GCE AS Mathematics (8MA0) – Paper 1
Pure Mathematics

Summer 2019 student-friendly mark scheme

Please note that this mark scheme is not the one used by examiners for making scripts. It is intended more as a guide to good practice, indicating where marks are given for model solutions. As such, it doesn't show follow-through marks (marks that are awarded despite errors being made) or special cases.

It should also be noted that for many questions, there may be alternative methods of finding correct solutions that are not shown here – they will be covered in the formal mark scheme.

Guidance on the use of codes within this document
M1 – method mark. This mark is generally given for an appropriate method in the context of the question. This mark is given for showing your working and may be awarded even if working is incorrect. A1 – accuracy mark. This mark is generally given for a correct answer following correct working. B1 – working mark. This mark is usually given when working and the answer cannot easily be separated. Some questions require all working to be shown; in such questions, no marks will be given for an answer with no working (even if it is a correct answer).

Question 1 (Total 4 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$y = -\frac{1}{2}x + \frac{3}{4}$	M1	This mark is given for a method to rearrange to find an equation for l_1 in terms of $y =$
	$m = 2$	A1	This mark is given for deducing the gradient of the perpendicular line l_2
(b)	Substituting $y = 2x + 7$ into $2x + 4y - 3 = 0$ gives $2x + 4(2x + 7) - 3 = 0$	M1	This mark is given for a method to substitute to form and solve an equation in a single variable.
	$10x + 25$ $x = -2.5$	A1	This mark is given for solving to find the value of the x-coordinate of the point P.

Question 2 (Total 8 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(i)	$16a^2 = 2\sqrt{a}$ so $\dfrac{16a^2}{2a^{\frac{1}{2}}} = 1$ $8a^{\frac{3}{2}} = 1$ so $a^{\frac{3}{2}} = \frac{1}{8}$	M1	This mark is given for a method to find an equation to solve with the terms in a on one side
	$a = \left(\frac{1}{8}\right)^{\frac{3}{2}}$	M1	This mark is given for finding a way to deal with the indices when solving the equation
	$a = \frac{1}{4}$	A1	This mark is given for finding one correct solution to the equation
	$a = 0$ is also a solution	B1	This mark is given for deducing that $a = 0$ is also a solution
(ii)	$b^4 + 7b^2 - 18 = 0$ factorises to $(b^2 + 9)(b^2 - 2) = 0$	M1	This mark is given for factorising the equation given
	$b^2 = -9, 2$	A1	This mark is given for finding two correct solutions for b^2
	For real solutions, $b^2 = 2$ only	M1	This mark is given for recognising that $b = \sqrt{-9}$ is not a real solution
	$b = \sqrt{2}, -\sqrt{2}$	A1	This mark is given for finding the two real solutions to the equation

Question 3 (Total 6 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$\int 4x^{-3} + kx \ \mathrm{d}x = -2x^{-2} + \frac{1}{2}kx$	M1	This mark is given for recognising that x^n becomes x^{n+1} when integrating
		A1	This mark is given for two correctly integrated terms (without c)
	$-\frac{2}{x^2} + \frac{kx^2}{2} + c$	A1	This mark is given for a full answer with a constant (in any correct form)
(b)	$\left[-\frac{2}{x^2} + \frac{k}{2}x^2\right]_{0.5}^{2} =$ $\left(-\frac{2}{2^2} + \frac{4k}{2}\right) - \left(-\frac{2}{0.5^2} + \frac{0.5^2 k}{2}\right) = 8$	M1	This mark is given for substituting the limits 2 and 0.5 and setting equal to 8
	$\left(-\frac{1}{2} + 2k\right) - \left(-8 + \frac{k}{8}\right) = 8$ $7.5 + \frac{15}{8}k = 8$	M1	This mark is given for a method to solve a linear equation in k
	$k = \frac{4}{15}$	A1	This mark is given for finding a correct value for k

Question 4 (Total 5 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$2.35 = 3m + b$ $3.28 = 6m + b$	M1	This mark is given for using the information to create a model of the form $H = mt + b$ where m is the rate of growth and b is the original height of the tree
	$0.93 = 3m$, $m = 0.31$	M1	This mark is given for finding a value for m
	$H = 0.31m$ so $b = 1.42$ $H = 0.31m + 1.42$	A1	This mark is given for finding a value for b
(b)	b represents the original height of the tree	B1	This mark is given for recognising what b represents
	140 cm = 1.4 m, very close to 1.42 m so supports the use of a linear model	B1	This mark is given for a valid statement to show the use of a linear model is justified

