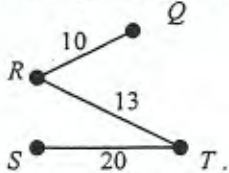
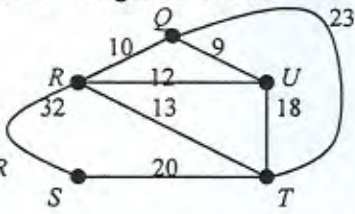


<p>1</p> <p>Original list 6 5 9 4 5 2</p> <p>After 1st pass 5 6 9 4 5 2</p> <p>After 2nd pass 5 6 9 4 5 2</p> <p>After 3rd pass 4 5 6 9 5 2</p> <p>After 4th pass 4 5 5 6 9 2</p> <p>After 5th pass 2 4 5 5 6 9</p> <p>May be shown vertically</p>	<p>M1</p> <p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p style="text-align: right;">5</p>	<p>Decreasing order can score method marks only</p> <p>For 1st pass correct with shuttle sort</p> <p>For 2nd pass correct with shuttle sort or follow through from previous list</p> <p>For 3rd pass correct with shuttle sort or follow through from previous list</p> <p>For 4th pass correct with shuttle sort or follow through from previous list</p> <p>For final list from correct shuttle sort, with results at end of each pass clearly shown</p>
<p>2</p> <p>(i) Number of arcs $\times 2 =$ sum of orders of vertices $\Rightarrow (3+3+4+4+4+4) \div 2 = 11$ arcs</p> <p>-----</p> <p>(ii) Semi-Eulerian, it has exactly two odd vertices</p> <p>-----</p> <p>(iii) Complete graph on five vertices has only 10 arcs, so 11 arcs means that all six vertices are connected.</p> <p>Or, a vertex of order 4 must join to four others so five vertices are connected. The sixth vertex has order at least three and cannot connect to itself so it must join to the other five.</p> <p>Or any equivalent reasoning.</p>	<p>M1</p> <p>A1</p> <p>B1</p> <p>B2</p> <p style="text-align: right;">5</p>	<p>For a general method</p> <p>For 11 calculated</p> <p><u>Drawing a specific case to get 11 scores B1 only</u></p> <p>For semi-Eulerian with a valid reason</p> <p>Accept 'two odd nodes' or 'two nodes of order 3' as minimal reasons</p> <p>For a good explanation of the general case by considering orders of vertices</p> <p>A weak explanation may score B1</p> <p>A diagram of a specific case is not sufficient</p>
<p>3</p> <p>(i) Minimum spanning tree with <i>U</i> removed</p>  <p>$QR + RT + TS = 43$ miles</p> <p>Join <i>U</i> back in using two shortest arcs</p> <p>$43 + 9 + 12$ $= 64$ miles</p> <p>-----</p> <p>(ii) Trying to apply nearest neighbour method</p>  <p>Start from <i>R</i></p> <p>to give <i>RQUTSR</i></p> <p>$= 89$ miles</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p style="text-align: right;">7</p>	<p>For 43 or arcs <i>QR, RT, TS</i> or a convincing attempt to find minimum spanning tree for $\{Q, R, S, T\}$</p> <p>For their $43 + 9 + 12$</p> <p>cao (miles may be implied)</p> <p>For a correct start to an application of nearest neighbour with any start vertex, ie at least: <i>QURTS, STRQU, TRQU</i> or <i>UQRTS</i></p> <p>For <i>R</i> as start vertex (may be implied from cycle)</p> <p>For <i>RQUTSR</i></p> <p>For 89 (miles may be implied) from valid method</p>

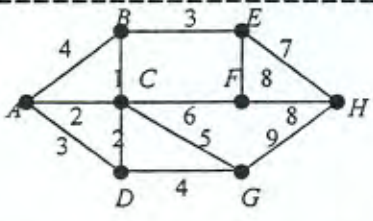
4

(i)

	1 st	3 rd	2 nd	4 th	5 th	7 th	6 th	8 th
	A	B	C	D	E	F	G	H
A	-	4	2	3	-	-	-	-
B	4	-	1	-	3	-	-	-
C	2	1	-	2	-	6	5	-
D	3	-	2	-	-	-	4	-
E	-	3	-	-	-	8	-	7
F	-	-	6	-	8	-	-	8
G	-	-	5	4	-	-	-	9
H	-	-	-	-	7	8	9	-

Quickest time is at least 25 hours

(ii)



(iii)

If AC is used then either B or D is excluded.
 Or must pass through C in getting between B and D, so AC is impossible.

