

Edexcel Maths C3

Past Paper Pack

2005-2013

Centre No.							Paper Reference					Surname	Initial(s)	
Candidate No.						6	6	6	5	/	0	1	Signature	

Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Advanced Level

Monday 20 June 2005 – Morning

Time: 1 hour 30 minutes

Materials required for examination

Mathematical Formulae (Green)

Items included with question papers

Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Examiner's use only

--	--	--

Team Leader's use only

--	--	--

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
Total	

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initial(s) and signature.
 Check that you have the correct question paper.
 You must write your answer for each question in the space following the question.
 When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
 Full marks may be obtained for answers to ALL questions.
 The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2).
 There are 7 questions in this question paper. The total mark for this paper is 75.
 There are 20 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
 You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

This publication may be reproduced only in accordance with Edexcel Limited copyright policy. ©2005 Edexcel Limited.

Printer's Log. No.

N23494B



Turn over

6.

Figure 1

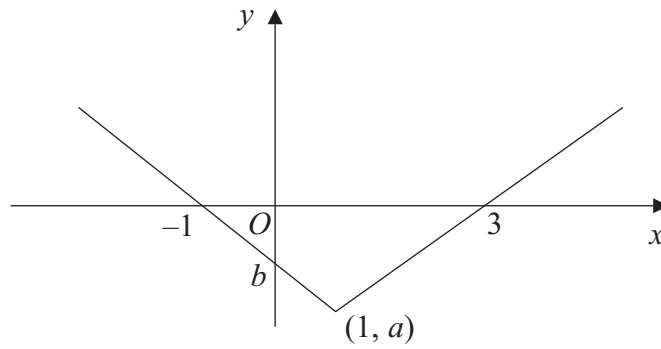


Figure 1 shows part of the graph of $y = f(x)$, $x \in \mathbb{R}$. The graph consists of two line segments that meet at the point $(1, a)$, $a < 0$. One line meets the x -axis at $(3, 0)$. The other line meets the x -axis at $(-1, 0)$ and the y -axis at $(0, b)$, $b < 0$.

In separate diagrams, sketch the graph with equation

(a) $y = f(x + 1)$, (2)

(b) $y = f(|x|)$. (3)

Indicate clearly on each sketch the coordinates of any points of intersection with the axes.

Given that $f(x) = |x - 1| - 2$, find

(c) the value of a and the value of b , (2)

(d) the value of x for which $f(x) = 5x$. (4)

Question 7 continued

Lined area for writing the answer to Question 7.

Q7

Small box for marking.

(Total 10 marks)

TOTAL FOR PAPER: 75 MARKS

END

Large box for marking at the bottom of the page.

1.

Figure 1

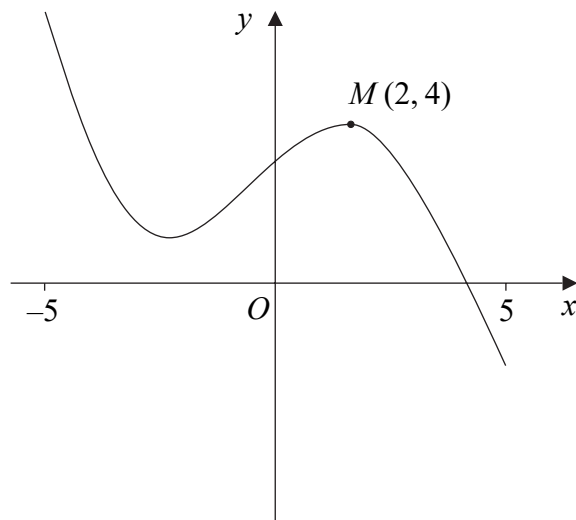


Figure 1 shows the graph of $y = f(x)$, $-5 \leq x \leq 5$.
The point $M(2, 4)$ is the maximum turning point of the graph.

Sketch, on separate diagrams, the graphs of

(a) $y = f(x) + 3$, (2)

(b) $y = |f(x)|$, (2)

(c) $y = f(|x|)$. (3)

Show on each graph the coordinates of any maximum turning points.

Question 1 continued

(Total 7 marks)

Q1

Question 6 continued

Lined writing area for the answer to Question 6. The page contains 31 horizontal lines.

(Total 12 marks)

Q6	
-----------	--

3.

Figure 1

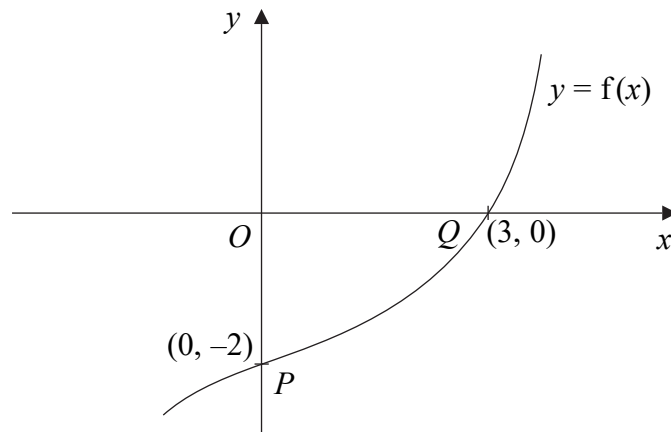


Figure 1 shows part of the curve with equation $y = f(x)$, $x \in \mathbb{R}$, where f is an increasing function of x . The curve passes through the points $P(0, -2)$ and $Q(3, 0)$ as shown.

In separate diagrams, sketch the curve with equation

(a) $y = |f(x)|$, **(3)**

(b) $y = f^{-1}(x)$, **(3)**

(c) $y = \frac{1}{2} f(3x)$. **(3)**

Indicate clearly on each sketch the coordinates of the points at which the curve crosses or meets the axes.

Question 3 continued

7. For the constant k , where $k > 1$, the functions f and g are defined by

$$\begin{aligned} f: x &\mapsto \ln(x+k), & x > -k, \\ g: x &\mapsto |2x-k|, & x \in \mathbb{R}. \end{aligned}$$

(a) On separate axes, sketch the graph of f and the graph of g .

