor' errors such as sign slips, absence of square in denominator, and absence of some brackets
		Obtain $\frac{3(x^3-4x^2+2)-(3x+4)(3x^2-8x)}{(x^3-4x^2+2)^2}$	A1		or equiv; allow A1 if brackets absent from $3x+4$ term or from $3x^2-8x$ term but not from both
		Equate numerator to 0 and attempt simplification	M1		at least as far as removing brackets, condoning sign or coeff slips; or equiv
		Obtain $-6x^3+32x+6=0$ or equiv and hence $x=\sqrt[3]{\frac{16}{3}x+1}$	A1	**4**	AG; necessary detail needed (i.e. at least one intermediate step) and following first derivative with correct numerator
	(ii)	Obtain correct first iterate having used initial value 2.4	B1		showing at least 3 dp (2.398 or 2.399 or greater accuracy 2.39861…)
		Apply iterative process	M1		to obtain at least 3 iterates in all; implied by plausible, converging sequence of values; having started with any initial non-negative value
		Obtain at least 3 correct iterates from their starting point	A1		allowing recovery after error
		Obtain 2.398	A1		value required to exactly 3 dp
		Obtain –1.552	A1	**5**	value required to exactly 3 dp; allow if apparently obtained by substitution of 2.4; answers only with no iterates shown gets 0/5

[$2.4 \to 2.3986103 \to 2.3981808 \to 2.3980480$]

$\boxed{\mathbf{9}}$

7	**(i)**	State $\ln(x^2+8)=8$	B1		or equiv such as $x^2+8=e^8$
		Attempt solution involving e^8	M1		by valid (exact) method at least as far as $x^2=...$
		Obtain $\sqrt{e^8-8}$	A1	**3**	or exact equiv; and no other answer
	(ii)	State f only	B1		
		State e^x or e^y	B1		or equiv; allow if g, or f and g, chosen
		Indicate domain is all real numbers	B1	**3**	however expressed
	(iii)	Attempt use of chain rule	M1		whether applied to gf or fg; or equiv such as use of product rule on $(\ln x)(\ln x)+8$
		Obtain $\frac{2\ln x}{x}$	A1		or equiv
		Obtain $6e^{-3}$	A1	**3**	or exact equiv but not including ln
	(iv)	Attempt evaluation using y attempts	M1		with coeffs 1, 4 and 2 occurring at least once each; whether fg or gf
		Obn $k(\ln 24+4\ln 12+2\ln 8+4\ln 12+\ln 24)$	A1		any constant k
		Use $k=\frac{2}{3}$ and obtain 20.3	A1	**3**	or greater accuracy (20.26…) but must round to 20.3

[Note that use of Simpson's rule between 0 and 4 with two strips, coeffs 1, 4, 1, followed by doubling of result is equiv;
SC: Use of Simpson's rule between 0 and 4 with four strips followed by doubling of result - allow 3/3 - answer is 20.2 (20.2327…)]

$\boxed{12}$

8 **(a)** **(i)** Draw at least two correctly shaped branches, one for $y > 0$, one for $y < 0$	M1		otherwise located anywhere including $x < 0$
Draw four correct branches	M1		now (more or less) correctly located;
Draw (more or less) correct graph	A1	**3**	with some indication of horiz scale (perhaps only 4π indicated); with asymptotic behaviour shown (but not too fussy about branch drifting slightly away from asymptotic value nor about branch touching asymptote) but branches must not obviously cross asymptotic value; with –1 and 1 shown (or implied by presence of sine curve or by presence of only one of them on a reasonably accurate sketch); no need for vertical (dotted) lines drawn to indicate asymptotic values
(ii) State expression of form $k\pi + \alpha$ or $k\pi - \alpha$ or $\alpha = k\pi + \beta$ or $\alpha = k\pi - \beta$	M1		any non-zero numerical value of *k*; M0 if degrees used
State $3\pi - \alpha$	A1	**2**	or unsimplified equiv
(b) **(i)** State $\dfrac{2\tan\theta}{1-\tan^2\theta}$	B1	**1**	or equiv such as $\dfrac{t+t}{1-t\times t}$ or $\dfrac{2\tan A}{1-\tan^2 A}$
(ii) State or imply $\tan\phi = \frac{1}{4}$	B1		or equiv such as $\frac{1}{\tan\phi} = 4$
Attempt to evaluate $\tan 2\phi$ or $\cot 2\phi$	M1		perhaps within attempt at complete expression but using correct identity
Obtain $\tan 2\phi = \frac{8}{15}$ or $\cot 2\phi = \frac{15}{8}$	A1		or (unsimplified) equiv; may be implied
Attempt to evaluate value of $\tan 4\phi$	M1		perhaps within attempt at complete expression; condone only minor slip(s) in use of relevant identity
Obtain $\frac{240}{161}$	A1		or (unsimplified) exact equiv; may be implied
Obtain final answer $\frac{225}{322}$	A1	**6**	or exact equiv

[SC – (use of calculator and little or no working)

State or imply $\tan\phi = \frac{1}{4}$ B1; Obtain $\tan 2\phi = \frac{8}{15}$ B1; Obtain $\frac{225}{322}$ B1 (max 3/6)

State or imply $\tan\phi = \frac{1}{4}$ B1; Obtain $\frac{225}{322}$ B2 (max 3/6)

$\boxed{\mathbf{12}}$

9 (i) (a)	Differentiate to obtain $k_1e^{2x} + k_2e^{-2x}$	M1		any constants k_1 and k_2 but derivative must be different from f(x); condone presence of + c
	Obtain $2e^{2x} + 6e^{-2x}$	A1		or unsimplified equiv; no + c now
	Refer to $e^{2x} > 0$ and $e^{-2x} > 0$ or to more general comment about exponential functions	A1	**3**	or equiv (which might be sketch of $y = \mathrm{f}(x)$ with comment that gradient is positive or might be sketch of $y = \mathrm{f}'(x)$ with comment that $y > 0$; AG
(b)	Differentiate to obtain $k_3e^{2x} + k_4e^{-2x}$	M1		any constants k_3 and k_4 but second derivative must be different from their first derivative; condone presence of + c
	Obtain $4e^{2x} - 12e^{-2x}$	A1		or unsimplified equiv; no + c now
	Attempt solution of $\mathrm{f}''(x) > 0$ or of $\mathrm{f}(x) > 0$ or of corresponding eqn	M1		at least as far as term involving e^{4x} or e^{-4x}
	Obtain $x > \frac{1}{4}\ln 3$	A1		
	Confirm both give same result	B1	**5**	AG; necessary detail needed; either by solving the other or by observing that same inequality involved (just noting that $\mathrm{f}''(x) = 4\mathrm{f}(x)$ is sufficient)
(ii)	Differentiate to obtain $2e^{2x} - 2ke^{-2x}$	B1		or unsimplified equiv
	Attempt to find x-coordinate of stationary pt	M1		equating to 0 and reaching $e^{4x} = ...$ or equiv
	Obtain $e^{4x} = k$ and hence $\frac{1}{4}\ln k$ or equiv	A1		or equiv such as $e^{2x} = \sqrt{k}$
	Substitute and attempt simplification	M1		using valid processes but allow if only limited progress [note that question can be successfully concluded (without actually finding x) by substitution of $e^{2x} = \sqrt{k}$ and $e^{-2x} = \frac{1}{\sqrt{k}}$]
	Obtain $\mathrm{g}(x) \geq 2\sqrt{k}$ or $y \geq 2\sqrt{k}$	A1	**5**	or similarly simplified equiv with $\geq$ not $>$
			$\boxed{13}$	

1	**(i)**	Obtain integral of form ke^{2x+1}	M1		any non-zero constant k different from 6; using substitution $u = 2x+1$ to obtain ke^u earns M1 (but answer to be in terms of x)
		Obtain correct $3e^{2x+1}$	A1		or equiv such as $\frac{6}{2}e^{2x+1}$
	(ii)	Obtain integral of form $k_1\ln(2x+1)$	M1		any non-zero constant k_1; allow if brackets absent; $k_1 \ln u$ (after sub'n) earns M1
		Obtain correct $5\ln(2x+1)$	A1		or equiv such as $\frac{10}{2}\ln(2x+1)$; condone brackets rather than modulus signs but brackets or modulus signs must be present (so that $5\ln 2x+1$ earns A0)
		Include … + c at least once	B1	**5**	anywhere in the whole of question 1; this mark available even if no marks awarded for integration
				$\boxed{5}$	
2		Apply one of the transformations correctly to their equation	B1		
		Obtain correct $-3\ln x + \ln 4$	B1		or equiv
		Show at least one logarithm property	M1		correctly applied to their equation of resulting curve (even if errors have been made earlier)
		Obtain $y = \ln(4x^{-3})$	A1	**4**	or equiv of required form; $\ln 4x^{-3}$ earns A1; correct answer only earns 4/4; condone absence of $y =$
				$\boxed{4}$	
3	**(a)**	State $14\sin\alpha\cos\alpha = 3\sin\alpha$	B1		or unsimplified equiv such as $7(2\sin\alpha\cos\alpha) = 3\sin\alpha$
		Attempt to find value of $\cos\alpha$	M1		by valid process; may be implied
		Obtain $\frac{3}{14}$	A1	**3**	exact answer required; ignore subsequent work to find angle
	(b)	Attempt use of identity for $\cos 2\beta$	M1		of form $\pm 2\cos^2\beta \pm 1$; initial use of $\cos^2\beta - \sin^2\beta$ needs attempt to express $\sin^2\beta$ in terms of $\cos^2\beta$ to earn M1
		Obtain $6\cos^2\beta + 19\cos\beta + 10$	A1		or unsimplified equiv or equiv involving $\sec\beta$
		Attempt solution of 3-term quadratic eqn	M1		for $\cos\beta$ or (after adjustment) for $\sec\beta$
		Use $\sec\beta = \dfrac{1}{\cos\beta}$ at some stage	M1		or equiv
		Obtain $-\frac{3}{2}$	A1	**5**	or equiv; and (finally) no other answer
				$\boxed{8}$	