(iv)

If EF is not used then passing through either E or F will take the team to H, the team will not be able to visit both E and F.

(v)

ABEFCDGH
 ADGCBEFH
 The second route is quicker (32 hours compared with 36 hours)

M1

M1

A1

B1

B1

M1

A1

B1

B1

M1

M1

A1

Answer should be on insert

For starting by choosing row C in column A
 For choosing more than one entry from column C

For a correct order (A), C, B, D, E, G, F, H

For correct entries chosen or a correct tree drawn



For 25
 Accept 'more than 25'

For a correct graph drawn

For correct weights shown

Follow through graph, if possible, provided same conclusion is valid

For explaining what happens if AC is used or why AC cannot be included.

Follow through graph, if possible, provided same conclusion is valid

For stating the effect of not using arc EF or for considering all possible routes into H

Follow through graph, if possible

For this route

For this route

For identifying ADGCBEFH as the quicker or for calculating 32

<p>5 (i) $x \geq 0, y \geq 0$ $y \leq 2x + 1$ $4x + 3y \leq 12$</p>	<p>B1 B1 B1</p>	<p>For both trivial constraints; allow > For this inequality, or equivalent; allow < For this inequality, or equivalent; allow <</p>																																																								
<p>(ii) (0, 0), (3, 0), (0, 1) (0.9, 2.8) (0, 0) $\rightarrow P = 0$; (0, 1) $\rightarrow P = 3$; (0.9, 2.8) $\rightarrow P = 12.9$; (3, 0) $\rightarrow P = 15$ $x = 3$ and $y = 0$ $P = 15$</p>	<p>B2 B1 M1 A1 A1</p>	<p>For these three vertices, any two correct \Rightarrow B1 For this vertex exact, in decimals or fractions For calculating $P = 5x + 3y$ for at least one of their vertices or clear evidence of using an appropriate line of constant profit For the correct values of x and y clearly identified For 15 clearly identified as the optimum value</p>																																																								
<p>(iii) Either consider the gradient of the profit line ($-\frac{1}{2}a$) and the gradients of the boundary lines (2 and $-1\frac{1}{2}$) or calculate Q at vertices $\Rightarrow 3, 0.9a+8.4, 3a$ Hence require $a \leq -6$</p>	<p>M1 M1 M1 A1</p>	<p>One method mark for each appropriate gradient calculated correctly or for each appropriate value of Q calculated correctly For the correct set of values identified [$a = -6$ or any valid proper subset of the correct answer with no method shown \Rightarrow B1 only]</p>																																																								
13																																																										
<p>6 (i) $5x + 3y - 5z + s = 15$ $2x + 6y + 8z + t = 24$</p>	<p>B1</p>	<p>For both equations correctly stated</p>																																																								
<p>(ii)</p> <table border="1" data-bbox="207 695 742 816"> <thead> <tr> <th>P</th> <th>x</th> <th>y</th> <th>z</th> <th>s</th> <th>t</th> <th>-</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>-2</td> <td>5</td> <td>1</td> <td>0</td> <td>0</td> <td>0</td> </tr> <tr> <td>0</td> <td>5</td> <td>3</td> <td>-5</td> <td>1</td> <td>0</td> <td>15</td> </tr> <tr> <td>0</td> <td>2</td> <td>6</td> <td>8</td> <td>0</td> <td>1</td> <td>24</td> </tr> </tbody> </table> <p>Pivot on 5 in x column</p> <table border="1" data-bbox="207 856 742 977"> <thead> <tr> <th>P</th> <th>x</th> <th>y</th> <th>z</th> <th>s</th> <th>t</th> <th>-</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>0</td> <td>6.2</td> <td>-1</td> <td>0.4</td> <td>0</td> <td>6</td> </tr> <tr> <td>0</td> <td>1</td> <td>0.6</td> <td>-1</td> <td>0.2</td> <td>0</td> <td>3</td> </tr> <tr> <td>0</td> <td>0</td> <td>4.8</td> <td>10</td> <td>-0.4</td> <td>1</td> <td>18</td> </tr> </tbody> </table>	P	x	y	z	s	t	-	1	-2	5	1	0	0	0	0	5	3	-5	1	0	15	0	2	6	8	0	1	24	P	x	y	z	s	t	-	1	0	6.2	-1	0.4	0	6	0	1	0.6	-1	0.2	0	3	0	0	4.8	10	-0.4	1	18	<p>B1 B1 B1 B1 M1 M1 A1</p>	<p>For $\pm(-2 \ 5 \ 1)$ in objective row Follow through from part (i) For 5 3 -5 1 0 15 and 2 6 8 0 1 24 or equivalent in constraint rows For correct pivot choice for their tableau For a correct method for their table and their pivot choice For increasing P For correct tableau or equivalent, cao</p>