On each sketch state, in terms of k , the coordinates of points where the graph meets the coordinate axes.

(5)

(b) Write down the range of f .

(1)

(c) Find $fg\left(\frac{k}{4}\right)$ in terms of k , giving your answer in its simplest form.

(2)

The curve C has equation $y = f(x)$. The tangent to C at the point with x -coordinate 3 is parallel to the line with equation $9y = 2x + 1$.

(d) Find the value of k .

(4)

5.

Figure 1

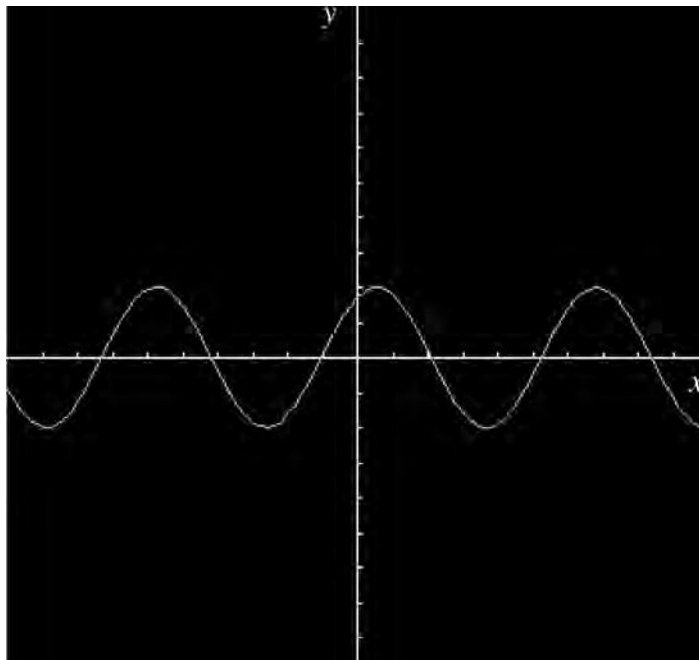


Figure 1 shows an oscilloscope screen.

The curve shown on the screen satisfies the equation

$$y = \sqrt{3} \cos x + \sin x.$$

- (a) Express the equation of the curve in the form $y = R \sin(x + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. (4)
- (b) Find the values of x , $0 \leq x < 2\pi$, for which $y = 1$. (4)

Question 5 continued

Ruled lines for writing the answer.

(Total 8 marks)

Q5

Question 6 continued

Question 7 continued

4.

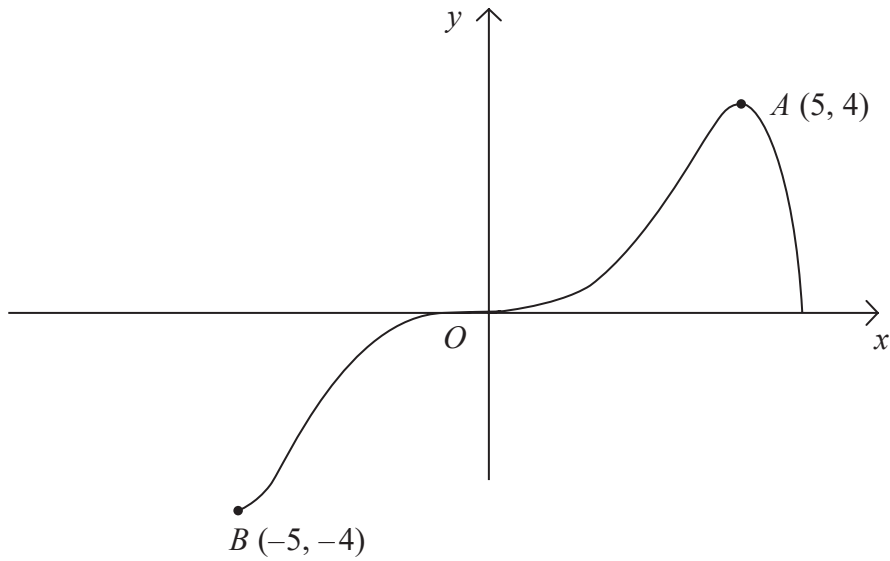


Figure 1

Figure 1 shows a sketch of the curve with equation $y = f(x)$.
The curve passes through the origin O and the points $A(5, 4)$ and $B(-5, -4)$.

In separate diagrams, sketch the graph with equation

(a) $y = |f(x)|$, (3)

(b) $y = f(|x|)$, (3)

(c) $y = 2f(x+1)$. (4)

On each sketch, show the coordinates of the points corresponding to A and B .

Question 4 continued

8. The functions f and g are defined by

$$f : x \mapsto 1 - 2x^3, \quad x \in \mathbb{R}$$

$$g : x \mapsto \frac{3}{x} - 4, \quad x > 0, \quad x \in \mathbb{R}$$

(a) Find the inverse function f^{-1} . (2)

(b) Show that the composite function gf is

$$gf : x \mapsto \frac{8x^3 - 1}{1 - 2x^3}. \quad (4)$$

(c) Solve $gf(x) = 0$. (2)

(d) Use calculus to find the coordinates of the stationary point on the graph of $y = gf(x)$. (5)

Question 4 continued

Handwritten response area consisting of 28 horizontal lines for text entry.

7.

$$f(x) = 3x^3 - 2x - 6$$

(a) Show that $f(x) = 0$ has a root, α , between $x = 1.4$ and $x = 1.45$ (2)

(b) Show that the equation $f(x) = 0$ can be written as

$$x = \sqrt{\left(\frac{2}{x} + \frac{2}{3}\right)}, \quad x \neq 0. \quad (3)$$

(c) Starting with $x_0 = 1.43$, use the iteration

$$x_{n+1} = \sqrt{\left(\frac{2}{x_n} + \frac{2}{3}\right)}$$

to calculate the values of x_1 , x_2 and x_3 , giving your answers to 4 decimal places. (3)

(d) By choosing a suitable interval, show that $\alpha = 1.435$ is correct to 3 decimal places. (3)

Centre No.						Paper Reference						Surname	Initial(s)
												Candidate No. 6 6 6 5 / 0 1	

Paper Reference(s)

6665/01

**Edexcel GCE
Core Mathematics C3
Advanced**

Thursday 15 January 2009 – Morning

Time: 1 hour 30 minutes

Examiner's use only

--	--	--

Team Leader's use only

--	--	--

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Materials required for examination
Mathematical Formulae (Green)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer **ALL** the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to **ALL** questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

This publication may be reproduced only in accordance with Edexcel Limited copyright policy.
©2009 Edexcel Limited.