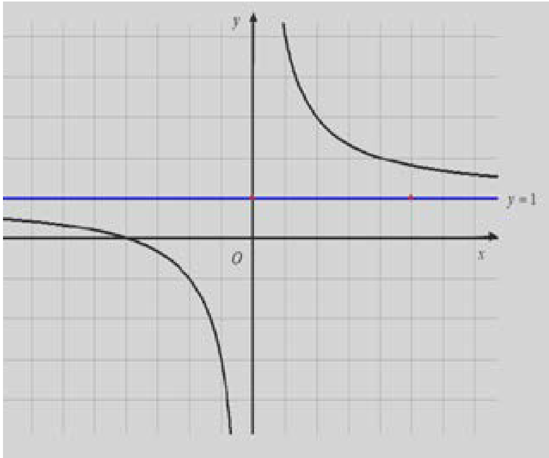
Question 5 (Total 5 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)		M1	This mark is given for recognising that x^n becomes x^{n-1} when differentiating
	$\frac{dy}{dx} = 6x - \frac{24}{x^2}$	A1	This mark is given for one of the two terms $6x$ or $-\frac{24}{x^2}$ given correctly
		A1	This mark is given for $\frac{dy}{dx}$ given fully correct
(b)	$6x - \frac{24}{x^2} > 0$	M1	This mark is given for setting $\frac{dy}{dx}$ greater than 0 (allow $\geq$)
	$6x^3 - 24 > 0$ $x^3 - 4 > 0$ $x > \sqrt[3]{4}$	A1	This mark is given for the exact range of values of x for which the curve is increasing (allow $\geq$)

Question 6 (Total 6 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$18\sqrt{3} = \frac{1}{2} \times 2x \times 3x \times \sin 60°$	M1	This mark is given for use of the formula $A = \frac{1}{2}ab \sin C$ for the area of the triangle
	$18\sqrt{3} = 3x^2 \times \frac{\sqrt{3}}{2}$ $x^2 = 12$	M1	This mark is given for using a value of sin 60° to find a value for x^2
	$x = \sqrt{12}$ $= \sqrt{(4 \times 3)}$ $= 2\sqrt{3}$	A1	This mark is given for a full solution to show that $x = 2\sqrt{3}$
(b)	$BC^2 =$ $(6\sqrt{3})^2 + (4\sqrt{3})^2 - 2 \times 6\sqrt{3} \times 4\sqrt{3} \times \cos 60°$	M1	This mark is given for using the cosine rule to start to find the length BC
	$BC^2 = 84$	A1	This mark is given for finding a value for BC^2
	$BC = 2\sqrt{21}$	A1	This mark is given for a correct answer presented as a simplified surd

Question 7 (Total 8 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)		M1	This mark is given for a graph with shape $\frac{1}{x}$ in the first quadrant
		A1	This mark is given for a fully correct sketch
		B1	This mark is given for the asymptote $y = 1$ correctly shown on the sletch
(b)	$\frac{k^2}{x} + 1 = -2x + 5$	M1	This mark is given for deducing the point of intersection
	$k^2 + x = -2x^2 + 5x$ $-2x^2 + 5x - x - k^2 = 0$ $2x^2 - 4x + k^2 = 0$	A1	This mark is given for correct working to show the result required
(c)	$16 = 4 \times 2 \times k^2$ $16 = 8k^2$	M1	This mark is given for deducing that the equation has a single root and setting $b^2 - 4ac = 0$
		A1	This mark is given for correctly finding $b^2 - 4ac = 0$
	$k^2 = \pm\sqrt{2}$	A1	This mark is given for finding the two exact values of k

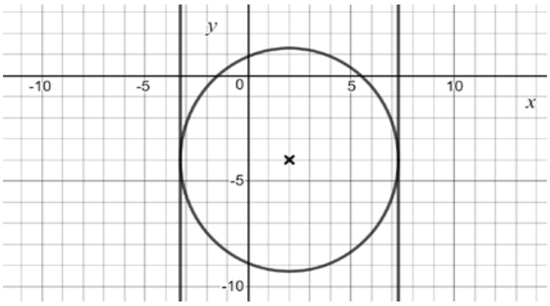
Question 8 (Total 5 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$2^6 = 64$	B1	This mark is given for finding the first term of the expansion
	$\left(2+\frac{3x}{4}\right)^6 =$ $2^6 + {}^6C_1\, 2^5 \left(\frac{3x}{4}\right)^1 + {}^6C_2\, 2^4 \left(\frac{3x}{4}\right)^2 + \ldots$	M1	This mark is given for a method to write out the binomial expansion
	$= 64 + (6 \times 32 \times \frac{3x}{4}) + (15 \times 16 \times \frac{9x^2}{16}) + \ldots$	A1	This mark is given for a correct binomial expansion up to the second and third terms
	$= 64 + 144x + 135x^2 + \ldots$	A1	This mark is given for a fully correct binomial expansion
(b)	$2 + \frac{3x}{4} = 1.925$ $\frac{3x}{4} = -0.075$ so $x = -0.1$ So find the value of $64 + 144x + 135x^2 + \ldots$ with $x = -0.1$	B1	This mark is given for a correct explanation of how the expansion could be used to find an estimate for 1.925^6

