

GCE

Edexcel GCE

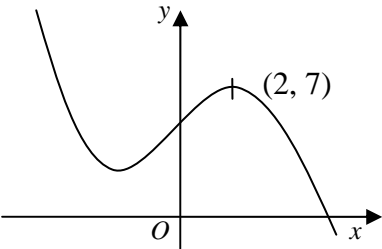
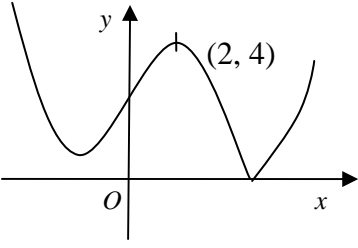
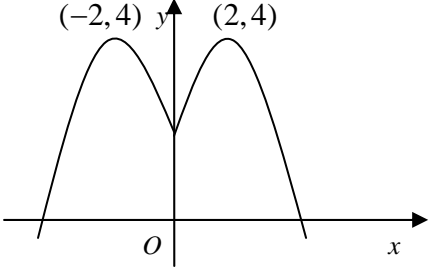
Core Mathematics C3 (6665)

January 2006

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Mark Scheme (Results)

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6665 Core Mathematics C3
Mark Scheme

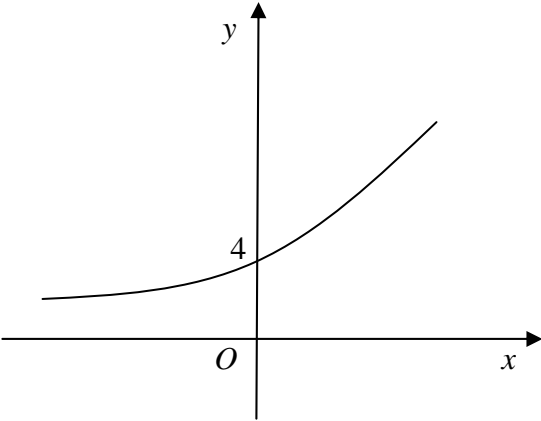
| Question Number | Scheme | Marks |
|------------------|--|--|
| <p>1.</p> | <p>(a)</p>  | <p>Shape unchanged Point</p> <p>B1 B1</p> <p>(2)</p> |
| | <p>(b)</p>  | <p>Shape Point</p> <p>B1 B1</p> <p>(2)</p> |
| | <p>(c)</p>  | <p>Shape (2, 4) (-2, 4)</p> <p>B1 B1 B1</p> <p>(3) [7]</p> |

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|-----------------|--|--|
| 2. | $x^2 - x - 2 = (x-2)(x+1)$ <p style="text-align: right;">At any stage</p> $\frac{2x^2 + 3x}{(2x+3)(x-2)} = \frac{x(2x+3)}{(2x+3)(x-2)} = \frac{x}{x-2}$ $\frac{2x^2 + 3x}{(2x+3)(x-2)} - \frac{6}{x^2 - x - 2} = \frac{x(x+1) - 6}{(x-2)(x+1)}$ $= \frac{x^2 + x - 6}{(x-2)(x+1)}$ $= \frac{(x+3)(x-2)}{(x-2)(x+1)}$ $= \frac{x+3}{x+1}$ | B1 B1 M1 A1 M1 A1 A1 (7) [7] |
| | <p>Alternative method</p> $x^2 - x - 2 = (x-2)(x+1)$ <p style="text-align: right;">At any stage</p> <p>(2x+3) appearing as a factor of the numerator at any stage</p> $\frac{2x^2 + 3x}{(2x+3)(x-2)} - \frac{6}{(x-2)(x+1)} = \frac{(2x^2 + 3x)(x+1) - 6(2x+3)}{(2x+3)(x-2)(x+1)}$ $= \frac{2x^3 + 5x^2 - 9x - 18}{(2x+3)(x-2)(x+1)}$ <p style="text-align: right;">can be implied</p> $= \frac{(x-2)(2x^2 + 9x + 9)}{(2x+3)(x-2)(x+1)} \quad \text{or} \quad \frac{(2x+3)(x^2 + x - 6)}{(2x+3)(x-2)(x+1)} \quad \text{or} \quad \frac{(x+3)(2x^2 - x - 6)}{(2x+3)(x-2)(x+1)}$ <p style="text-align: right;">M1</p> <p style="text-align: center;">Any one linear factor × quadratic</p> $= \frac{(2x+3)(x-2)(x+3)}{(2x+3)(x-2)(x+1)}$ <p style="text-align: right;">Complete factors</p> $= \frac{x+3}{x+1}$ <p style="text-align: right;">A1 (7)</p> | B1 B1 M1 A1 M1 A1 (7) |