Turn over

3.

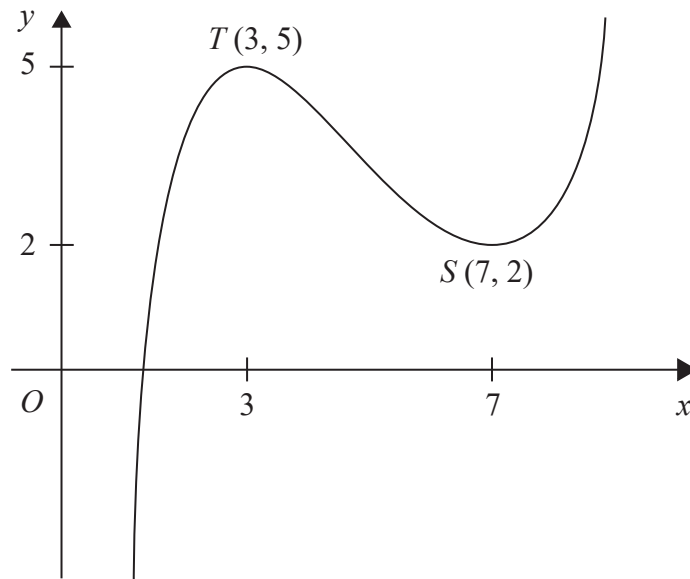


Figure 1

Figure 1 shows the graph of $y = f(x)$, $1 < x < 9$.
The points $T(3, 5)$ and $S(7, 2)$ are turning points on the graph.

Sketch, on separate diagrams, the graphs of

(a) $y = 2f(x) - 4$, **(3)**

(b) $y = |f(x)|$. **(3)**

Indicate on each diagram the coordinates of any turning points on your sketch.

7.

$$f(x) = 3xe^x - 1$$

The curve with equation $y = f(x)$ has a turning point P .

- (a) Find the exact coordinates of P . (5)

The equation $f(x) = 0$ has a root between $x = 0.25$ and $x = 0.3$

- (b) Use the iterative formula

$$x_{n+1} = \frac{1}{3}e^{-x_n}$$

with $x_0 = 0.25$ to find, to 4 decimal places, the values of x_1 , x_2 and x_3 . (3)

- (c) By choosing a suitable interval, show that a root of $f(x) = 0$ is $x = 0.2576$ correct to 4 decimal places. (3)

Question 7 continued

Lined writing area for the answer to Question 7.

Question 8 continued

Lined area for writing the answer to Question 8.

Q8

(Total 12 marks)

TOTAL FOR PAPER: 75 MARKS

END

5.

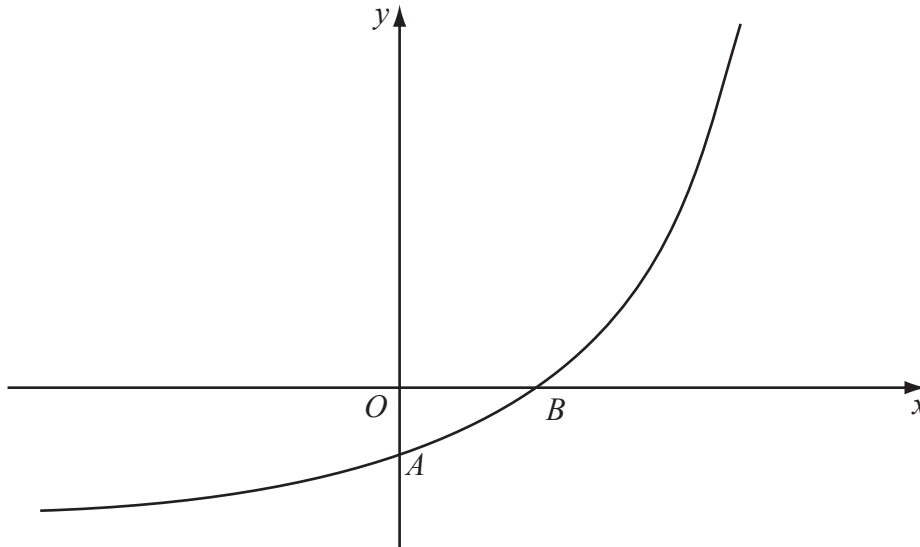


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = f(x)$, $x \in \mathbb{R}$.

The curve meets the coordinate axes at the points $A(0, 1-k)$ and $B(\frac{1}{2} \ln k, 0)$, where k is a constant and $k > 1$, as shown in Figure 2.

On separate diagrams, sketch the curve with equation

(a) $y = |f(x)|$, (3)

(b) $y = f^{-1}(x)$. (2)

Show on each sketch the coordinates, in terms of k , of each point at which the curve meets or cuts the axes.

