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**Pearson Edexcel Level 3 GCE**

Centre Number | Candidate Number

# Mathematics

## Advanced Subsidiary
## Paper 1: Pure Mathematics

Sample Assessment Material for first teaching September 2017

**Time: 2 hours**

Paper Reference **8MA0/01**

**You must have:**
Mathematical Formulae and Statistical Tables, calculator

Total Marks

**Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 17 questions in this question paper. The total mark for this paper is 100.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

*Turn over* ▶

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**Answer ALL questions. Write your answers in the spaces provided.**

**1.** The line $l$ passes through the points $A$ (3, 1) and $B$ (4, −2).

Find an equation for $l$.

**(3)**

**(Total for Question 1 is 3 marks)**

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**2.** The curve $C$ has equation

$$y = 2x^2 - 12x + 16$$

Find the gradient of the curve at the point $P$ (5, 6).

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

**(4)**

**(Total for Question 2 is 4 marks)**

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**3.** Given that the point $A$ has position vector $3\mathbf{i} - 7\mathbf{j}$ and the point $B$ has position vector $8\mathbf{i} + 3\mathbf{j}$,

(a) find the vector $\overrightarrow{AB}$

**(2)**

(b) Find $|\overrightarrow{AB}|$. Give your answer as a simplified surd.

**(2)**

**(Total for Question 3 is 4 marks)**

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**4.**

$$f(x) = 4x^3 - 12x^2 + 2x - 6$$

(a) Use the factor theorem to show that $(x - 3)$ is a factor of $f(x)$.

**(2)**

(b) Hence show that 3 is the only real root of the equation $f(x) = 0$

**(4)**

**(Total for Question 4 is 6 marks)**

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**5.** Given that

$$f(x) = 2x + 3 + \frac{12}{x^2}, \quad x > 0$$

show that $\int_1^{2\sqrt{2}} f(x)\,dx = 16 + 3\sqrt{2}$

**(5)**

**(Total for Question 5 is 5 marks)**

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**6.** Prove, from first principles, that the derivative of $3x^2$ is $6x$.

**(4)**

**(Total for Question 6 is 4 marks)**

**7.** (a) Find the first 3 terms, in ascending powers of $x$, of the binomial expansion of $\left(2 - \frac{x}{2}\right)^7$, giving each term in its simplest form.

**(4)**

(b) Explain how you would use your expansion to give an estimate for the value of $1.995^7$

**(1)**

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**Question 7 continued**

**(Total for Question 7 is 5 marks)**

**8.**

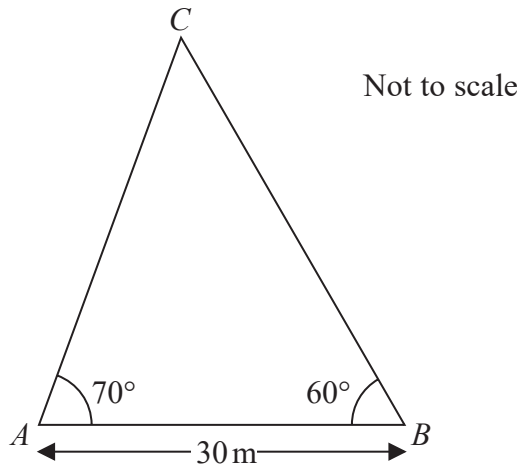


**Figure 1**

A triangular lawn is modelled by the triangle $ABC$, shown in Figure 1. The length $AB$ is to be 30 m long.

Given that angle $BAC = 70°$ and angle $ABC = 60°$,

(a) calculate the area of the lawn to 3 significant figures.

**(4)**

(b) Why is your answer unlikely to be accurate to the nearest square metre?

**(1)**

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**Question 8 continued**

**(Total for Question 8 is 5 marks)**

**9.** Solve, for $360° \leqslant x < 540°$,

$$12\sin^2 x + 7\cos x - 13 = 0$$

Give your answers to one decimal place.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

**(5)**

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**Question 9 continued**

**(Total for Question 9 is 5 marks)**

**10.** The equation $kx^2 + 4kx + 3 = 0$, where $k$ is a constant, has no real roots.

Prove that

$$0 \leqslant k < \frac{3}{4}$$

**(4)**

**(Total for Question 10 is 4 marks)**

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**11.** (a) Prove that for all positive values of $x$ and $y$

$$\sqrt{xy} \leqslant \frac{x + y}{2}$$

**(2)**

(b) Prove by counter example that this is not true when $x$ and $y$ are both negative.