4	**(i)**	Draw sketch of $y=(x-2)^4$	*B1		touching positive x-axis and extending at least as far as the y-axis; no need for 2 or 16 to be marked; ignore wrong intercepts
		Draw straight line with positive gradient	*B1		at least in first quadrant and reaching positive y-axis; assess the two graphs independently of each other
		Indicate two roots	B1	**3**	AG; dep *B *B and two correct graphs which meet on the y-axis; indicated in words or by marks on sketch
		[SC: Draw sketch of $y=(x-2)^4-x-16$ and indicate the two roots : B1 (i.e. max 1 mark)]			
	(ii)	State 0 or $x=0$	B1	**1**	not merely for coordinates (0, 16)
	(iii)	Obtain correct first iterate	B1		to at least 3 dp; any starting value (> –16)
		Show correct iteration process	M1		producing at least 3 iterates in all; may be implied by plausible converging values
		Obtain at least 3 correct iterates	A1		allowing recovery after error; iterates given to only 3 d.p. acceptable; values may be rounded or truncated
		Obtain 4.118	A1	**4**	answer required to exactly 3 dp; A0 here if number of iterates is not enough to justify 4.118; attempt consisting of answer only earns 0/4

$$\begin{array}{l}[\,0 \to 4 \to 4.114743 \to 4.117769 \to 4.117849\,; \\ 1 \to 4.030543 \to 4.115549 \to 4.117790 \to 4.117849\,; \\ 2 \to 4.059767 \to 4.116321 \to 4.117811 \to 4.117850\,; \\ 3 \to 4.087798 \to 4.117060 \to 4.117830 \to 4.117850\,; \\ 4 \to 4.114743 \to 4.117769 \to 4.117849 \to 4.117851\,; \\ 5 \to 4.140695 \to 4.118452 \to 4.117867 \to 4.117851\,]\end{array}$$

$\boxed{8}$

5	Attempt use of product rule	*M1		to produce $k_1 x\ln(4x-3)+\dfrac{k_2x^2}{4x-3}$ form
	Obtain $2x\ln(4x-3)$	A1		
	Obtain $\ldots+\dfrac{4x^2}{4x-3}$	A1		or equiv
	Attempt second use of product rule	*M1		
	Attempt use of quotient (or product) rule	*M1		allow numerator the wrong way round
	Obtain $2\ln(4x-3)+\dfrac{8x}{4x-3}+\dfrac{8x(4x-3)-16x^2}{(4x-3)^2}$	A1		or equiv
	Substitute 2 into attempt at second deriv	M1		dep *M *M *M
	Obtain $2\ln 5+\frac{96}{25}$	A1	**8**	or exact equiv consisting of two terms

$\boxed{8}$

6 Method 1: (Differentiation; assume value $\frac{10}{3}$; eqn of tangent; through origin)

Differentiate to obtain $k(3x-5)^{-\frac{1}{2}}$	M1	any constant k
Obtain $\frac{3}{2}(3x-5)^{-\frac{1}{2}}$	A1	or equiv
Attempt to find equation of tangent at P and attempt to show tangent passing through origin	M1	assuming value $\frac{10}{3}$; or equiv
Obtain $y=\frac{3}{2\sqrt{5}}x$ and confirm that tangent passes through O	A1	AG; necessary detail needed

Method 2: (Differentiation; equate $\frac{y\text{ change}}{x\text{ change}}$ to deriv; solve for x)

Differentiate to obtain $k(3x-5)^{-\frac{1}{2}}$	M1	any constant k
Obtain $\frac{3}{2}(3x-5)^{-\frac{1}{2}}$	A1	or equiv
Equate $\frac{y\text{ change}}{x\text{ change}}$ to deriv and attempt solution	M1	
Obtain $\frac{\sqrt{3x-5}}{x}=\frac{3}{2}(3x-5)^{-\frac{1}{2}}$ and solve to obtain $\frac{10}{3}$ only	A1	

Method 3: (Differentiation; find x from $y=\mathrm{f}'(x)\,x$ and $y=\sqrt{3x-5}$)

Differentiate to obtain $k(3x-5)^{-\frac{1}{2}}$	M1	any constant k
Obtain $\frac{3}{2}(3x-5)^{-\frac{1}{2}}$	A1	or equiv
State $y=\frac{3}{2}(3x-5)^{-\frac{1}{2}}x$, $y=\sqrt{3x-5}$, eliminate y and attempt solution	M1	condone this attempt at 'eqn of tangent'
Obtain $\frac{10}{3}$ only	A1	

Method 4: (No differentiation; general line through origin to meet curve at one point only)

Eliminate y from equations $y=kx$ and $y=\sqrt{3x-5}$ and attempt formation of quadratic eqn	M1	
Obtain $k^2x^2-3x+5=0$	A1	or equiv
Equate discriminant to zero to find k	M1	
Obtain $k=\frac{3}{2\sqrt{5}}$ or equiv and confirm $x=\frac{10}{3}$	A1	

Method 5: (No differentiation; use coords of P to find eqn of OP; confirm meets curve once)

Use coordinates $(\frac{10}{3},\sqrt{5})$ to obtain $y=\frac{3\sqrt{5}}{10}x$ or equiv as equation of OP	B1	
Eliminate y from this eqn and eqn of curve and attempt quadratic eqn	M1	should be $9x^2-60x+100=0$ or equiv
Attempt solution or attempt discriminant	M1	
Confirm $\frac{10}{3}$ only or discriminant = 0	A1	

Either:			
Integrate to obtain $k(3x-5)^{\frac{3}{2}}$	*M1		any constant k
Obtain correct $\frac{2}{9}(3x-5)^{\frac{3}{2}}$	A1		
Apply limits $\frac{5}{3}$ and $\frac{10}{3}$	M1		dep *M; the right way round
Make sound attempt at triangle area and calculate (triangle area) minus (their area under curve)	M1		or equiv
Obtain $\frac{10}{6}\sqrt{5}-\frac{10}{9}\sqrt{5}$ and hence $\frac{5}{9}\sqrt{5}$	A1	**9**	or exact equiv involving single term
Or:			
Arrange to $x = \ldots$ and integrate to obtain $k_1y^3+k_2y$ form	*M1		
Obtain $\frac{1}{9}y^3+\frac{5}{3}y$	A1		
Apply limits 0 and $\sqrt{5}$	M1		dep *M; the right way round
Make sound attempt at triangle area and calculate (their area from integration) minus (triangle area)	M1		
Obtain $\frac{20}{9}\sqrt{5}-\frac{5}{3}\sqrt{5}$ and hence $\frac{5}{9}\sqrt{5}$	A1	(**9**)	or exact equiv involving single term
		$\boxed{\mathbf{9}}$	

7	**(i)** Either: Attempt solution of at least one linear eq'n of form $ax+b=12$	M1		
	Obtain $\frac{1}{3}$	A2	**3**	and (finally) no other answer
	Or: Attempt solution of 3-term quadratic eq'n obtained by squaring attempt at $\mathrm{g}(x+2)$ on LHS and squaring 12 or –12 on RHS	M1		
	Obtain $\frac{1}{3}$	A2	(**3**)	and (finally) no other answer
	(ii) Either: Obtain $3(3x+5)+5$ for h	B1		
	Attempt to find inverse function	M1		of function of form $ax+b$
	Obtain $\frac{1}{9}(x-20)$	A1	**3**	or equiv in terms of x
	Or: State or imply g^{-1} is $\frac{1}{3}(x-5)$	B1		
	Attempt composition of g^{-1} with g^{-1}	M1		
	Obtain $\frac{1}{9}(x-5)-\frac{5}{3}$	A1	(**3**)	or more simplified equiv in terms of x
	(iii) State $x \le 0$	B2	**2**	give B1 for answer $x<0$
			$\boxed{\mathbf{8}}$	

8	**(i)**	Differentiate to obtain form $ke^{-0.014t}$	M1		any constant k different from 400
		Obtain $5.6e^{-0.014t}$ or $-5.6e^{-0.014t}$	A1		or (unsimplified) equiv
		Obtain 4.9 or –4.9 or 4.87 or –4.87	A1	**3**	but not greater accuracy; allow if final statement seems contradictory; answer only earns 0/3 – differentiation is needed
	(ii)	Either: State or imply $M_2 = 75e^{kt}$	B1		or equiv
		Attempt to find formula for M_2	M1		
		Obtain $M_2 = 75e^{0.047t}$	A1		or equiv such as $75e^{(\frac{1}{10}\ln\frac{8}{5})t}$
		Equate masses and attempt rearrangement	M1		as far as equation with e appearing once
		Obtain $e^{0.061t} = \frac{16}{3}$	A1	**5**	or equiv of required form which might involve 5.33 or greater accuracy on RHS; final two marks might be earned in part iii
		Or: State or imply $M_2 = 75 \times r^{0.1t}$	B1		for positive value r
		Obtain $75 \times 1.6^{0.1t}$	B1		
		Attempt to find M_2 in terms of e	M1		
		Equate masses and attempt rearrangement	M1		
		Obtain $e^{0.061t} = \frac{16}{3}$	A1	**5**	or equiv of required form which might involve 5.33 or greater accuracy on RHS; final two marks might be earned in part iii
	(iii)	Attempt solution involving logarithm of any equation of form $e^{mt} = c_1$	M1		whether the conclusion of part ii or not
		Obtain 27.4	A1	**2**	or greater accuracy 27.4422…; correct answer only earns both marks
				$\boxed{10}$	

9	(i)	Use at least one identity correctly	B1		angle-sum or angle-difference identity
		Attempt use of relevant identities in single rational expression	M1		not earned if identities used in expression where step equiv to $\frac{A+B+C}{D+E+F}=\frac{A}{D}+\frac{B}{E}+\frac{C}{F}$ or similar has been carried out; condone (for M1A1) if signs of identities apparently switched (so that, for example, denominator appears as $\cos\theta\cos\alpha-\sin\theta\sin\alpha+3\cos\theta+\cos\theta\cos\alpha+\sin\theta\sin\alpha$)
		Obtain $\frac{2\sin\theta\cos\alpha+3\sin\theta}{2\cos\theta\cos\alpha+3\cos\theta}$	A1		or equiv but with the other two terms from each of num'r and den'r absent
		Attempt factorisation of num'r and den'r	M1		
		Obtain $\frac{\sin\theta}{\cos\theta}$ and hence $\tan\theta$	A1	5	AG; necessary detail needed
	(ii)	State or imply form $k\tan 150°$	M1		obtained without any wrong method seen
		State or imply $\frac{4}{3}\tan 150°$	A1		or equiv such as $\frac{12\sin 150°}{9\cos 150°}$
		Obtain $-\frac{4}{9}\sqrt{3}$	A1	3	or exact equiv (such as $-\frac{4}{3\sqrt{3}}$); correct answer only earns 3/3
	(iii)	State or imply $\tan 6\theta = k$	B1		
		State $\frac{1}{6}\tan^{-1}k$	B1		
		Attempt second value of θ	M1		using $6\theta = \tan^{-1}k +$ (multiple of 180)
		Obtain $\frac{1}{6}\tan^{-1}k + 30°$	A1	4	and no other value
				$\boxed{12}$	