Given that $f(x) = e^{2x} - k$,

(c) state the range of f , (1)

(d) find $f^{-1}(x)$, (3)

(e) write down the domain of f^{-1} . (1)

Question 5 continued

7. The function f is defined by

$$f(x) = 1 - \frac{2}{(x + 4)} + \frac{x - 8}{(x - 2)(x + 4)}, \quad x \in \mathbb{R}, x \neq -4, x \neq 2$$

(a) Show that $f(x) = \frac{x - 3}{x - 2}$

(5)

The function g is defined by

$$g(x) = \frac{e^x - 3}{e^x - 2}, \quad x \in \mathbb{R}, x \neq \ln 2$$

(b) Differentiate $g(x)$ to show that $g'(x) = \frac{e^x}{(e^x - 2)^2}$

(3)

(c) Find the exact values of x for which $g'(x) = 1$

(4)

Centre No.						Paper Reference						Surname	Initial(s)	
Candidate No.						6	6	6	5	/	0	1	Signature	

Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Advanced

Wednesday 20 January 2010 – Afternoon
Time: 1 hour 30 minutes

Examiner's use only

--	--	--

Team Leader's use only

--	--	--

Materials required for examination
Mathematical Formulae (Pink or Green)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature. Check that you have the correct question paper. Answer ALL the questions. You must write your answer to each question in the space following the question. When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided. Full marks may be obtained for answers to ALL questions. The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2). There are 9 questions in this question paper. The total mark for this paper is 75. There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled. You should show sufficient working to make your methods clear to the Examiner. Answers without working may not gain full credit.



Turn over

5. Sketch the graph of $y = \ln|x|$, stating the coordinates of any points of intersection with the axes.

(3)

6.

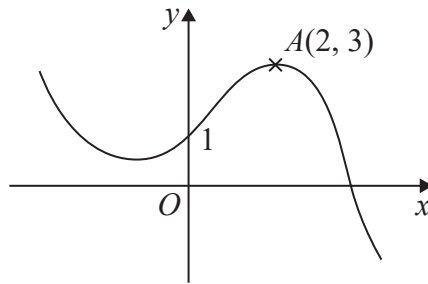


Figure 1

Figure 1 shows a sketch of the graph of $y = f(x)$.

The graph intersects the y -axis at the point $(0, 1)$ and the point $A(2, 3)$ is the maximum turning point.

Sketch, on separate axes, the graphs of

- (i) $y = f(-x) + 1$,
- (ii) $y = f(x + 2) + 3$,
- (iii) $y = 2f(2x)$.

On each sketch, show the coordinates of the point at which your graph intersects the y -axis and the coordinates of the point to which A is transformed.

(9)

Question 6 continued

Centre No.							Paper Reference				Surname	Initial(s)
Candidate No.					6	6	6	5	/	0	1	Signature

Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Advanced

Tuesday 15 June 2010 – Morning

Time: 1 hour 30 minutes

Examiner's use only

--	--	--

Team Leader's use only

--	--	--

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
Total	

<u>Materials required for examination</u>	<u>Items included with question papers</u>
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature. Check that you have the correct question paper.
 Answer ALL the questions.
 You must write your answer to each question in the space following the question.
 When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
 Full marks may be obtained for answers to ALL questions.
 The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2).
 There are 8 questions in this question paper. The total mark for this paper is 75.
 There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
 You should show sufficient working to make your methods clear to the Examiner.
 Answers without working may not gain full credit.

This publication may be reproduced only in accordance with Edexcel Limited copyright policy.
 ©2010 Edexcel Limited.

Printer's Log. No.
H35385A



4. The function f is defined by

$$f : x \mapsto |2x - 5|, \quad x \in \mathbb{R}$$

(a) Sketch the graph with equation $y = f(x)$, showing the coordinates of the points where the graph cuts or meets the axes. (2)

(b) Solve $f(x) = 15 + x$. (3)

The function g is defined by

$$g : x \mapsto x^2 - 4x + 1, \quad x \in \mathbb{R}, \quad 0 \leq x \leq 5$$

(c) Find $fg(2)$. (2)

(d) Find the range of g . (3)

6.

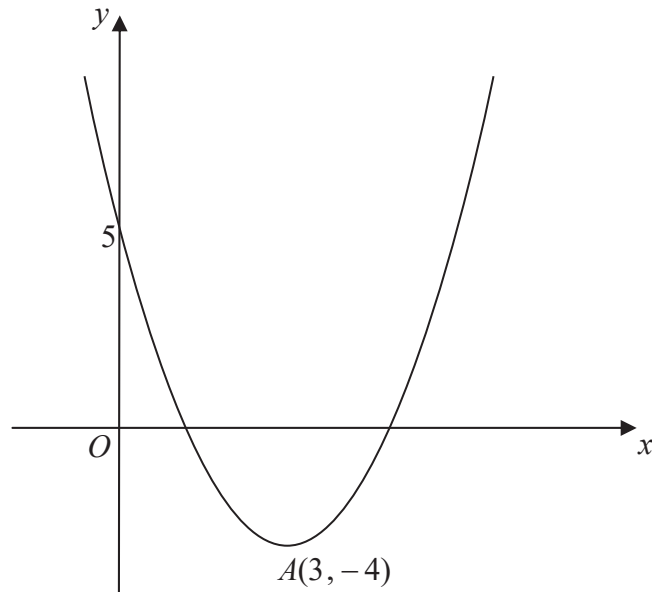


Figure 2

Figure 2 shows a sketch of the curve with the equation $y = f(x)$, $x \in \mathbb{R}$.
The curve has a turning point at $A(3, -4)$ and also passes through the point $(0, 5)$.

(a) Write down the coordinates of the point to which A is transformed on the curve with equation

(i) $y = |f(x)|$,

(ii) $y = 2f(\frac{1}{2}x)$.

(4)

(b) Sketch the curve with equation

$$y = f(|x|)$$

On your sketch show the coordinates of all turning points and the coordinates of the point at which the curve cuts the y -axis.

(3)

The curve with equation $y = f(x)$ is a translation of the curve with equation $y = x^2$.

(c) Find $f(x)$.

(2)

(d) Explain why the function f does not have an inverse.

(1)

7. (a) Express $2\sin\theta - 1.5\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 4 decimal places. **(3)**

(b) (i) Find the maximum value of $2\sin\theta - 1.5\cos\theta$.

(ii) Find the value of θ , for $0 \leq \theta < \pi$, at which this maximum occurs. **(3)**

Tom models the height of sea water, H metres, on a particular day by the equation

$$H = 6 + 2\sin\left(\frac{4\pi t}{25}\right) - 1.5\cos\left(\frac{4\pi t}{25}\right), \quad 0 \leq t < 12,$$

where t hours is the number of hours after midday.