Question 9 (Total 6 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$1200 - 3(1 - 20)^2$ $= 1200 - 3(-19)^2$ $= 1200 - 1083$ $= 117$ tonnes	B1	This mark is given for the correct answer
(b)	1200 tonnes	B1	This mark is given for deducing that $(n - 20)^2$ is always positive, and so deducing the maximum value for T Units (tonnes) must be stated
(c)	$[1200 - 3(5 - 20)^2] - [1200 - 3(4 - 20)^2]$ $= 525 - 432$	M1	This mark is given for a method to find the mass of tin that will be mined in 2023
	$= 93$ tonnes	A1	This mark is given for the correct answer (units need not be given)
(d)	$n \leq 20$	B1	This mark is given for an appreciation that the model only predicts the mass of tin mined for the next 20 years
	This model predicts that the mass of tin mined will increase each year	B1	This mark is given for an appreciation that the total mass of tin mined cannot decrease but that for $n > 20$ the value of T decreases as n increases.

Question 10 (Total 5 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$(x - 2)^2 + (y + 4)^2 - 4 - 16 - 8 = 0$	M1	This mark is given for a method to complete the square
	Centre at $x = 2$ and $y = -4$, $(2, -4)$	A1	This mark is given for finding the correct coordinates of the centre of the circle
	$(x - 2)^2 + (y + 4)^2 - 28 = 0$ Radius $= \sqrt{28} = 2\sqrt{7}$	A1	This mark is given for finding the exact radius of the circle
(b)	 Tangent of $x = k$ touches circle at $2 + \sqrt{28}$ and $2 - \sqrt{28}$	M1	This mark is given for adding or subtracting the length of the radius of the circle from 2
		A1	This mark is given for deducing both values of k

Question 11 (Total 10 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$f(4) = (2 \times 4^3) - (13 \times 4^2) + (8 \times 4) + 48$ $= 128 - 208 + 32 + 48$	M1	This mark is given for a method to find f(4)
	$f(4) = 0$ so $(x - 4)$ is a factor	A1	
(b)	$2x^3 - 13x^2 + 8x + 48 =$ $(x - 4)(2x^2 - 5x - 12)$	M1	This mark is given for attempting to factorise the expression ($2x^2$ and -12 seen)
		A1	This mark is given for a fully correct factorisation
	$2x^2 - 5x - 12 = (x - 4)(2x + 3)$	M1	This mark is given for factorisng the expression $2x^2 - 5x - 12$
	$f(x) = (x - 4)^2(2x + 3)$ Thus f(x) = only has two roots 4 and $-\frac{3}{2}$	A1	This mark is given for a valid explanation of why the expression only has two roots
(c)	The curve will move two units down	M1	This mark is given for deducing that the curve will be translated by two units
	The curve will cross the axis at three places and so have three roots	A1	This mark is given for deducing that the curve will intersect the x-axis in three places and so the expression will have three roots
(d)	Since f(x) passes through the origin, $f(0) = 0$ so	M1	This mark is given for deducing that $f(0) = 0$ when $f(x + k) = 0$
	$f(x + k) = 0$ when $k = 4, \ -\frac{3}{2}$	A1	This mark is given for the correct two values of k

Question 12 (Total 7 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$\dfrac{10\sin^2\theta - 7\cos\theta + 2}{3 + 2\cos\theta}$ $\equiv \dfrac{10(1-\cos^2\theta) - 7\cos\theta + 2}{3 + 2\cos\theta}$	M1	This mark is given for using the identity sin θ = 1 – cos^2 θ in the fraction
	$\equiv \dfrac{12 - 7\cos\theta - 10\cos^2\theta}{3 + 2\cos\theta}$	A1	This mark is given for finding a simplified expression in terms of cos θ only
	$\equiv \dfrac{(3+2\cos\theta)(4-5\cos\theta)}{3+2\cos\theta}$	M1	This mark is given for factorising the numerator of the expression
	$\equiv 4 - 5\cos\theta$	A1	This mark is given for a fully correct proof with correct notation and no errors.
(b)	$4 - 5\cos x = 4 + 3\sin x$ $\tan x = -\dfrac{5}{3}$	M1	This mark is given for substituting for the fraction and rearranging the equation, using $\dfrac{\sin x}{\cos x} = \tan x$
	$x = 121°$	A1	This mark is given for one correct value of x
	$x = 301°$	A1	This mark is given for the other correct value of x