Question			Answer	Marks	Guidance	
1			State $2\ln x$ Use both relevant logarithm properties correctly Obtain $\ln 3$	B1 M1 A1 **[3]**	may be implied by immediate use of limits either or both may be implied, eg by $2\ln\sqrt{6} = \ln 6$ or by $\ln 6 - \ln 2 = \ln 3$ AG; with at least one property shown explicitly	
2			State volume is $\int \frac{36\pi}{(2x+1)^4}\,dx$ Obtain integral of form $k(2x+1)^n$ Obtain $-6\pi(2x+1)^{-3}$ or $-6(2x+1)^{-3}$ Substitute correct limits and subtract Obtain $\frac{52}{9}\pi$	B1 M1 A1 M1 A1 **[5]**	or equiv in terms of x; no need for limits; condone absence of dx; condone absence of π here if it appears later in solution (even as part of a wrong answer) for any $n \le -1$; with or without π; or ku^n following substitution; allow if $n = -5$; allow M1 if one slight slip occurs in $(2x+1)$ or (unsimplified) equiv the correct way round for integral of form $k(2x+1)^{-3}$; allow if one slight slip occurs in $(2x+1)$; not earned if limit 0 leads to $... - 0$ or similarly simplified exact equiv	

Question			Answer	Marks	Guidance	
3			Attempt use of quotient rule	M1	condone *u*/*v* muddles but needs $(x+2)^2$ in denominator; condone numerator back to front; or product rule to produce terms involving $(x+2)^{-1}$ and $(x+2)^{-2}$	
			Obtain $\dfrac{2x(x+2)-(x^2+4)}{(x+2)^2}$	A1	or equiv; brackets may be implied by subsequent recovery	
			Substitute 1 into attempt at first derivative	M1	also allow if sign slip leads to derivative cancelling to 1	
			Obtain $\frac{1}{9}$	A1		
			Use –9 as gradient of normal	A1ft	following their value of first derivative	
			Attempt to find equation of normal	M1	not equation of tangent; needs use of negative reciprocal of their derivative value	
			Obtain $27x+3y-32=0$	A1	or equiv of requested form	
				[7]		
4	(i)		State $\tan\alpha=2$	B1	ignoring subsequent work to find angle	
			Use identity $\sec^2\beta=1+\tan^2\beta$	B1		
			Attempt solution of quad eqn for $\tan\beta$	M1	3 term quad eqn; using reasonable attempt at factorisation to find value or use of quadratic formula (with no more than one slip)	
			Obtain $\tan\beta=5$	A1	ignoring subsequent work to find angle; value 5 must be obtained legitimately	
				[4]		

Question			Answer	Marks	Guidance	
4	(ii)		Substitute their values of $\tan\alpha$ and $\tan\beta$ in formula … Obtain $\frac{2+5}{1-2\times5}$ Obtain $-\frac{7}{9}$	M1 A1ft A1 **[3]**	… of form $\frac{\pm\tan\alpha\pm\tan\beta}{\pm1\pm\tan\alpha\tan\beta}$ following their values from part **(i)** or correct simplified exact equiv including $\frac{7}{-9}$; A0 if $\tan\beta=5$ obtained incorrectly in part **(i)** SC: use of calculator for $\tan(\tan^{-1}2+\tan^{-1}5)$ to give $-\frac{7}{9}$ earns all 3 marks (but 0 out of 3 if answer is not exact); with either or both of 2 and 5 wrong, 2 out of 3 available for this approach if result is exact and correct given their two values	
5	(i)		State 26 State 4	B1 B1 **[2]**		
5	(ii)		Sketch (more or less) correct curve Refer to reflection in $y=x$ or symmetrical about $y=x$ or mirrored in $y=x$	B1 B1 **[2]**	with approx correct curvatures and curve going through second quadrant but not fourth quadrant; allow if sketch does not meet given curve on line $y=x$ explicit reference needed, not just line $y=x$ shown on sketch	

Question			Answer	Marks	Guidance	
5	(iii)		Attempt calculation $k(y+4y+2y+...)$ Obtain $k(1+32+28+76+46+100+26)$ Use $k=\frac{1}{3}\times 2$ Obtain 206	M1 A1 A1 A1 **[4]**	any constant *k*; with *y*-values from table and coefficients 1, 2 and 4 occurring at least once each; brackets may be implied by subsequent calculation or (unsimplified) equiv	
6	(i)		Obtain rational expression of form $\frac{\mathrm{f}(y)}{y^3+2y}$ Obtain $\frac{3y^2+2}{y^3+2y}$	M1 A1 **[2]**	where $\mathrm{f}(y)$ is not constant; ignore how expression is labelled	
6	(ii)		Recognise that $\frac{\mathrm{d}y}{\mathrm{d}x}=1\div\frac{\mathrm{d}x}{\mathrm{d}y}$ for rational expression of form $\frac{\mathrm{f}(y)}{y^3+2y}$ Obtain $\frac{y^3+2y}{3y^2+2}=4$ or $\frac{3y^2+2}{y^3+2y}=\frac{1}{4}$ Confirm $y=\frac{12y^2+8}{y^2+2}$	M1 A1ft A1 **[3]**	may be implied following their rational expression from **(i)** AG; following correct work and with at least one step between $\frac{y^3+2y}{3y^2+2}=4$ or equiv and answer	

Question			Answer	Marks	Guidance	
6	(iii)		Obtain correct first iterate 11.89	B1	or greater accuracy; having started with 12; accept if 12 used in part **(ii)** to produce next value and 11.89 used as starting value here	
			Attempt iteration process to produce at least 3 iterates in all	M1	implied by plausible sequence of values; having started anywhere; if formula clearly not based on equation from part **(ii)**, award M0	
			Obtain at least 2 more correct iterates	A1	showing at least 3 decimal places	
			Obtain 11.888 for y	A1	answer needed to exactly 3 decimal places	
			Obtain 7.441 for x	A1	answer needed to exactly 3 decimal places; award final A0 if not clear which is x and which is y [12 $\rightarrow$ 11.89041 $\rightarrow$ 11.88841 $\rightarrow$ 11.88837]	
				[5]		

Question			Answer	Marks	Guidance	
7	(i)	(a)	State or imply $e^{-0.132t} = 0.25$ Attempt solution of eqn of form $e^{-0.132t} = k$ Obtain 10.5	B1 M1 A1 **[3]**	or equiv such as $40e^{-0.132t} = 10$ using sound process; implied by correct ans; allow trial and improvement attempt or greater accuracy	
7	(i)	(b)	Differentiate to obtain $ke^{-0.132t}$ Obtain $5.28e^{-0.132t}$ or $-5.28e^{-0.132t}$ Substitute 5 to obtain 2.73 or –2.73	M1 A1 A1 **[3]**	where k is a constant not equal to 40 (allow even if process looks like integration) or (unsimplified) equiv accept 2.7 or –2.7 or greater accuracy; allow 2.73 or –2.73 whatever it is claimed to be	
7	(ii)		<u>EITHER</u> Attempt to solve $40e^{2\lambda} = 31.4$ or $40e^{-2\lambda} = 31.4$ Obtain or imply $40e^{-0.121t}$ Substitute 3 to obtain 27.8 <u>OR</u> Attempt calculation involving multiplication of power of $\frac{31.4}{40}$ Obtain $31.4\times(\frac{31.4}{40})^{0.5}$ or $40\times(\frac{31.4}{40})^{1.5}$ Obtain 27.8	 M1 A1 A1 **[3]** M1 A1 A1	 using sound process; method implied by correct formula for mass of B obtained or greater accuracy (–0.12103..) or $0.5\ln 0.785$ accept 28 or greater accuracy accept 28 or greater accuracy	

Question			Answer	Marks	Guidance	
8	(i)		State $\cos 4\theta = 1 - 2\sin^2 2\theta$ State or clearly imply $\sin 2\theta = 2\sin\theta\cos\theta$ Obtain $1 - 8\sin^2\theta\cos^2\theta$	B1 B1 B1 **[3]**	 possibly substituted in incorrect expression	
8	(ii)		Produce expression involving $\cos\frac{4}{24}\pi$ as only trigonometrical ratio Obtain $\frac{1}{8} - \frac{1}{16}\sqrt{3}$	M1 A1 **[2]**	or exact equiv (including, eg $\frac{1-\frac{1}{2}\sqrt{3}}{8}$)	
8	(iii)		Use $2\cos^2 2\theta = 1 + \cos 4\theta$ Attempt to express in terms of $\cos 4\theta$ Obtain $\frac{2}{3} + \frac{4}{3}\cos 4\theta$ Substitute at least one of –1 and 1 for $\cos 4\theta$ in expression where $\cos 4\theta$ is only trigonometrical ratio Obtain 2 and $-\frac{2}{3}$	B1 M1 A1 M1 A1 **[5]**	or use $2\cos^2 2\theta = 2 - 8\sin^2\theta\cos^2\theta$ or unsimplified equiv or at least one of $\theta = \frac{1}{4}\pi$ and $\theta = 0$	

Question			Answer	Marks	Guidance	
9	(i)		Attempt differentiation to find x-coordinate of stationary point or attempt completion of square as far as $(x+...)^2$	M1	or equiv; first two marks of part **(i)** may be earned by work seen in part **(ii)**; $x=-2$ only stated earns M1A1	
			Obtain $x=-2$ or $(x+2)^2$	A1	first two marks of part **(i)** are implied by correct answer to translation in x-direction	
			State translation by 2 in negative x-direction	A1	or (clear) equiv; allow correct vector	
			State translation by 4 in negative y-direction	A1	or (clear) equiv; allow correct vector	
			State stretch parallel to y-axis, scale factor k	B1 **[5]**	or equiv at least mentioning y and k	
9	(ii)		State one of $y<4k,\ y\le 4k,\ y<-4k,\ y\le -4k$ $y>4k,\ y\ge 4k,\ y>-4k,\ y\ge -4k$	B1		
			State $y\ge -4k$	B1 **[2]**	allow alternative notation such as $\mathrm{f}(x)\ge -4k$ or range $\ge -4k$	
9	(iii)		Attempt to relate y-value involving k at their stationary point to 20 or –20 or consider discriminant of $k(x^2+4x)=20$ or of $k(x^2+4x)=-20$	*M1		
			Obtain $k=5$	A1	earned unless there is clear evidence of error in working	
			State one root $x=-2$	B1		
			Attempt solution of $k(x^2+4x)=20$	M1	dep *M; for their value of k provided positive	
			Obtain $\dfrac{-4\pm\sqrt{32}}{2}$	A1ft	or (unsimplified) exact equivs; following their value of k	
			Obtain $-2\pm 2\sqrt{2}$ or $-2\pm\sqrt{8}$	A1 **[6]**	dependent on previous A1 A1ft marks being awarded	