(c) Calculate the maximum value of H predicted by this model and the value of t , to 2 decimal places, when this maximum occurs. **(3)**

(d) Calculate, to the nearest minute, the times when the height of sea water is predicted, by this model, to be 7 metres. **(6)**

Question 7 continued

Lined writing area for question 7.

5.

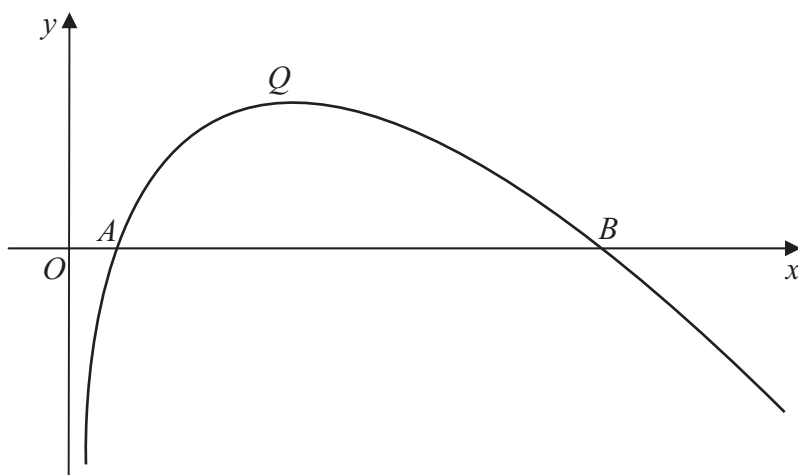
**Figure 1**

Figure 1 shows a sketch of part of the curve with equation $y = f(x)$, where

$$f(x) = (8 - x) \ln x, \quad x > 0$$

The curve cuts the x -axis at the points A and B and has a maximum turning point at Q , as shown in Figure 1.

(a) Write down the coordinates of A and the coordinates of B . (2)

(b) Find $f'(x)$. (3)

(c) Show that the x -coordinate of Q lies between 3.5 and 3.6 (2)

(d) Show that the x -coordinate of Q is the solution of

$$x = \frac{8}{1 + \ln x} \quad (3)$$

To find an approximation for the x -coordinate of Q , the iteration formula

$$x_{n+1} = \frac{8}{1 + \ln x_n}$$

is used.

(e) Taking $x_0 = 3.55$, find the values of x_1 , x_2 and x_3 .
Give your answers to 3 decimal places. (3)

6. The function f is defined by

$$f: x \mapsto \frac{3 - 2x}{x - 5}, \quad x \in \mathbb{R}, \quad x \neq 5$$

(a) Find $f^{-1}(x)$.

(3)

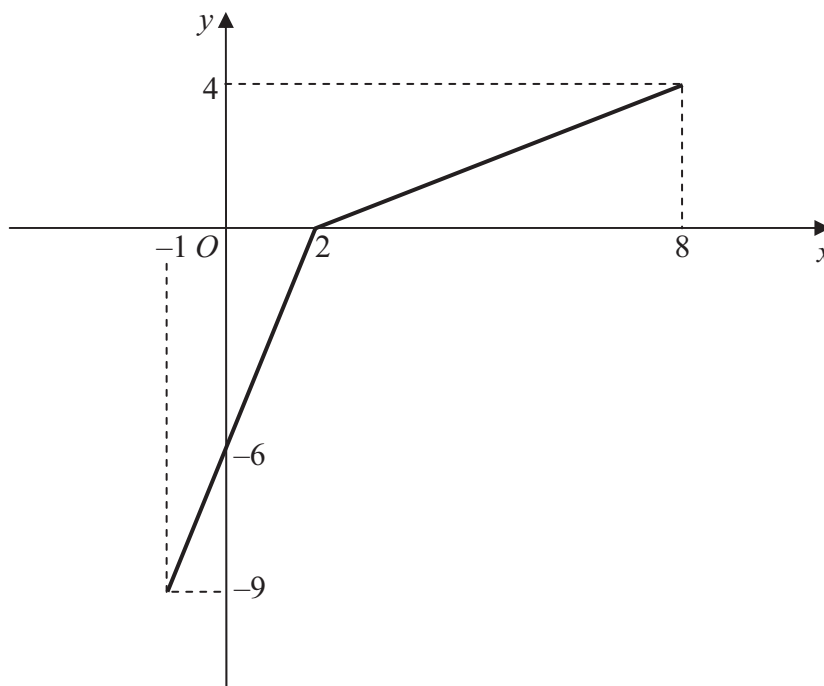


Figure 2

The function g has domain $-1 \leq x \leq 8$, and is linear from $(-1, -9)$ to $(2, 0)$ and from $(2, 0)$ to $(8, 4)$. Figure 2 shows a sketch of the graph of $y = g(x)$.

(b) Write down the range of g .

(1)

(c) Find $gg(2)$.

(2)

(d) Find $fg(8)$.

(2)

(e) On separate diagrams, sketch the graph with equation

(i) $y = |g(x)|$,

(ii) $y = g^{-1}(x)$.

Show on each sketch the coordinates of each point at which the graph meets or cuts the axes.

(4)

(f) State the domain of the inverse function g^{-1} .

(1)

3.

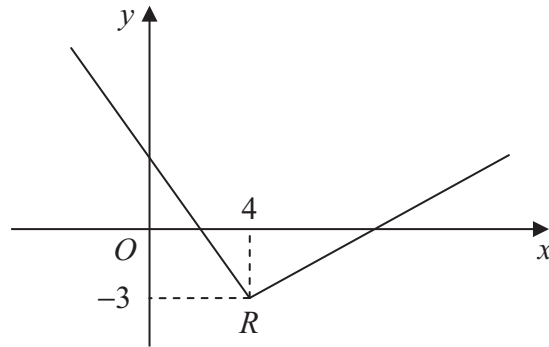


Figure 1

Figure 1 shows part of the graph of $y = f(x)$, $x \in \mathbb{R}$.