**(1)**

**(Total for Question 11 is 3 marks)**

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**12.** A student was asked to give the exact solution to the equation

$$2^{2x+4} - 9(2^x) = 0$$

The student's attempt is shown below:

$$2^{2x+4} - 9(2^x) = 0$$

$$2^{2x} + 2^4 - 9(2^x) = 0$$

Let $\quad 2^x = y$

$$y^2 - 9y + 8 = 0$$

$$(y - 8)(y - 1) = 0$$

$$y = 8 \text{ or } y = 1$$

So $x = 3$ or $x = 0$

(a) Identify the two errors made by the student.

**(2)**

(b) Find the exact solution to the equation.

**(2)**

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**Question 12 continued**

**(Total for Question 12 is 4 marks)**

**13.** (a) Factorise completely $x^3 + 10x^2 + 25x$

**(2)**

(b) Sketch the curve with equation

$$y = x^3 + 10x^2 + 25x$$

showing the coordinates of the points at which the curve cuts or touches the $x$-axis.

**(2)**

The point with coordinates $(-3, 0)$ lies on the curve with equation

$$y = (x + a)^3 + 10(x + a)^2 + 25(x + a)$$

where $a$ is a constant.

(c) Find the two possible values of $a$.

**(3)**

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**Question 13 continued**

**(Total for Question 13 is 7 marks)**

**14.**

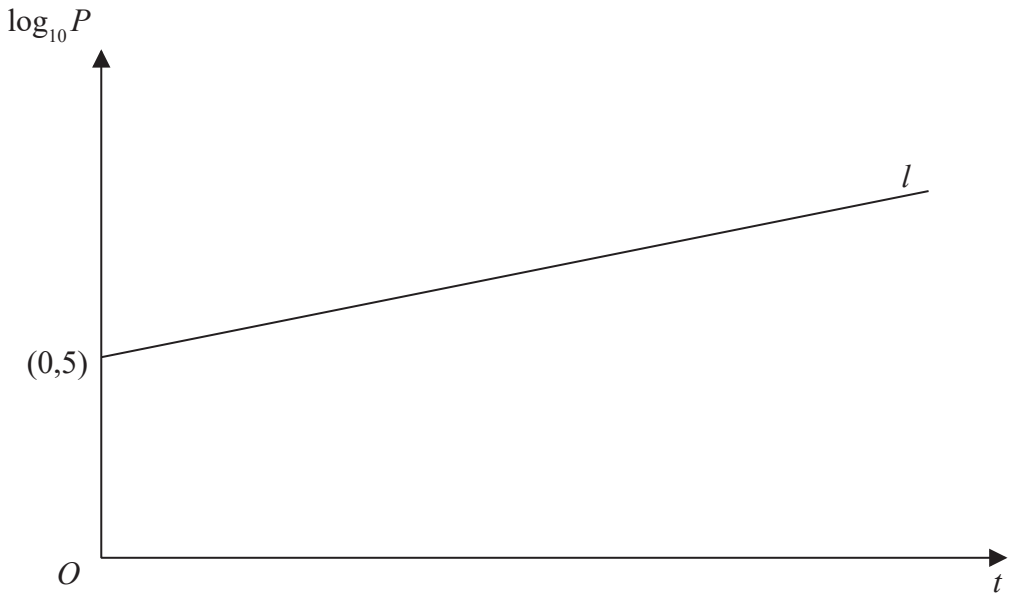


**Figure 2**

A town's population, $P$, is modelled by the equation $P = ab^t$, where $a$ and $b$ are constants and $t$ is the number of years since the population was first recorded.

The line $l$ shown in Figure 2 illustrates the linear relationship between $t$ and $\log_{10} P$ for the population over a period of 100 years.

The line $l$ meets the vertical axis at (0, 5) as shown. The gradient of $l$ is $\frac{1}{200}$.

(a) Write down an equation for $l$.

**(2)**

(b) Find the value of $a$ and the value of $b$.