Question			Answer	Marks	Guidance	
1			Attempt process for finding critical values	M1	squaring both sides, 2 linear eqns, ineqs, …	If using quadratic, need to go as far as factorising or substituting in formula for M1; if using two linear eqns or ineqs, signs of $2x$ and x must be same in one, different in the other for M1
			Obtain $\frac{4}{3}$	A1		
			Obtain 6	A1		
			Attempt process for inequality involving two critical values	M1	sketch, table, …; implied by plausible soln	
			Obtain $x < \frac{4}{3}$, $x > 6$	A1	A0 for use of $\le$ and/or $\ge$	
				[5]		
2	(i)		<u>EITHER</u> Attempt use of at least one logarithm property correctly applied to $\ln(\frac{ep^2}{q})$	M1	not including $\ln e = 1$; such as $\ldots = \ln ep^2 - \ln q$ for example	
			Obtain 261 legitimately with necessary detail seen	A2	AG; award A1 if nothing wrong but not quite enough detail or if there is one slip on way to 261	
				[3]		
			<u>OR</u> Express $\dfrac{ep^2}{q}$ in form e^n	M1	with correct treatment of powers	
			Obtain e^{261} and hence 261	A2	AG; award A1 if nothing wrong but not quite enough detail to be fully convincing	
2	(ii)		Introduce logarithms and bring power down	M1	relating $n\ln 5$ to a constant; if using base 5 or base 10, no powers must remain on right-hand side	
			Obtain $n\ln 5 > 580$	A1	or equiv (such as $n > 580\log_5 e$ or $n\log 5 > 580\log e$); allow eqn at this stage	
			State single integer 361	A1	not $n > 360$ nor $n \ge 361$	
				[3]		

Question			Answer	Marks	Guidance	
3	(i)		Use $\sec\theta = \dfrac{1}{\cos\theta}$	B1		
			Attempt to express in terms of $\tan\theta$ only	M1		
			Obtain $\tan^2\theta = 36$ and hence $\tan\theta = 6$	A1	AG; necessary detail needed (but no need to justify exclusion of $\tan\theta = -6$)	
				[3]		
3	(ii)	(a)	Substitute 6 in attempt at formula	M1	of form $\dfrac{\tan\theta \pm \tan 45°}{1 \mp \tan\theta \tan 45°}$ with different signs in numerator and denominator	any apparent use of angle 80.5.. means M0
			Obtain $\frac{5}{7}$	A1	or exact equiv	answer only: 0/2
				[2]		
3	(ii)	(b)	Substitute 6 in attempt at formula	M1	of form $\dfrac{\tan\theta + \tan\theta}{1 \pm \tan\theta \tan\theta}$	any apparent use of angle 80.5.. means M0
			Obtain $-\frac{12}{35}$	A1	or exact equiv; allow $\frac{12}{-35}$	answer only: 0/2
				[2]		
4	(a)		Obtain integral of form $k(6x+1)^{\frac{1}{2}}$	*M1	any constant k	
			Obtain $6(6x+1)^{\frac{1}{2}}$	A1	or (unsimplified) equiv	
			Substitute both limits and subtract	M1	dep *M	
			Obtain 30 – 6 and hence 24	A1	AG; necessary detail needed	
				[4]		
4	(b)		Attempt expansion of integrand	M1	to obtain (at least) 3 terms	
			Integrate e^{kx} to obtain $\frac{1}{k}e^{kx}$	M1	for any constant k other than 1	
			Obtain $\frac{1}{2}e^{2x} + 4e^{x} + 4x$	A1	allow $+c$ at this stage	
			Obtain $\frac{1}{2}e^{2} + 4e - \frac{1}{2}$	A1	or equiv in terms of e simplified to three terms; no $+c$ now	
				[4]		

Question			Answer	Marks	Guidance
5	(i)		Sketch (more or less) correct $y = 14 - x^2$	B1	assessed separately from other graph; must exist in all four quadrants; ignore any intercepts given
			Sketch (more or less) correct $y = k \ln x$	B1	assessed separately from other graph; must exist in first and fourth quadrants; if clearly meets y-axis award B0; if clear maximum point in first quadrant award B0
			Indicate one root ('blob' on sketch or written reference to one intersection or …)	B1	dependent on both curves being correct in first quadrant and there being no possibility, from their graphs, of further points of intersection elsewhere
				[3]	
5	(ii)	(a)	Calculate values for at least 2 integers	M1	
			Obtain correct values for $x = 3$ and $x = 4$	A1	$14 - x^2 - 3\ln x$: 1.7 – 6.2 $14 - x^2$, $3\ln x$: 5, 3.3 –2, 4.2
			State 3 and 4	A1	following correct calculations
				[3]	
5	(ii)	(b)	Obtain correct first iterate	B1	having started with any positive value; B1 available if 'iteration' never goes beyond a first iterate;
			Attempt iteration process	M1	implied by plausible sequence of values
			Obtain at least 3 correct iterates in all	A1	showing at least 2 d.p.
			Obtain 3.24	A1	answer required to exactly 2 d.p; not given for 3.24 as the final iterate in a sequence, i.e. needs an indication (perhaps just underlining) that value of α found [$3 \to 3.27172 \to 3.23173 \to 3.23743 \to 3.23661$ $3.5 \to 3.20027 \to 3.24196 \to 3.23596 \to 3.23682$ $4 \to 3.13706 \to 3.25118 \to 3.23465 \to 3.23701$]
				[4]	

Question			Answer	Marks	Guidance
6	(i)		Attempt use of chain rule	*M1	to obtain derivative of form $kh(3h^2+4)^n$, any non-zero constants k and n
			Obtain $9h(3h^2+4)^{\frac{1}{2}}$	A1	condone retention of -8 or (unsimplified) equiv; no – 8 here
			Substitute 0.6 in attempt at first derivative	M1	dep *M; condone retention of – 8 here; implied by their value following wrong derivative if no working seen
			Obtain 12.17	A1	or greater accuracy
				[4]	
6	(ii)		State or imply that $\frac{dh}{dt}=-0.015$ or 0.015	B1	implied by use in calculation with part (i) answer
			Carry out multiplication of $(\pm)0.015$ and answer from part (i)	M1	
			Obtain 0.18 or -0.18 (whatever this value is claimed to be)	A1	or greater accuracy; condone absence or misuse of negative signs throughout; ignore units; allow for answer rounding to 0.18 following slight inaccuracy due to use of 12.18 or 12.2 or …
				[3]	
7			Show composition of functions	M1	the right way round; or equiv
			Obtain $2\sqrt[3]{12-a}+5=9$	A1	or equiv
			Obtain $a=4$	A1	
			<u>EITHER</u>		
			Attempt to find $\mathrm{g}(x)$	*M1	obtaining px^3+q or py^3+q form
			Obtain $(2x+5)^3+4=68$	A1ft	following their value of a
			Attempt solution of equation	M1	dep *M; earned at stage $2x+5=...$; if expanding to produce cubic equation, earned with attempt at linear and quadratic factors
			Obtain $-\frac{1}{2}$	A1	and no others; dependent on correct work throughout
				[7]	
			<u>OR</u>		
			State or imply $\mathrm{f}(x)=\mathrm{g}^{-1}(68)$	B2	
			Attempt solution of equation of form $2x+5=\sqrt[3]{68-4}$	M1	
			Obtain $-\frac{1}{2}$	A1	

Question			Answer	Marks	Guidance	
8	(i)		State $R = 5$ Attempt to find value of α Obtain 53.1	B1 M1 A1 **[3]**	 implied by correct value or its complement allow $\tan^{-1}\frac{4}{3}$	
8	(ii)	(a)	Attempt to find at least one value of $\theta + \alpha$ Obtain 1 correct value of θ (–64.7 or 138) Attempt correct process to find the second value Obtain second value of θ (138 or –64.7)	M1 A1 M1 A1 **[4]**	(should be –168.5 or –11.5 or 191.5 or …) allow ± 0.1 in answer and greater accuracy involving a positive value of $\sin^{-1}(-\frac{1}{5})$ and subtraction of their α allow ± 0.1 in answer and greater accuracy; and no others between –180 and 180	 note that 138 needs to be obtained legitimately from positive value of $\sin^{-1}(-\frac{1}{5})$ and not from $180 - 41.6$ answers only: 0/4
8	(ii)	(b)	Use –1 as minimum or 1 as maximum value of $\sin(\theta + \alpha)$ Relate $-5k + c$ to -37 and $5k + c$ to 43 Attempt solution of pair of linear eqns Obtain $k = 8$ and $c = 3$	 *M1 A1 M1 A1 **[4]**	 as equations or inequalities dep *M; must be equations now **SC**: both $k = 8$ and $c = 3$ obtained with no working or from unconvincing working, award B2 (i.e. max 2/4)	 Note that alternative solutions may occur. If mathematically sound, all 4 marks are available; if work is not fully convincing, apply SC

Question			Answer	Marks	Guidance	
9	(i)		Attempt use of product rule to produce the form $\ln 2y + y \times \frac{a}{by}$	M1		Note that product rule may be applied to expression in form $y(\ln 2y - 1)$
			Obtain correct $\ln 2y + y \times \frac{2}{2y}$ ...	A1	or equiv	
			Obtain complete $\ln 2y + 1 - 1$ and confirm	A1	AG; necessary detail needed	
				[3]		
9	(ii)		Attempt to rearrange eqn to $x = \ldots$ or $x^2 = \ldots$	M1	obtaining form $p \ln qy$	
			Obtain $x = \sqrt{\ln 2y}$ or $x^2 = \ln 2y$	A1		
			State or imply volume is $\int \pi \ln 2y \, dy$	A1ft	following their $x = \ldots$ or $x^2 = \ldots$; condone absence of dy; condone presence of dx; no need for limits here; π may be implied by its first appearance later in solution	
			Integrate using result of part (i)	M1		
			Attempt to use limits $\frac{1}{2}$ and $\frac{1}{2}e^4$ correctly with expression involving y	M1		
			Obtain $\frac{1}{2}\pi(3e^4 + 1)$	A1	or equiv involving two terms; dependent on correct work throughout part (ii)	
				[6]		
9	(iii)		Subtract answer to part (ii) from $2\pi e^4$ …	M1	… or its decimal equivalent	
			Obtain $\frac{1}{2}\pi(e^4 - 1)$	A1	or exact equiv involving two terms	
				[2]		