The graph consists of two line segments that meet at the point $R(4, -3)$, as shown in Figure 1.

Sketch, on separate diagrams, the graphs of

(a) $y = 2f(x+4)$, **(3)**

(b) $y = |f(-x)|$. **(3)**

On each diagram, show the coordinates of the point corresponding to R .

2.

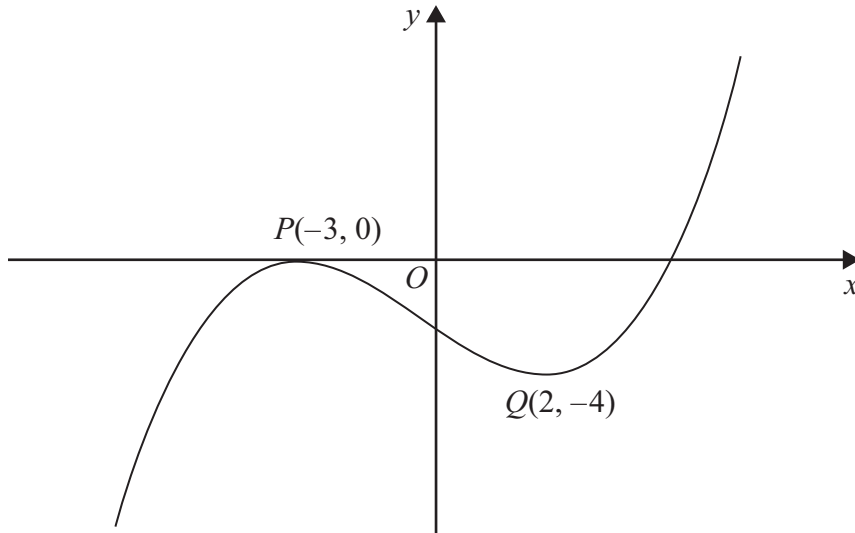


Figure 1

Figure 1 shows the graph of equation $y = f(x)$.

The points $P(-3, 0)$ and $Q(2, -4)$ are stationary points on the graph.

Sketch, on separate diagrams, the graphs of

(a) $y = 3f(x + 2)$

(3)

(b) $y = |f(x)|$

(3)

On each diagram, show the coordinates of any stationary points.

Question 8 continued

Lined writing area for the response to Question 8. The area consists of 24 horizontal lines for writing.

(Total 13 marks)

Q8

--	--

TOTAL FOR PAPER: 75 MARKS

END

4.

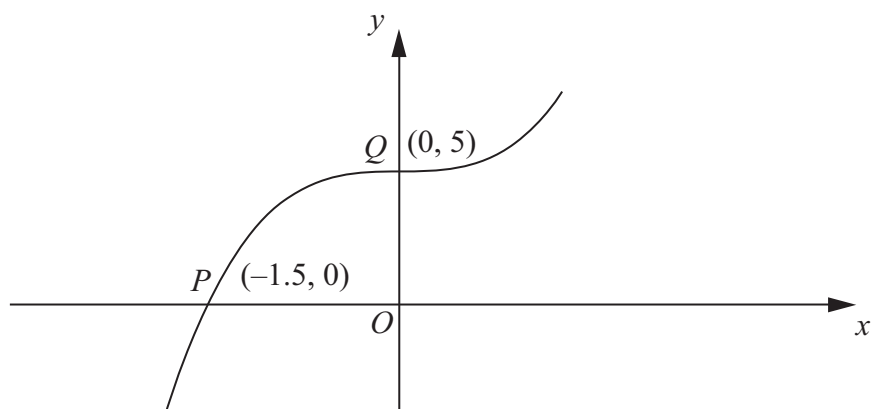


Figure 2

Figure 2 shows part of the curve with equation $y = f(x)$
The curve passes through the points $P(-1.5, 0)$ and $Q(0, 5)$ as shown.

On separate diagrams, sketch the curve with equation

(a) $y = |f(x)|$ **(2)**

(b) $y = f(|x|)$ **(2)**

(c) $y = 2f(3x)$ **(3)**

Indicate clearly on each sketch the coordinates of the points at which the curve crosses or meets the axes.

Question 4 continued

3.

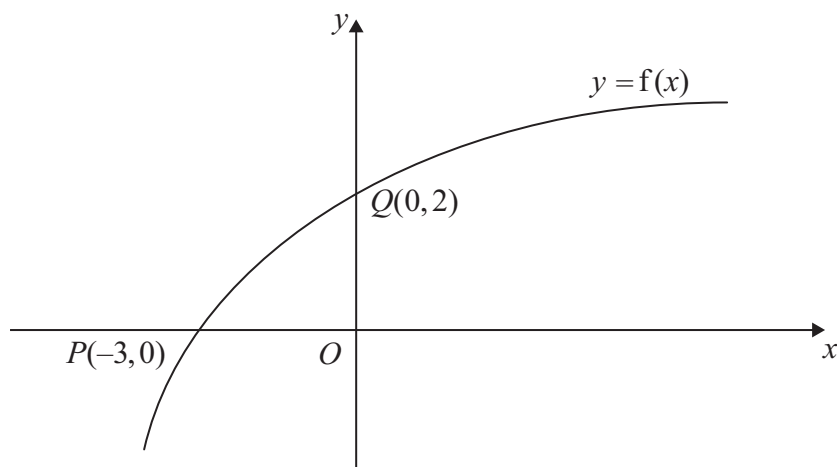


Figure 1

Figure 1 shows part of the curve with equation $y = f(x)$, $x \in \mathbb{R}$.

The curve passes through the points $Q(0, 2)$ and $P(-3, 0)$ as shown.

- (a) Find the value of $ff(-3)$. (2)

On separate diagrams, sketch the curve with equation

- (b) $y = f^{-1}(x)$, (2)

- (c) $y = f(|x|) - 2$, (2)

- (d) $y = 2f\left(\frac{1}{2}x\right)$. (3)

Indicate clearly on each sketch the coordinates of the points at which the curve crosses or meets the axes.