P	x	y	z	s	t	-																																																				
1	-2	5	1	0	0	0																																																				
0	5	3	-5	1	0	15																																																				
0	2	6	8	0	1	24																																																				
P	x	y	z	s	t	-																																																				
1	0	6.2	-1	0.4	0	6																																																				
0	1	0.6	-1	0.2	0	3																																																				
0	0	4.8	10	-0.4	1	18																																																				
<p>(iii)</p> <p>Pivot on 10 in z column</p> <table border="1" data-bbox="207 1068 742 1189"> <thead> <tr> <th>P</th> <th>x</th> <th>y</th> <th>z</th> <th>s</th> <th>t</th> <th>-</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>0</td> <td>6.68</td> <td>0</td> <td>0.36</td> <td>0.1</td> <td>7.8</td> </tr> <tr> <td>0</td> <td>1</td> <td>1.08</td> <td>0</td> <td>0.16</td> <td>0.1</td> <td>4.8</td> </tr> <tr> <td>0</td> <td>0</td> <td>0.48</td> <td>1</td> <td>-0.04</td> <td>0.1</td> <td>1.8</td> </tr> </tbody> </table> <p>$x = 4.8, y = 0, z = 1.8$ $P = 7.8$</p>	P	x	y	z	s	t	-	1	0	6.68	0	0.36	0.1	7.8	0	1	1.08	0	0.16	0.1	4.8	0	0	0.48	1	-0.04	0.1	1.8	<p>M1 A1 A1 B1 B1</p>	<p>ft their tableau provided not yet optimal For correct pivot choice For correct tableau or equivalent, cao For all three correct values for their final tableau For correct value for their final tableau</p>																												
P	x	y	z	s	t	-																																																				
1	0	6.68	0	0.36	0.1	7.8																																																				
0	1	1.08	0	0.16	0.1	4.8																																																				
0	0	0.48	1	-0.04	0.1	1.8																																																				
<p>(iv) We must now pivot on the 2 in the x column, this gives</p> <table border="1" data-bbox="207 1340 742 1461"> <thead> <tr> <th>P</th> <th>x</th> <th>y</th> <th>z</th> <th>s</th> <th>t</th> <th>-</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>0</td> <td>11</td> <td>9</td> <td>0</td> <td>1</td> <td>24</td> </tr> <tr> <td>0</td> <td>0</td> <td>-12</td> <td>-25</td> <td>1</td> <td>-2.5</td> <td>$k-60$</td> </tr> <tr> <td>0</td> <td>1</td> <td>3</td> <td>4</td> <td>0</td> <td>0.5</td> <td>12</td> </tr> </tbody> </table> <p>Hence $y = 0$ Accept 'no change to y'</p>	P	x	y	z	s	t	-	1	0	11	9	0	1	24	0	0	-12	-25	1	-2.5	$k-60$	0	1	3	4	0	0.5	12	<p>M1 A1</p>	<p>For showing what happens to tableau, only need to show enough to be able to deduce answer (eg top row: 0 11 9 0 1 or y column) For correctly deducing $y = 0$ in general case. Only using a specific value of k (eg $k = 60$) with no general argument \Rightarrow M1, A0 Do not imply method mark from statement '$y = 0$' with no method seen.</p>																												
P	x	y	z	s	t	-																																																				
1	0	11	9	0	1	24																																																				
0	0	-12	-25	1	-2.5	$k-60$																																																				
0	1	3	4	0	0.5	12																																																				
13																																																										