Question			Answer	Marks	Guidance
1	(i)		Either Attempt use of quotient rule	M1	allow numerator wrong way round but needs minus sign in numerator and both terms in numerator involving x; for M1 condone minor errors such as absence of square in denominator, absence of brackets, …
			Obtain $\dfrac{3(2x+1)-6x}{(2x+1)^2}$ or equiv	A1	give A0 if necessary brackets absent unless subsequent calculation indicates their 'presence'
			Substitute 2 to obtain $\frac{3}{25}$ or 0.12	A1	or simplified equiv but A0 for final $\frac{3}{5^2}$
				[3]	
			Or Attempt use of product rule for $3x(2x+1)^{-1}$	M1	allow sign error; condone no use of chain rule
			Obtain $3(2x+1)^{-1}-6x(2x+1)^{-2}$ or equiv	A1	
			Substitute 2 to obtain $\frac{3}{25}$ or 0.12	A1	or simplified equiv
1	(ii)		Differentiate to obtain form $kx(4x^2+9)^n$	M1	any non-zero constants k and n (including 1 or $\frac{1}{2}$ for n)
			Obtain $4x(4x^2+9)^{-\frac{1}{2}}$	A1	or (unsimplified) equiv
			Substitute 2 to obtain $\frac{8}{5}$ or 1.6	A1	or simplified equiv but A0 for final $\frac{8}{\sqrt{25}}$
				[3]	
2	(i)		Either Attempt to find exact value of $\sin A$	M1	using right-angled triangle or identity or …
			Obtain $\frac{1}{2}\sqrt{5}$ or $\sqrt{\frac{5}{4}}$ or exact equiv	A1	final $\pm\frac{1}{2}\sqrt{5}$ is A0; correct answer only earns M1A1
				[2]	
			Or Attempt use of identity $1+\cot^2 A=\operatorname{cosec}^2 A$	M1	using $\cot A=\frac{1}{2}$; allow sign error in attempt at identity
			Obtain $\frac{1}{2}\sqrt{5}$ or $\sqrt{\frac{5}{4}}$ or exact equiv	A1	final $\pm\frac{1}{2}\sqrt{5}$ is A0; correct answer only earns M1A1
2	(ii)		State or imply $\dfrac{2+\tan B}{1-2\tan B}=3$	B1	
			Attempt solution of equation of form $\dfrac{\text{linear in } t}{\text{linear in } t}=3$	M1	by sound process at least as far as $k\tan B=c$
			Obtain $\tan B=\frac{1}{7}$	A1	answer must be exact; ignore subsequent attempt to find angle B
				[3]	

Question			Answer	Marks	Guidance
3	(a)		Substitute $t=3$ in $\lvert 2t-1 \rvert$ and obtain value 5	B1	not awarded for final $\lvert 5 \rvert$ nor for ± 5
			Substitute $t=-3$ in $\lvert 2t-1 \rvert$ and apply modulus correctly to any negative value to obtain a positive value	M1	with no modulus signs remaining
			Obtain value 7 as final answer	A1	not awarded for final $\lvert 7 \rvert$ nor for ± 7 NB: substitutions in $\lvert 2t+1 \rvert$ will give 5 and 7 – this is 0/3, not MR; a further step to 5 < t < 7 – B1 M1 A0; answers $\pm 5, \pm 7$ – this is B0 M0 A0
				[3]	
3	(b)		<u>Either</u> Attempt solution of linear equation or inequality with signs of x different	M1	
			Obtain critical value $-\sqrt{2}$	A1	or equiv (exact or decimal approximation)
			<u>Or 1</u> Attempt to square both sides	M1	obtaining at least 3 terms on each side
			Obtain $x^2-2\sqrt{2}\,x+2>x^2+6\sqrt{2}\,x+18$	A1	or equiv; or equation; condone $>$ here
			<u>Or 2</u> Attempt sketches of $y=\lvert x-\sqrt{2}\rvert$, $y=\lvert x+3\sqrt{2}\rvert$	M1	
			Obtain $x=-\sqrt{2}$ at point of intersection	A1	or equiv
			Conclude with inequality of one of the following types:		
			$x<k\sqrt{2},\quad x>k\sqrt{2},\quad x<\frac{k}{\sqrt{2}},\quad x>\frac{k}{\sqrt{2}}$	M1	any integer k
			Obtain $x<-\sqrt{2}$ or $-\sqrt{2}>x$ as final answer	A1	final answer $x<-\frac{2}{\sqrt{2}}$ (or similar unsimplified version) is A0
				[4]	

Question			Answer	Marks	Guidance
4	(i)		Attempt process involving logarithm to solve $e^{0.021t} = 2$	M1	with t the only variable; at least as far as $0.021t = \ln 2$; must be $\ldots = 2$
			Obtain 33	A1	or greater accuracy; ignore absence of, or wrong, units; final answer $\frac{\ln 2}{0.021}$ is A0
			State (or calculate separately to obtain) 99	B1√	following previous answer; no need to include units
				[3]	
4	(ii)		Differentiate to obtain $ke^{0.021t}$	M1	where k is any constant not equal to 250
			Obtain $250 \times 0.021\, e^{0.021t}$	A1	or simplified equiv $5.25e^{0.021t}$
			Substitute to obtain 8.4 or $\frac{42}{5}$	A1	or value rounding to 8.4 with no obvious error
				[3]	
5	(i)		Integrate to obtain form $k(3x+1)^{\frac{1}{2}}$	*M1	any non-zero constant k
			Obtain $4(3x+1)^{\frac{1}{2}}$	A1	or (unsimplified) equiv; or $4u^{\frac{1}{2}}$ following substitution
			Apply the limits and subtract the right way round	M1	dep *M
			Obtain $4\sqrt{28} - 4\sqrt{7}$ and show at least one intermediate step in confirming $4\sqrt{7}$	A1	AG; necessary detail required; decimal verification is A0; $[\ldots]_2^9 = 4\sqrt{28} - 4\sqrt{7} = 4\sqrt{7}$ is A0; $[\ldots]_2^9 = 8\sqrt{7} - 4\sqrt{7} = 4\sqrt{7}$ is A0
				[4]	
5	(ii)		State or imply volume is $\int \pi \left(\frac{6}{\sqrt{3x+1}}\right)^2 \mathrm{d}x$ or equiv	B1	merely stating $\int \pi y^2 \,\mathrm{d}x$ not enough; condone absence of dx; no need for limits yet; π may be implied by its later appearance
			Integrate to obtain $k\ln(3x+1)$	M1	any non-zero constant with or without π
			Obtain $12\pi\ln(3x+1)$ or $12\ln(3x+1)$	A1	or unsimplified equiv
			Substitute limits correct way round and show each logarithm property correctly applied	M1	allowing correct applications to incorrect result of integration providing natural logarithm involved; evidence of $\ln 28 - \ln 7 = \frac{\ln 28}{\ln 7}$ error means M0
			Obtain $24\pi\ln 2$	A1	no need for explicit statement of value of k
				[5]	

Question			Answer	Marks	Guidance
6	(i)		Sketch more or less correct $y = \ln x$	B1	existing for positive and negative y; no need to indicate (1, 0); ignore any scales given on axes; condone graph touching y-axis but B0 if it crosses y-axis
			Sketch more or less correct $y = 8 - 2x^2$	B1	(roughly) symmetrical about y-axis; extending, if minimally, into quadrants for which $y < 0$; no need to indicate ($\pm$2, 0), (0, 8); assess each curve separately
			Indicate intersection by some mark on diagram (just a 'blob' sufficient) of by statement in words away from diagram	B1 **[3]**	needs each curve to be (more or less) correct in the first quadrant and on curves being related to each other correctly there
6	(ii)		Refer, in some way, to graphs crossing x-axis at $x = 1$ and $x = 2$ and that intersection is between these values	B1 **[1]**	AG; the values 1 and 2 may be assumed from part (i) if clearly marked there; dependent on curves being (more or less) correct in first quadrant; carrying out the sign-change routine is B0
6	(iii)		Obtain correct first iterate	B1	to at least 3 dp (except in the case of starting value 1 leading to 2)
			Show correct iterative process	M1	involving at least 3 iterates in all; may be implied by plausible converging values
			Obtain at least 3 correct iterates	A1	allowing recovery after error; iterates given to at least 3 dp; values may be rounded or truncated
			Conclude with 1.917	A1 **[4]**	answer required to exactly 3 dp; answer only with no evidence of process is 0/4
			$1 \to 2 \to 1.91139 \to 1.91731... \to 1.91690... \to 1.91693...$ $1.5 \to 1.94865... \to 1.91479... \to 1.91707... \to 1.91692...$ $2 \to 1.91139... \to 1.91731... \to 1.91690... \to 1.91693...$		
6	(iv)		Obtain 3.92 or greater accuracy	B1√	following their answer to part (iii)
			Attempt $4 \times \ln$(part (iii) answer)	M1	
			Obtain y-coordinate 2.60	A1 **[3]**	value required to exactly 2 dp (so A0 for 2.6 and 2.603)

Question			Answer	Marks	Guidance
7	(i)		Attempt use of product rule	M1	to produce expression of form (something non-zero) $\ln(2y+3)$ + $\dfrac{\text{linear in } y}{\text{linear in } y}$; ignore what they call their derivative
			Obtain $\ln(2y+3)$ …	A1	with brackets included
			Obtain … + $\dfrac{2(y+4)}{2y+3}$	A1	with brackets included as necessary
				[3]	
7	(ii)		Substitute $y=0$ into attempt from part (i) or into their attempt (however poor) at its reciprocal	M1	
			Obtain 0.27 for gradient at A	A1	or greater accuracy 0.26558…; beware of 'correct' answer coming from incorrect version $\ln(2y+3)+\frac{8}{3}$ of answer in part (i)
			Attempt to find value of y for which $x=0$	M1	allowing process leading only to $y=-4$
			Substitute $y=-1$ into attempt from part (i) or into their attempt (however poor) at its reciprocal	M1	
			Obtain 0.17 or $\frac{1}{6}$ for gradient at B	A1	or greater accuracy 0.16666…; value following from correct working
				[5]	
8	(i)		Attempt completion of square at least as far as $(x+2a)^2$ or differentiation to find stationary point at least as far as linear equation involving two terms	*M1	or equiv but a must be present
			Obtain $(x+2a)^2-3a^2$ or $(-2a,-3a^2)$	A1	
			Attempt inequality involving appropriate y-value	M1	dep *M; allow <, > or $\le$ here; allow use of x; or unsimplified equiv
			State $y \ge -3a^2$ or $\mathrm{f}(x) \ge -3a^2$	A1	now with $\ge$; here $x \ge -3a^2$ is A0
				[4]	