**(4)**

(c) With reference to the model interpret

(i) the value of the constant $a$,

(ii) the value of the constant $b$.

**(2)**

(d) Find

(i) the population predicted by the model when $t = 100$, giving your answer to the nearest hundred thousand,

(ii) the number of years it takes the population to reach 200 000, according to the model.

**(3)**

(e) State two reasons why this may not be a realistic population model.

**(2)**

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**Question 14 continued**

**(Total for Question 14 is 13 marks)**

**15.**

Diagram not drawn to scale

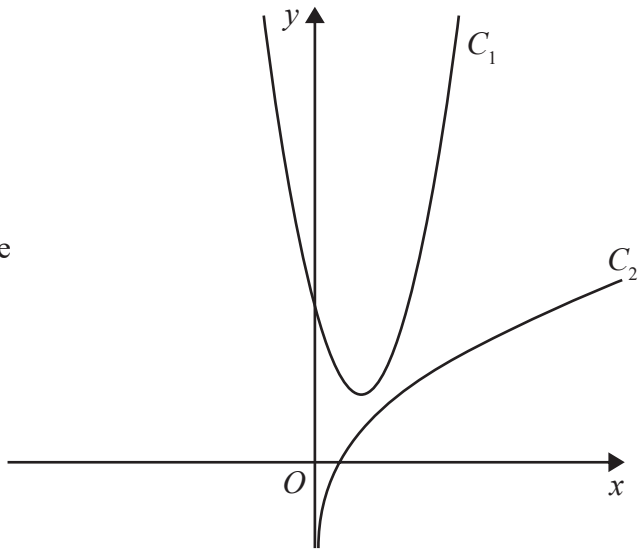


**Figure 3**

The curve $C_1$, shown in Figure 3, has equation $y = 4x^2 - 6x + 4$.

The point $P\left(\frac{1}{2}, 2\right)$ lies on $C_1$

The curve $C_2$, also shown in Figure 3, has equation $y = \frac{1}{2}x + \ln(2x)$.

The normal to $C_1$ at the point $P$ meets $C_2$ at the point $Q$.

Find the exact coordinates of $Q$.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

**(8)**

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**Question 15 continued**

**(Total for Question 15 is 8 marks)**

**16.**

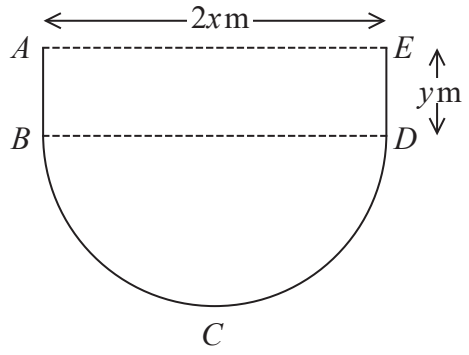


**Figure 4**

Figure 4 shows the plan view of the design for a swimming pool.

The shape of this pool *ABCDEA* consists of a rectangular section *ABDE* joined to a semicircular section *BCD* as shown in Figure 4.

Given that $AE = 2x$ metres, $ED = y$ metres and the area of the pool is $250\,\text{m}^2$,

(a) show that the perimeter, $P$ metres, of the pool is given by

$$P = 2x + \frac{250}{x} + \frac{\pi x}{2}$$

**(4)**

(b) Explain why $0 < x < \sqrt{\frac{500}{\pi}}$

**(2)**

(c) Find the minimum perimeter of the pool, giving your answer to 3 significant figures.

**(4)**

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**Question 16 continued**

**(Total for Question 16 is 10 marks)**

**17.** A circle $C$ with centre at $(-2, 6)$ passes through the point $(10, 11)$.

(a) Show that the circle $C$ also passes through the point $(10, 1)$.

**(3)**

The tangent to the circle $C$ at the point $(10, 11)$ meets the $y$ axis at the point $P$ and the tangent to the circle $C$ at the point $(10, 1)$ meets the $y$ axis at the point $Q$.

(b) Show that the distance $PQ$ is 58 explaining your method clearly.

**(7)**

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**Question 17 continued**

**Question 17 continued**

**(Total for Question 17 is 10 marks)**

**TOTAL FOR PAPER IS 100 MARKS**

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