Question 3 continued

2.

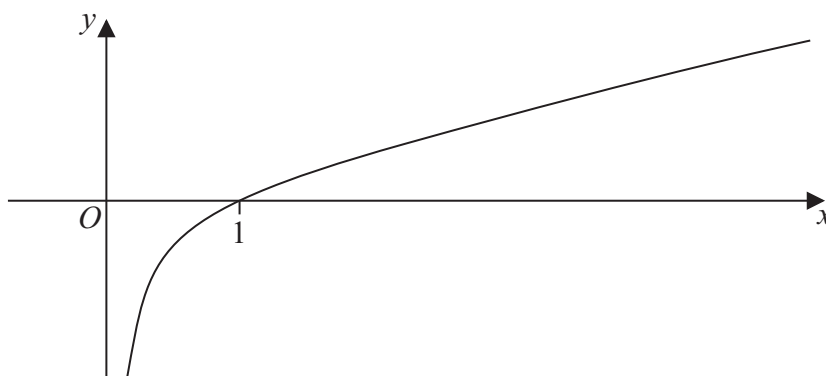
**Figure 1**

Figure 1 shows a sketch of the curve with equation $y = f(x)$, $x > 0$, where f is an increasing function of x . The curve crosses the x -axis at the point $(1, 0)$ and the line $x = 0$ is an asymptote to the curve.

On separate diagrams, sketch the curve with equation

(a) $y = f(2x)$, $x > 0$ **(2)**

(b) $y = |f(x)|$, $x > 0$ **(3)**

Indicate clearly on each sketch the coordinates of the point at which the curve crosses or meets the x -axis.

Question 4 continued

A series of horizontal lines provided for writing the answer to Question 4.

7.

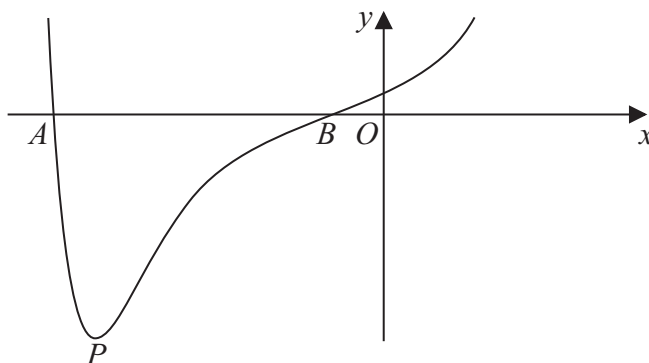


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = f(x)$ where

$$f(x) = (x^2 + 3x + 1)e^{x^2}$$

The curve cuts the x -axis at points A and B as shown in Figure 2.

- (a) Calculate the x coordinate of A and the x coordinate of B , giving your answers to 3 decimal places. (2)

- (b) Find $f'(x)$. (3)

The curve has a minimum turning point at the point P as shown in Figure 2.

- (c) Show that the x coordinate of P is the solution of

$$x = -\frac{3(2x^2 + 1)}{2(x^2 + 2)}$$
(3)

- (d) Use the iteration formula

$$x_{n+1} = -\frac{3(2x_n^2 + 1)}{2(x_n^2 + 2)}, \quad \text{with } x_0 = -2.4,$$

to calculate the values of x_1 , x_2 and x_3 , giving your answers to 3 decimal places. (3)

The x coordinate of P is α .

- (e) By choosing a suitable interval, prove that $\alpha = -2.43$ to 2 decimal places. (2)

Question 7 continued

A series of 22 horizontal lines for writing an answer.

8.

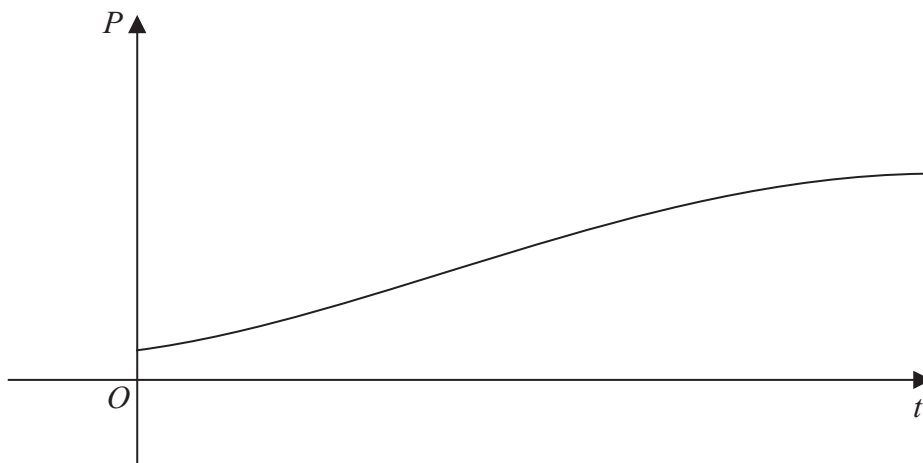


Figure 3

The population of a town is being studied. The population P , at time t years from the start of the study, is assumed to be

$$P = \frac{8000}{1 + 7e^{-kt}}, \quad t \geq 0,$$

where k is a positive constant.

The graph of P against t is shown in Figure 3.

Use the given equation to

(a) find the population at the start of the study, (2)

(b) find a value for the expected upper limit of the population. (1)

Given also that the population reaches 2500 at 3 years from the start of the study,

(c) calculate the value of k to 3 decimal places. (5)

Using this value for k ,

(d) find the population at 10 years from the start of the study, giving your answer to 3 significant figures. (2)

(e) Find, using $\frac{dP}{dt}$, the rate at which the population is growing at 10 years from the start of the study. (3)

2. Given that

$$f(x) = \ln x, \quad x > 0$$

sketch on separate axes the graphs of

(i) $y = f(x)$,

(ii) $y = |f(x)|$,

(iii) $y = -f(x - 4)$.

Show, on each diagram, the point where the graph meets or crosses the x -axis.
In each case, state the equation of the asymptote.