Question			Answer	Marks	Guidance
8	(ii)		Attempt composition of f and g the right way round	*M1	algebraic or (part) numerical; need to see $4x-2a$ replacing x at least once
			Obtain or imply $16x^2-3a^2$ or $144-3a^2$	A1	or less simplified equiv but with at least the brackets expanded correctly
			Attempt to find a from $\text{fg}(3)=69$	M1	dep *M
			Obtain at least $a=5$	A1	
			Attempt to solve $4x-10=x$ or $\frac{1}{4}(x+10)=x$ or $4x-10=\frac{1}{4}(x+10)$	M1	for their a; must be linear equation in one variable; condone sign slip in finding inverse of g
			Obtain $\frac{10}{3}$	A1	and no other answer
				[6]	
9	(i)		State $\cos\theta\cos 45-\sin\theta\sin 45$	B1	or equiv including use of decimal approximation for $\frac{1}{\sqrt{2}}$
			Use correct identity for $\sin 2\theta$ or $\cos 2\theta$	B1	must be used; not earned for just a separate statement
			Attempt complete simplification of left-hand side	M1	with relevant identities but allowing sign errors, and showing two terms involving $\sin\theta\cos\theta$
			Obtain $\sin^2\theta$	A1	AG; necessary detail needed
				[4]	
9	(ii)		Use identity to produce equation of form $\sin\frac{1}{2}\theta=c$	M1	condoning single value of constant c here (including values outside the range –1 to 1); M0 for $\sin\theta=c$ unless value(s) are subsequently doubled
			Obtain 70.5 or 70.6	A1	or greater accuracy 70.528…
			Obtain –70.5 or –70.6	A1√	or greater accuracy –70.528…; following first answer; and no other answer between –90 and 90; answer(s) only : 0/3
				[3]	
9	(iii)		State or imply $6\sin^2\frac{1}{3}\theta=k$	B1	
			Attempt to relate k to at least $6\sin^2 30°$	M1	
			Obtain $0<k<\frac{3}{2}$	A1	condone use of $\leq$
				[3]	

Question			Answer	Marks	Guidance
1	**(i)**		Obtain integral of form $k(4-3x)^8$	M1	any non-zero constant k; using substitution to obtain ku^8 earns M1
			Obtain $-\frac{1}{24}(4-3x)^8$	A1	or unsimplified equiv; must be in terms of x
1	**(ii)**		Obtain integral of form $k\ln(4-3x)$	M1	any non-zero constant k; allow M1 if brackets missing; using substitution to obtain $k\ln u$ earns M1; $\log(4-3x)$ with base e not specified is M1A0
			Obtain $-\frac{1}{3}\ln(4-3x)$	A1	now with either brackets or modulus signs; must be in terms of x; note that $-\frac{1}{3}\ln(x-\frac{4}{3})$ and $-\frac{1}{3}\ln(\frac{4}{3}-x)$ are correct alternatives
			Include + c or + k at least once	B1	anywhere in solution to question 1; this mark available even if no other marks earned
				[5]	
2	**(i)**		Use $2\cos^2\alpha-1$ or $\cos^2\alpha-\sin^2\alpha$ or $1-2\sin^2\alpha$	B1	
			Obtain equation in which $\sin^2\alpha$ appears once	M1	condoning sign slips or arithmetic slips; for solution which gives equation involving $\tan^2\alpha$, M1 is not earned until valid method for reaching $\sin\alpha$ is used; attempt involving $4(1-s^2)=s^2$ is M0
			Obtain $\pm\frac{2}{3}$	A1	both values needed; ±0.667 is A0; $\pm\sqrt{\frac{4}{9}}$ is A0; ignore subsequent work to find angle(s)
				[3]	
2	**(ii)**		<u>Either</u> Attempt use of identity	M1	of form $\tan^2\beta=\pm\sec^2\beta\pm1$
			Obtain $2\sec^2\beta-9\sec\beta-5=0$	A1	condone absence of $=0$
			Attempt solution of 3-term quadratic in $\sec\beta$ to obtain at least one value of $\sec\beta$	M1	if factorising, factors must be such that expansion gives their first and third terms; if using formula, this must be correct for their values
			Obtain 5 with no errors in solution	A1	and, finally, no other value; no need to justify rejection of $-\frac{1}{2}$
				[4]	
			<u>Or</u> Attempt to express equation in terms of $\cos\beta$	M1	using identities which are correct apart maybe for sign slips
			Obtain $5\cos^2\beta+9\cos\beta-2=0$	A1	condone absence of $=0$
			Attempt solution of 3-term quadratic and show switch at least once to a secant value	M1	if factorising, factors must be such that expansion gives their first and third terms; if using formula, this must be correct for their values
			Obtain 5 with no errors in solution	A1 [4]	and, finally, no other value; no need to justify rejection of $-\frac{1}{2}$

Question			Answer	Marks	Guidance
3	(i)		Use α (possibly implicitly) to state that radius of 'base' is $\frac{1}{2}x$	*B1	or to obtain equiv such as $2r = x$ or $\frac{r}{x} = \frac{1}{2}$ or $\frac{x}{r} = 2$
			Substitute into formula to obtain $\frac{1}{3}\pi(\frac{1}{2}x)^2 x$ or $\frac{1}{3}\pi\frac{1}{4}x^2 x$ and obtain $\frac{1}{12}\pi x^3$	B1	dep *B; AG; necessary detail needed Note: comparing formulae $\frac{1}{3}\pi r^2 h$ and $\frac{1}{12}\pi x^3$ to 'deduce' is B0B0
				[2]	
3	(ii)		Differentiate to obtain $\frac{1}{4}\pi x^2$ or equiv	B1	whatever they call it
			Attempt division involving 14 and their value of derivative when $x = 8$	M1	ie $14 \div$ deriv or deriv $\div 14$ with $x = 8$
			Obtain 0.28	A1	allow 0.279 but not greater accuracy Alternatives: 1. $14t = \frac{1}{12}\pi x^3$ Obtain $\frac{dt}{dx} = \frac{1}{56}\pi x^2$ <u>B1</u> Sub 8 and invert <u>M1</u> Ans <u>A1</u> 2. $x^3 = \frac{168t}{\pi}$ Obtain $3x^2 \frac{dx}{dt} = \frac{168}{\pi}$ <u>B1</u> Sub 8 <u>M1</u> Ans <u>A1</u>
				[3]	
4			Differentiate first term to obtain form $k(4x-7)^{-\frac{1}{2}}$	*M1	any non-zero constant k ; M0 if this differentiation is carried out in the midst of some incorrect involved expression
			Obtain $2(4x-7)^{-\frac{1}{2}}$	A1	or (unsimplified) equiv
			Attempt use of quotient rule or, after adjustment, product rule	*M1	for QR, allow numerator wrong way round but needs – sign in numerator; condone a single error such as absence of square in denominator, absence of brackets, …; for PR, condone no use of chain rule M0 if this differentiation is carried out in the midst of some incorrect involved expression
			Obtain $\dfrac{4(2x+1)-8x}{(2x+1)^2}$ or $4(2x+1)^{-1} - 8x(2x+1)^{-2}$	A1	or (unsimplified) equivs; give A0 if brackets absent unless subsequent calculation indicates their 'presence'
			Substitute 4 into expression for first derivative so that (initially at least) exactness is retained	M1	dep *M *M
			Obtain $\frac{58}{81}$	A1	answer must be exact Note: using $y = \sqrt{4x-7} + \dfrac{4}{2x+1}$: do not apply MR
				[6]	

Question			Answer	Marks	Guidance
5	**(i)**		Refer to translation and stretch	M1	in either order; ignore details here; allow any equiv wording (such as move or shift for translation) to describe geometrical transformation but not statements such as add 3 to x
			<u>Either</u> State translation in negative x-direction by 3	A1	or state translation by $\begin{pmatrix}-3\\0\end{pmatrix}$; accept horizontal to indicate direction; term 'translate' or 'translation' needed for award of A1
			State stretch by factor 2 in y-direction	A1	or parallel to y-axis or vertically; term 'stretch' needed for award of A1; these two transformations can be given in either order SC: if M0 but details of one transformation correct, award B1 for 1/3 (in <u>Either</u>, <u>Or 1</u>, <u>Or 2</u> cases)
				[3]	
			<u>Or 1</u> State stretch by factor $\frac{1}{2}$ in x-direction	A1	or parallel to x-axis; term 'stretch' needed for award of A1
			State translation in negative x-direction by 3	A1 [3]	or state translation by $\begin{pmatrix}-3\\0\end{pmatrix}$; term 'translate' or 'translation' needed for award of A1; these two transformations must be in this order – if details correct for M1A1A1 but order wrong, award M1A1A0
			<u>Or 2</u> State translation in negative x-direction by 6	A1	or state translation by $\begin{pmatrix}-6\\0\end{pmatrix}$; term 'translate' or 'translation' needed for award of A1
			State stretch by factor $\frac{1}{2}$ in x-direction	A1 [3]	or parallel to x-axis; term 'stretch' needed for award of A1; these two transformations must be in this order – if details correct for M1A1A1 but order wrong, award M1A1A0
5	**(ii)**		<u>Either</u> Solve linear eqn/ineq to obtain critical value –6	B1	
			Attempt solution of linear eqn/ineq where signs of x and $2x$ are different	M1	
			Obtain critical value –2	A1	
			Attempt solution of inequality	M1	using table, sketch, …; implied by correct answer or answer of form $a < x < b$ or of form $x < a, x > b$ (where $a < b$); allow $\leq$ here
			Obtain $-6 < x < -2$	A1	as final answer; must be < not $\leq$; allow "$x > -6$ and $x < -2$"
				[5]	