7 (a)	(i)	<p>A</p> <table border="1" style="display: inline-table; vertical-align: middle;"> <tr><td>1</td><td>0</td></tr> <tr><td colspan="2"> </td></tr> </table>	1	0			<p>B</p> <table border="1" style="display: inline-table; vertical-align: middle;"> <tr><td>2</td><td>2</td></tr> <tr><td colspan="2">2</td></tr> </table>	2	2	2		M1	Answer should be on insert
1	0												
2	2												
2													
		<p>C</p> <table border="1" style="display: inline-table; vertical-align: middle;"> <tr><td>3</td><td>4</td></tr> <tr><td colspan="2">4</td></tr> </table>	3	4	4		<p>D</p> <table border="1" style="display: inline-table; vertical-align: middle;"> <tr><td>4</td><td>7</td></tr> <tr><td colspan="2">8 7</td></tr> </table>	4	7	8 7		A1	For correct temporary labels at D and E (condone extras here) For all temporary labels correct (with no extras)
3	4												
4													
4	7												
8 7													
		<p>E</p> <table border="1" style="display: inline-table; vertical-align: middle;"> <tr><td>6</td><td>9</td></tr> <tr><td colspan="2">12 11 9</td></tr> </table>	6	9	12 11 9		<p>F</p> <table border="1" style="display: inline-table; vertical-align: middle;"> <tr><td>7</td><td>14</td></tr> <tr><td colspan="2">15 14</td></tr> </table>	7	14	15 14		M1 A1	For value 38 at J For all permanent labels correct
6	9												
12 11 9													
7	14												
15 14													
		<p>G</p> <table border="1" style="display: inline-table; vertical-align: middle;"> <tr><td>5</td><td>8</td></tr> <tr><td colspan="2">8</td></tr> </table>	5	8	8		<p>H</p> <table border="1" style="display: inline-table; vertical-align: middle;"> <tr><td>8</td><td>18</td></tr> <tr><td colspan="2">20 18</td></tr> </table>	8	18	20 18		B1	For the correct order of assigning permanent labels: A, B, C, D, G, E, F, H, J
5	8												
8													
8	18												
20 18													
		<p>J</p> <table border="1" style="display: inline-table; vertical-align: middle;"> <tr><td>9</td><td>38</td></tr> <tr><td colspan="2">39 38</td></tr> </table>	9	38	39 38								
9	38												
39 38													
		Shortest route: A - B - G - E Length = 900		B1	For correct route and length. Accept route reversed and accept length = 9								
		Shortest route: A - C - D - F - H - J Length = 3800		B1	For correct route and length. Accept route reversed and accept length = 38								
	(ii)	Length: 4700 metres		B1	Follow through from (i), if possible								
		E - G - B - A - C - D - F - H - J		M1	For 47 or 4700								
				M1	For E - G - B - A, or reversed, as part of a longer route								
				M1	For A - C - D - F - H - J, or reversed, as part of a longer route								
				A1	For whole route correct								
	(iii)	Explanation: G - B - A - C - D - F - H E and J will be left out (either is sufficient)		M1	May be implied								
		Odd nodes are A, C, D, E, F, G Need to pair C, D, F, G in the shortest way		A1	For identifying that route will not visit every vertex.								
	(b)	CD = 3 and FG = 7 ⇒ 10 (CF = 10, DG = 11 and CG = 8, DF = 7)		M1	For trying to pair C, D, F, G (and no others)								
		Sum of all weights = 147		A1	For CD, FG or 10 (or 1000)								
		Length = 15700 metres		M1	For 147 (or 14700) or a good attempt seen or implied								
				A1	For 15700 metres (or 15700 m or 157 hundred metres or 15.7 km). But 157 ⇒ M1, A0								