Question 13 (Total 7 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 6x^2 - 34x + 40$	B1	This mark is given for the equation correctly differentiated
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ when $6x^2 - 34x + 40 = 0$	M1	This mark is given for setting the equation equal to zero
	$2(3x - 5)(x - 4) = 0$	M1	This mark is given for factorising the expression
	$(x = \frac{5}{3})$, $x = 4$,	A1	This mark is given for finding two solutions and choosing $x = 4$ as the upper limit of the integral
	$R = \int_0^4 2x^3 - 17x^2 + 40x \ \mathrm{d}x$ $= \left[\frac{1}{2}x^4 - \frac{17}{3}x^3 + 20x^2\right]_0^4$	B1	This mark is given for integrating the expression from 0 to 4
	$R = (\frac{1}{2} \times 4^4) - (\frac{17}{3} \times 4^3) + (20 \times 4^2)$	M1	This mark is given for a calculation for find the area
	$R = 127 - \frac{1088}{3} + 320 = \frac{256}{3}$	A1	This mark is given for a full proof with correct notation and no errors

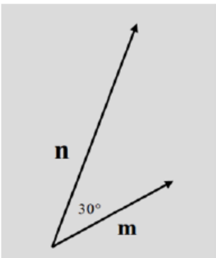
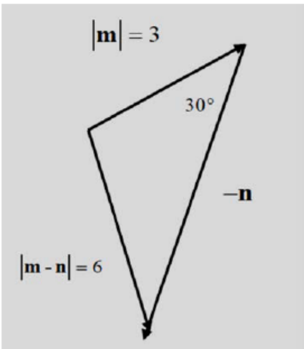
Question 14 (Total 9 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$(15\,700 \times e^0) + 2300 = 18\,000$	B1	This mark is given for a correct value for the initial value of the car
(b)(i)	$\frac{dV}{dt} = (-0.25 \times 15\,700)\ e^{-0.25t}$ $= -3925e^{-0.25t}$	M1	This mark is given for making the link between gradient and rate of change and finding $\frac{dV}{dt} = ke^{-0.25t}$
		A1	This mark is given for a fully correct expression for $\frac{dV}{dt}$
	$-3925e^{-0.25T} = -500$ thus $3925e^{-0.25T} = 500$	A1	This mark is given fully correct working to show that $3925e^{-0.25T} = 500$
(b)(ii)	$e^{-0.25T} = \frac{500}{3925}$ $-0.25T = \ln \frac{500}{3925}$	M1	This mark is given for the start of a method to find the age of the car using logarithms
	$T = \frac{\ln 0.127...}{-0.25} = \frac{-2.0605...}{-0.25} = 8.24\ldots$	A1	This mark is given for rearranging and solving for T
	8 years and 3 months	A1	This mark is given for finding the answer in years and months to the nearest month
(c)	£2300	B1	This mark is given for deducing from the original equation that as $e^{-0.25t}$ tends to zero, V tends to 2300
(d)	Other factors can affect the price such as mileage or condition The price may rise as the car becomes rare	B1	This mark is given for any valid limitation to the model stated

Question 15 (Total 4 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	If n is even, $n = 2k$ and $n^3 + 2 = (2k)^3 + 2$ $= 8k^3 + 2$	M1	This mark is given for finding expressions for n and $n^3 + 2$ when n is even
	$8k^3 + 2$ is two more than a multiple of 8 and so not divisible by 8	A1	This mark is given for a correct conclusion following correct working
	If n is odd, $n = 2k + 1$ and $n^3 + 2 = (2k + 1)^3 + 2$ $= 8k^3 + 12k^2 + 6k + 3$	M1	This mark is given for finding expressions for n and $n^3 + 2$ when n is odd
	$8k^3 + 12k^2 + 6k + 3$ is an odd number and so not divisible by 8	A1	This mark is given for a correct conclusion following correct working

Question 16 (Total 5 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(i)	**a** and **b** lie in the same direction	B1	This mark is given for a valid explanation
(ii)		M1	This mark is given for showing the vector problem graphically (may be implied)
	$\frac{\sin 30°}{6} = \frac{\sin\theta}{3}$ $\sin\theta = \frac{1.5}{6} = \frac{1}{4}$	M1	This mark is given for using the sine rule as a method to find the angle between –**n** and **n** – **m**
	$\theta = 14.5°$	A1	This mark is given for finding the the angle between –**n** and **n** – **m**
	Angle between **m** and **m** – **n** = (180 – 30 – 14.5) = 135.5°	A1	This mark is given for the angle between vector **m** and vector **m** – **n**