Question			Answer	Marks	Guidance
			Or Square both sides to obtain $x^2 > 4(x^2 + 6x + 9)$	B1	or equiv
			Attempt solution of 3-term quadratic eqn/ineq	M1	with same guidelines as in Q2(ii) for factorising and formula
			Obtain critical values –6 and –2	A1	
			Attempt solution of inequality	M1	using table, sketch, …; implied by correct answer or answer of form $a < x < b$ or of form $x < a, x > b$ (where $a < b$); allow $\le$ here
			Obtain $-6 < x < -2$	A1 [5]	as final answer; must be < not $\le$; allow '$x > -6$ and $x < -2$'
6	**(i)**		Attempt evaluation involving *y* values	M1	with coefficients 1, 4 and 2 each occurring at least once; allow for wrong *y*-values; solution must include sufficient evidence of method
			Obtain $k(\ln 3 + 4\ln 7 + 2\ln 19 + 4\ln 39 + \ln 67)$	A1	any constant *k*; or decimal equivs; correct use of brackets required unless subsequent working shows their 'presence'
			Identify value of *k* as $\frac{2}{3}$	A1	as factor for their complete expression
			Obtain 22.4	A1 **[4]**	allow any value rounding to 22.4; answer only is 0/4
6	**(ii)**		State $9 + 6x^2 + x^4 = (3 + x^2)^2$	B1	or, if proceeding numerically, demonstrate in at least three cases that $\ln 9 = \ln 3^2, \ln 49 = \ln 7^2, \ln 361 = \ln 19^2, \ldots$
			Show relevant property $\ln(3 + x^2)^2 = 2\ln(3 + x^2)$ and conclude with value $2A$	B1 **[2]**	AG; necessary detail needed; if proceeding numerically, needs all five cases with relevant property Note: using Simpson's rule again here is B0B0
6	**(iii)**		Recognise $\ln(3\mathrm{e} + \mathrm{e}x^2)$ as $1 + \ln(3 + x^2)$	B1	
			Indicate in some way that $\int_0^8 1 \, \mathrm{d}x$ is 8 and conclude with value $A + 8$	B1 **[2]**	AG; necessary detail needed Note: using Simpson's rule again here is B0B0
7	**(i)**		State $y > 3$ or $\mathrm{f}(x) > 3$ or $\mathrm{f} > 3$ or 'greater than 3'	B1 **[1]**	must be > not $\ge$; allow $3 < y < \infty$

Question			Answer	Marks	Guidance
7	(ii)		Obtain expression or eqn involving $\ln(\frac{y-3}{4})$ or $\ln(\frac{x-3}{4})$	M1	or equivs such as $\ln(\frac{4}{y-3})$ or $\ln(\frac{4}{x-3})$
			Obtain $\ln(\frac{4}{x-3})$ or $-\ln(\frac{x-3}{4})$	A1	or equiv
			State domain is $x>3$ or equiv	B1FT	following answer to part **(i) (**but with adjustment so that reference is to *x*)
			State range is all real numbers or equiv	B1 **[4]**	
7	(iii)		Obtain correct first iterate	B1	showing at least 3 dp; B0 if initial value not 3 but then M1A1A1 available
			Show correct iteration process	M1	showing at least 3 iterates in all; may be implied by plausible converging values; M1available if based on equation with just a slip in $x = \mathrm{f}(x)$ but M0 if based on clearly different equation
			Obtain at least 3 correct iterates	A1	allowing recovery after error; iterates to only 3 dp acceptable; values may be rounded or truncated
			Obtain (3.168, 3.168)	A1	each coordinate required to exactly 3 dp; award A0 if fewer than 4 iterates shown; part (iii) consisting of answer only gets 0 out of 4
			[$3 \rightarrow 3.199148.. \rightarrow 3.163187.. \rightarrow 3.169162.. \rightarrow 3.168155.. \rightarrow 3.168324..$]	**[4]**	
7	(iv)		State *P* is point where the curves meet	B1 **[1]**	or equiv
8	(i)		Obtain $R=\sqrt{20}$ or $R = 4.47$	B1	
			Attempt to find value of α	M1	implied by correct value or its complement; allow sin/cos muddles; allow use of radians for M1; condone use of $\cos\alpha = 4, \sin\alpha = 2$ here but not for A1
			Obtain 26.6	A1 **[3]**	or greater accuracy 26.565…; with no wrong working seen
8	(ii)	(a)	Show correct process for finding one answer	M1	allowing for case where the answer is negative
			Obtain 21.3	A1FT	or greater accuracy 21.3045…; or anything rounding to 21.3 with no obvious error; following a wrong value of α but not wrong *R*
			Show correct process for finding second answer	M1	ie attempting fourth quadrant value minus α value
			Obtain 286 or 285.6	A1FT **[4]**	or greater accuracy 285.5653…; or anything rounding to 286 with no obvious error; following a wrong value of α but not wrong *R* ;and no others between $0°$ and $360°$

Question			Answer	Marks	Guidance
8	(ii)	(b)	State greatest value is 25	B1	allow if α incorrect
			Obtain value 63.4 clearly associated with correct greatest value	B1FT	or greater accuracy 63.4349…; following a wrong value of α
			State least value is 5	B1	allow if α incorrect
			Attempt to find θ from $\cos(\theta + \text{their}\,\alpha) = -1$	M1	and clearly associated with correct least value
			Obtain 153 or 153.4	A1FT	or greater accuracy 153.4349…; following a wrong value of α
				[5]	
9	(i)		Differentiate to obtain $2e^{2x} - 18$	B1	
			Equate first derivative to zero and use legitimate method to reach equation without e involved	M1	
			Confirm $x = \ln 3$	A1	AG; necessary detail needed (in particular, for solutions concluding $x = \frac{1}{2}\ln 9 = \ln 3$ or equiv award A0)
				[3]	
9	(ii)		Attempt integration	*M1	confirmed by at least one correct term
			Obtain $\frac{1}{2}e^{2x} - 9x^2 + 15x$	A1	or equiv
			Apply limits 0 and $\ln 3$ to obtain exact unsimplified expression	M1	dep *M
			Obtain $4 - 9(\ln 3)^2 + 15\ln 3$	A1	or exact (maybe unsimplified) equiv perhaps still involving e
			Attempt area of trapezium or equiv, retaining exactness throughout	M1	using $\frac{1}{2}\ln 3 \times (y_1 + y_2)$ where y_1 is 15 or 16 and y_2 is attempt at y-coordinate of Q; if using alternative approach involving rectangle and triangle, complete attempt needs to be seen for M1; another alternative approach involves equation of PQ ($y = \frac{8-18\ln 3}{\ln 3}x + 16$) with integration: M1 for attempting equation and integration, A1 for correct answer
			Obtain $\frac{1}{2}\ln 3 \times (16 + 24 - 18\ln 3)$	A1	or equiv perhaps still including e
			Subtract areas the right way round, retaining exactness	M1	dep on award of all three M marks
			Obtain $5\ln 3 - 4$	A1	or similarly simplified exact equiv
				[8]	

Question			Answer	Marks	Guidance	
1			Attempt use of product rule to find first derivative	M1	producing form … ± …where one term involves $\ln x$ and the other does not	
			Obtain $8x\ln x + 4x$	A1	or unsimplified equiv	
			Attempt use of correct product rule to find second derivative	M1	with one term involving $\ln x$	
			Obtain $8\ln x + 12$	A1	or unsimplified equiv	
			Obtain 28	A1		
				[5]		
2			State or imply $\operatorname{cosec} q = 1$ ¸ $\sin q$	B1	allow $\cos \text{ec} = 1$ ¸ $\sin$	
			Attempt to express equation in terms of $\sin q$ only	M1	using identity of form $\pm 1 \pm 2\sin^2 q$ for $\cos 2q$	
			Obtain $10\sin^2 q + 2\sin q - 5 = 0$	A1	or unsimplified equiv involving $\sin q$ only but with no $\sin q$ remaining in denominator	
			Attempt use of formula to find $\sin q$ from 3-term quadratic equation involving $\sin q$ (using formula or completing square even if their equation can be solved by factorisation)	M1	use implied by at least one correct value of $\sin q$ or q; if correct quadratic formula quoted, condone one sign error for M1; if formula not first quoted, any error leads to M0	if completion of square used to solve equation, this must be correct for M1 to be earned
			Obtain 37.9°	A1	or greater accuracy 37.8896…	no working and answers only (max 2/6): 37.9 (or greater accuracy) B1 142 (or greater accuracy) and no others … B1
			Obtain 142°	A1	or greater accuracy 142.1103…; and no others between 0 and 180; ignore any answers, right or wrong, outside 0 - 180	
				[6]		

Question			Answer	Marks	Guidance	
3	**(i)**		Attempt calculation $k(y+4y+2y+...)$	M1	any constant *k*; using *y* values with coefficients 1, 2, 4 each occurring at least once; brackets may be implied by subsequent calculation	allow M1 for attempt using *y* values based on wrong *x* values such as 0, 1, 2, 3, 4; attempt based on $k(y_0+y_4)+4y_1+2y_2+4y_3$ is M0 unless subsequent calculation shows missing brackets are ' present'
			Obtain $k(e^0+4e^{\sqrt{0.5}}+2e+4e^{\sqrt{1.5}}+e^{\sqrt{2}})$	A1	or equiv perhaps involving decimal values 1, 2.02811..,.2.71828..,. 3.40329..,.4.11325…	
			Use $k=\frac{1}{3}\times\frac{1}{2}$	A1		
			Obtain 5.38	A1	allow 5.379 but not, in final answer, greater ' accuracy'; answer $5.38+c$ is final A0	answer only: 0/4
				[4]		
3	**(ii)**		Attempt calculation of form $10\times$ (answer to part i) $+k$	M1	implied by correct answer only or by answer following correctly from their incorrect part **(i)** ; any non-zero constant *k*	allow attempt involving second use of Simpson's rule: M1 for complete correct expression, A1 for answer
			Obtain 55.8 or greater accuracy based on their part **(i)** –more than 3 s.f. acceptable	A1ft	following their answer to part **(i)** but A0 for $55.8+c$	answer only 54.8 with no working earns M1A0 (as does 10(their ans) + 1); otherwise incorrect answer with no working earns 0/2
				[2]		
4	**(i)**		Either: State $2x^3+4=-50$	B1		
			State -3 and no other	B1		
			Or: Obtain $\sqrt[3]{\frac{1}{2}(x-4)}$ for inverse of f	B1	or equiv; using any letter	
			State -3 and no other	B1		
				[2]		
4	**(ii)**		Show composition of functions the right way round	M1		
			Obtain $2x-16$	A1	AG; necessary detail needed	first step $2(x-10)+4$ acceptable but then two more steps needed
				[2]		