(7)

7. The function f has domain $-2 \leq x \leq 6$ and is linear from $(-2, 10)$ to $(2, 0)$ and from $(2, 0)$ to $(6, 4)$. A sketch of the graph of $y = f(x)$ is shown in Figure 1.

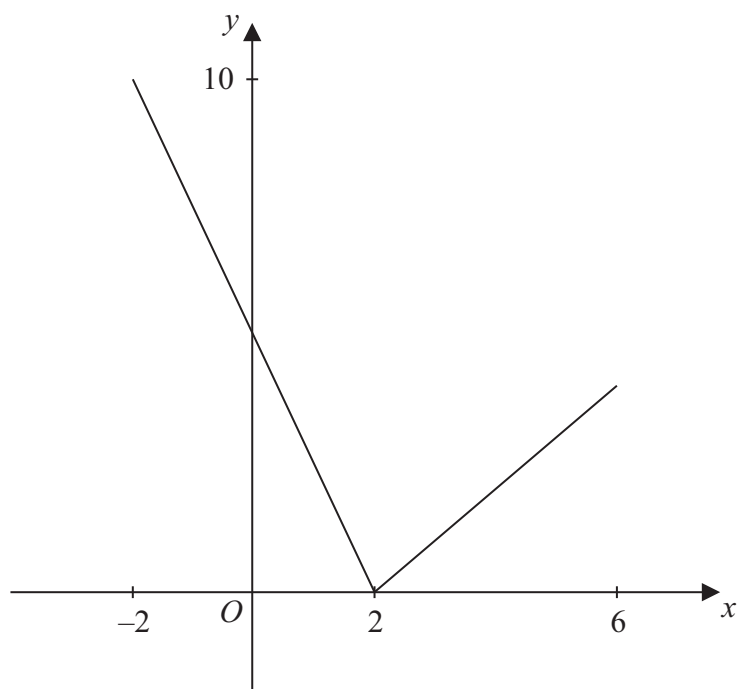


Figure 1

- (a) Write down the range of f . (1)
- (b) Find $ff(0)$. (2)

The function g is defined by

$$g : x \rightarrow \frac{4 + 3x}{5 - x}, \quad x \in \mathbb{R}, \quad x \neq 5$$

- (c) Find $g^{-1}(x)$ (3)
- (d) Solve the equation $gf(x) = 16$ (5)

8.

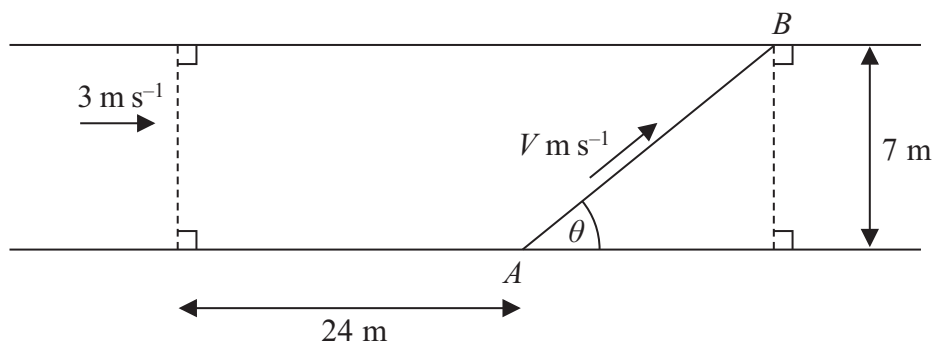


Figure 2

Kate crosses a road, of constant width 7 m, in order to take a photograph of a marathon runner, John, approaching at 3 m s^{-1} .

Kate is 24 m ahead of John when she starts to cross the road from the fixed point A .

John passes her as she reaches the other side of the road at a variable point B , as shown in Figure 2.

Kate's speed is $V \text{ m s}^{-1}$ and she moves in a straight line, which makes an angle θ , $0 < \theta < 150^\circ$, with the edge of the road, as shown in Figure 2.

You may assume that V is given by the formula

$$V = \frac{21}{24 \sin \theta + 7 \cos \theta}, \quad 0 < \theta < 150^\circ$$

- (a) Express $24 \sin \theta + 7 \cos \theta$ in the form $R \cos(\theta - \alpha)$, where R and α are constants and where $R > 0$ and $0 < \alpha < 90^\circ$, giving the value of α to 2 decimal places. (3)

Given that θ varies,

- (b) find the minimum value of V . (2)

Given that Kate's speed has the value found in part (b),

- (c) find the distance AB . (3)

Given instead that Kate's speed is 1.68 m s^{-1} ,

- (d) find the two possible values of the angle θ , given that $0 < \theta < 150^\circ$. (6)

Question 8 continued

Lined area for writing the answer to Question 8.

Q8

--	--

(Total 14 marks)

TOTAL FOR PAPER: 75 MARKS

END

--

Core Mathematics C3

Candidates sitting C3 may also require those formulae listed under Core Mathematics C1 and C2.

Logarithms and exponentials

$$e^{x \ln a} = a^x$$

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad (A \pm B \neq (k + \frac{1}{2})\pi)$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

Differentiation

f(x)	f'(x)
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\frac{f(x)}{g(x)}$	$\frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

Core Mathematics C2

Candidates sitting C2 may also require those formulae listed under Core Mathematics C1.

Cosine rule

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Binomial series

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \dots + \binom{n}{r} a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N})$$

$$\text{where } \binom{n}{r} = {}^n C_r = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1 \times 2} x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{1 \times 2 \times \dots \times r} x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

Logarithms and exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Geometric series

$$u_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Numerical integration

The trapezium rule: $\int_a^b y \, dx \approx \frac{1}{2} h \{(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})\}$, where $h = \frac{b-a}{n}$

Core Mathematics C1

Mensuration

$$\text{Surface area of sphere} = 4\pi r^2$$

$$\text{Area of curved surface of cone} = \pi r \times \text{slant height}$$

Arithmetic series

$$u_n = a + (n - 1)d$$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n[2a + (n - 1)d]$$