Question			Answer	Marks	Guidance	
4	(iii)		Obtain $\sqrt[3]{2x^3-6}$ or $(2x^3-6)^{\frac{1}{3}}$ for $\mathrm{gf}(x)$	B1	or unsimplified equiv	
			Apply chain rule to function which is cube root of a non-linear expression	M1	condone incorrect constant; otherwise use of chain rule for their function must be correct	may use $u=2x^3-6$; M1 earned for expression involving u
			Obtain $2x^2(2x^3-6)^{-\frac{2}{3}}$	A1	or similarly simplified equiv; do not accept final answer with $\frac{6}{3}$ unsimplified	..in terms of x
				[3]		
5	(a)		Differentiate to produce $k\mathrm{e}^{-0.33t}$	M1	where constant k is different from 58	method must involve differentiation
			Obtain $-19.14\mathrm{e}^{-0.33t}$ or $19.14\mathrm{e}^{-0.33t}$	A1	or unsimplified equiv	
			Obtain -5.1 or 5.1	A1	whatever they claim value represents; accept 5.11 but not greater accuracy	
				[3]		
5	(b)		<u>Either</u>:			
			State or imply formula $42\mathrm{e}^{kt}$ or $42a^t$	B1	$42\mathrm{e}^{-kt}$, $42\mathrm{e}^{-kx}$, etc. also acceptable	
			Attempt to find k from $42\mathrm{e}^{6k}=51.8$ or a from $42a^6=51.8$	M1	using sound process involving logarithms at least as far as $6k = \ldots$ or $a = \ldots$	
			Obtain $k=0.035$ or $a=1.0356$	A1	or greater accuracy 0.03495... or exact equiv $\frac{1}{6}\ln\frac{37}{30}$	
			Substitute 24 to obtain value between 97.1 and 97.3 inclusive	A1	allow greater accuracy than 3 s.f.	
			<u>Or</u>:			
			Use ratio $\frac{51.8}{42}$ in calculation	B1		
			Attempt calculation of form $42\times r^n$	M1		
			Obtain $42\times(\frac{51.8}{42})^4$ or $51.8\times(\frac{51.8}{42})^3$	A1		
			Obtain value between 97.1 and 97.3 inclusive	A1	allow greater accuracy than 3 s.f.	
				[4]		

Question			Answer	Marks	Guidance	
6	**(i)**		Draw inverted parabola roughly symmetrical about the *y*-axis and with maximum point more or less on *y*-axis	M1	drawing enough of the parabola that two intersections occur, ignoring their locations at this stage	
			State $y = 9 - x^2$ and indicate two intersections by marks on diagram or written reference to two intersections	A1	now needs second curve drawn so that right-hand intersection occurs in first quadrant	
				[2]		
6	**(ii)**	**(a)**	Calculate values of quartic expression for 2.1 and 2.2	M1	if no explicit working seen, M1 is implied by at least one correct value; but if no explicit working seen and both values wrong, award M0	
			Obtain $-1.9...$ and 1.6…and draw attention to sign change or clear equiv	A1		
				[2]		
6	**(ii)**	**(b)**	Obtain correct first iterate	B1	starting anywhere between –1 and 9 and showing at least 3 d.p.	
			Carry out process to produce at least three iterates in all	M1	implied by plausible sequence of values; allow recovery after error	2.1® 2.15056® 2.15531® 2.15575® 2.15579 2.15® 2.15526® 2.15574® 2.15579
			Obtain at least two more correct iterates	A1	showing at least 3 decimal places	
			Obtain 2.156	A1	final answer needed to exactly 3 d.p.; not given for 2.156 as final iterate in sequence, i.e. needs indication (perhaps just underlining) that value of a found	2.2® 2.15980® 2.15616® 2.15583® 2.15580 answer only: 0/4
				[4]		

Question			Answer	Marks	Guidance	
7	**(i)**		Integrate to obtain $k(4x+1)^{\frac{1}{2}}$ or $ku^{\frac{1}{2}}$	*M1	any constant k	
			Obtain correct $\frac{1}{2}\sqrt{3}(4x+1)^{\frac{1}{2}}$ or $\frac{1}{2}\sqrt{3}u^{\frac{1}{2}}$	A1	or exact equiv	
			Apply limits 0 and 20 and attempt subtraction of area of rectangle (or limits 1 and 81 if u involved)	M1	dep *M; or equiv such as including term $-\frac{1}{9}\sqrt{3}$ in the integration or finding $\int_0^1 \sqrt{3}\,\mathrm{d}x$ separately; allow M1 if decimal values used here	<u>Alternative</u>: (region between curve and y-axis) Obtain equation $x=\frac{3}{4}y^{-2}-\frac{1}{4}$ B1 Integrate to obtain form $k_1y^{-1}+k_2y$ *M1 Apply limits $\frac{1}{9}\sqrt{3}$ and $\sqrt{3}$ the right way round M1 d*M
			Obtain $4\sqrt{3}-\frac{20}{9}\sqrt{3}$ and hence $\frac{16}{9}\sqrt{3}$	A1	answer must be exact and a single term; $\frac{16}{9}\sqrt{3}+c$ as answer is final A0	Obtain $\frac{6}{\sqrt{3}}-\frac{8}{36}\sqrt{3}$ or better A1
				[4]		
	(ii)		State volume is $\pi\int\frac{3}{4x+1}\,\mathrm{d}x$	B1	no need for limits here; condone absence of dx; condone absence of π here if it appears later in solution	allow B1 for $\int\pi y^2$ <u>and</u> $y^2=\frac{3}{4x+1}$ stated
			Obtain integral of form $k\ln(4x+1)$	M1	any constant k with or without π	if brackets missing, and subsequent calculation does not show their ' presence', marks are max B1M1A0A0M1A0
			Obtain $\frac{3}{4}\pi\ln(4x+1)$ or $\frac{3}{4}\ln(4x+1)$	A1		
			Apply limits to obtain $\frac{3}{4}\pi\ln 81$ or $\frac{3}{4}\ln 81$	A1	or exact equiv perhaps with $\ln 1$ present	
			Attempt to subtract volume of cylinder, using correct radius and ' height'	M1	with exact volume of cylinder attempted	do not treat rotation around y-axis as mis-read: this is 0/6
			Obtain $3\pi\ln 3-\frac{20}{27}\pi$ or $\pi(\frac{3}{4}\ln 81-\frac{20}{27})$	A1	or exact equiv involving two terms	
				[6]		

Question			Answer	Marks	Guidance	
8	**(i)**		Attempt use of quotient rule or equiv	M1	condone one slip only but must be subtraction in numerator; condone absence of necessary brackets; or equiv	
			Obtain $\dfrac{2(x^2+5)-2x(2x+4)}{(x^2+5)^2}$	A1	or correct equiv; now with brackets as necessary	correct numerator but error in denominator: max M1A0A1M1A1A1; numerator wrong way round: max M0A0A0M1A1A1
			Obtain $-2x^2-8x+10=0$	A1	or equiv involving three terms	
			Attempt solution of three-term quadratic equation based on numerator of derivative (even if their equation has no real roots)	M1	implied by no working but 2 correct values obtained	M1 for factorisation awarded if attempt is such that x^2 term and one other term correct upon expansion; if formula used, M1 awarded as per Qn 2
			Obtain -5 and 1	A1		
			Obtain $(-5, -\frac{1}{5})$ and (1, 1)	A1	Allow $-\frac{6}{30}$	
				[6]		
	(ii)	**(a)**	Sketch (more or less) correct curve	B1	showing negative part reflected in x-axis and positive part unchanged; ignore intercept values on axes, right or wrong	
			State values between 0 and their y-value of maximum point lying in first quadrant	M1	accept $\le$ or < signs here	
			State correct $0 \le y \le 1$	A1ft	following their y-value of maximum point in first quadrant; now with $\le$ signs; or equiv perhaps involving g or $\mathrm{g}(x)$	for "$y \ge 0$ and $y \le 1$", award M1A1; for separate statements $y \ge 0$, $y \le 1$, award M1A0
				[3]		
	(ii)	**(b)**	Indicate, in some way, values between y-coordinates of maximum point and reflected minimum point (provided their y-coordinate of minimum point is negative)	M1	allow $\le$ sign(s) here; could be clear indication on graph	for "$k > \frac{1}{5}$ and $k < 1$", award M1A1; for separate statements, award M1A0
			State $\frac{1}{5} < k < 1$	A1	or correct equiv; not $\le$ now; correct answer only earns M1A1	
				[2]		

Question			Answer	Marks	Guidance	
9	**(i)**		Simplify to obtain $\frac{11}{2}\cos\theta + \frac{5\sqrt{3}}{2}\sin\theta$	B1	or equiv with two terms perhaps with $\sin 60$ retained	accept decimal values
			Attempt correct process to find R	M1	for expression of form $a\cos\theta + b\sin\theta$	obtained after initial simplification
			Attempt correct process to find α	M1	for expression of form $a\cos\theta + b\sin\theta$; condone $\sin\alpha = \frac{11}{2},\ \cos\alpha = \frac{5}{2}\sqrt{3}$	obtained after initial simplification
			Obtain $7\sin(\theta + 51.8)$	A1	or greater accuracy 51.786…	
				[4]		
	(ii)	**(a)**	State stretch and translation in either order	M1	or equiv but using correct terminology, not move, squash, …	SC: if M0 but one transformation completely correct, award B1 for 1/3
			State stretch parallel to y-axis with factor $\frac{1}{7}$	A1ft	following their R and clearly indicating correct direction	
			State translation parallel to θ-axis or x-axis by 51.8 in positive direction or state translation by vector $\begin{pmatrix} -51.8 \\ 0 \end{pmatrix}$	A1ft	following their α and clearly indicating correct direction; or equiv such as 308.2 parallel to x-axis in negative direction	
				[3]		
		(b)	State left-hand side (their R) $\sin(\frac{1}{3}\beta + \gamma)$ where $\gamma \neq \pm$(their α), $\gamma \neq \pm 40$, $\gamma \neq \pm 20$	M1	or equiv such as stating $\theta = \frac{1}{3}\beta + 20$	
			Obtain (their R) $\sin(\frac{1}{3}\beta + \text{their } \alpha + 20) = 3$	A1ft	(and, in this case, allowing A1ft provided value of $\frac{1}{3}\beta$ attempted later)	
			Attempt correct process to find any value of $\frac{1}{3}\beta$	M1	for equation of form $\sin(\frac{1}{3}\beta + \gamma) = k$ where $\lvert k \rvert < 1,\ k \neq 0$	
			Attempt complete process to find positive value of β	M1	including choosing second quadrant value of their $\sin^{-1}\frac{3}{7}$	
			Obtain 248 or 249 or 248.5	A1	or greater accuracy 248.508…	
				[5]		