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| 3. | $\frac{dy}{dx} = \frac{1}{x}$ <p style="text-align: right;">accept $\frac{3}{3x}$</p> $\text{At } x = 3, \frac{dy}{dx} = \frac{1}{3} \Rightarrow m' = -3$ <p style="text-align: right;">Use of $mm' = -1$</p> $y - \ln 1 = -3(x - 3)$ $y = -3x + 9$ <p style="text-align: right;">Accept $y = 9 - 3x$</p> <p>$\frac{dy}{dx} = \frac{1}{3x}$ leading to $y = -9x + 27$ is a maximum of M1 A0 M1 M1 A0 = 3/5</p> | M1 A1 M1 M1 A1 (5) [5] |
| 4. | <p>(a) (i)</p> $\frac{d}{dx}(e^{3x+2}) = 3e^{3x+2} \quad (\text{or } 3e^2 e^{3x})$ <p style="text-align: right;">At any stage</p> $\frac{dy}{dx} = 3x^2 e^{3x+2} + 2x e^{3x+2}$ <p style="text-align: right;">Or equivalent</p> <p>(ii)</p> $\frac{d}{dx}(\cos(2x^3)) = -6x^2 \sin(2x^3)$ <p style="text-align: right;">At any stage</p> $\frac{dy}{dx} = \frac{-18x^3 \sin(2x^3) - 3 \cos(2x^3)}{9x^2}$ <p style="text-align: right;">M1 A1 (4)</p> <p>Alternatively using the product rule for second M1 A1</p> $y = (3x)^{-1} \cos(2x^3)$ $\frac{dy}{dx} = -3(3x)^{-2} \cos(2x^3) - 6x^2 (3x)^{-1} \sin(2x^3)$ <p style="text-align: right;">Accept equivalent unsimplified forms</p> <p>(b)</p> $1 = 8 \cos(2y + 6) \frac{dy}{dx} \quad \text{or} \quad \frac{dx}{dy} = 8 \cos(2y + 6)$ $\frac{dy}{dx} = \frac{1}{8 \cos(2y + 6)}$ $\frac{dy}{dx} = \frac{1}{8 \cos\left(\arcsin\left(\frac{x}{4}\right)\right)} \quad \left(= (\pm) \frac{1}{2\sqrt{(16-x^2)}} \right)$ | B1 M1 A1+A1 (4) M1 A1 M1 A1 (4) M1 M1 A1 M1 A1 (5) [13] |

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| 5. | <p>(a) $2x^2 - 1 - \frac{4}{x} = 0$ Dividing equation by x M1 $x^2 = \frac{1}{2} + \frac{4}{2x}$ Obtaining $x^2 = \dots$ M1 $x = \sqrt{\left(\frac{2}{x} + \frac{1}{2}\right)}$ * cso A1 (3)</p> <p>(b) $x_1 = 1.41, x_2 = 1.39, x_3 = 1.39$ B1, B1, B1 If answers given to more than 2 dp, penalise first time then accept awrt above. (3)</p> <p>(c) Choosing (1.3915, 1.3925) or a tighter interval M1 $f(1.3915) \approx -3 \times 10^{-3}, f(1.3925) \approx 7 \times 10^{-3}$ Both, awrt A1 Change of sign (and continuity) $\Rightarrow \alpha \in (1.3915, 1.3925)$ $\Rightarrow \alpha = 1.392$ to 3 decimal places * cso A1 (3)</p> | <p>M1 M1 A1 (3)</p> <p>B1, B1, B1 (3)</p> <p>M1 A1 A1 (3) [9]</p> |
| 6. | <p>(a) $R \cos \alpha = 12, R \sin \alpha = 4$ $R = \sqrt{(12^2 + 4^2)} = \sqrt{160}$ Accept if just written down, awrt 12.6 M1 A1 $\tan \alpha = \frac{4}{12}, \Rightarrow \alpha \approx 18.43^\circ$ awrt 18.4° M1, A1(4)</p> <p>(b) $\cos(x + \text{their } \alpha) = \frac{7}{\text{their } R}$ (≈ 0.5534) M1 $x + \text{their } \alpha = 56.4^\circ$ awrt 56° A1 $= \dots, 303.6^\circ$ $360^\circ - \text{their principal value}$ M1 $x = 38.0^\circ, 285.2^\circ$ Ignore solutions out of range A1, A1 (5) If answers given to more than 1 dp, penalise first time then accept awrt above.</p> <p>(c)(i) minimum value is $-\sqrt{160}$ ft their R B1ft</p> <p>(ii) $\cos(x + \text{their } \alpha) = -1$ M1 $x \approx 161.57^\circ$ cao A1 (3)</p> | <p>M1 A1 M1, A1(4)</p> <p>M1 A1 M1 A1, A1 (5)</p> <p>B1ft</p> <p>M1 A1 (3) [12]</p> |

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| 7. | (a) (i) Use of $\cos 2x = \cos^2 x - \sin^2 x$ in an attempt to prove the identity. $\frac{\cos 2x}{\cos x + \sin x} = \frac{\cos^2 x - \sin^2 x}{\cos x + \sin x} = \frac{(\cos x - \sin x)(\cos x + \sin x)}{\cos x + \sin x} = \cos x - \sin x \quad *$ | M1 A1 (2) |
| | (ii) Use of $\cos 2x = 2\cos^2 x - 1$ in an attempt to prove the identity. Use of $\sin 2x = 2\sin x \cos x$ in an attempt to prove the identity. $\frac{1}{2}(\cos 2x - \sin 2x) = \frac{1}{2}(2\cos^2 x - 1 - 2\sin x \cos x) = \cos^2 x - \cos x \sin x - \frac{1}{2} \quad *$ | M1 M1 A1 (3) |
| | (b) $\cos \theta (\cos \theta - \sin \theta) = \frac{1}{2}$ $\cos^2 \theta - \cos \theta \sin \theta - \frac{1}{2} = 0$ | Using (a)(i) M1 |
| | $\frac{1}{2}(\cos 2\theta - \sin 2\theta) = 0$ $\cos 2\theta = \sin 2\theta \quad *$ | Using (a)(ii) M1 A1 (3) |
| | (c) $\tan 2\theta = 1$ $2\theta = \frac{\pi}{4}, \left(\frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}\right)$ | M1 any one correct value of 2θ A1 |
| | $\theta = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$ | Obtaining at least 2 solutions in range M1 The 4 correct solutions A1 (4) |
| | If decimals (0.393, 1.963, 3.534, 5.105) or degrees (22.5°, 112.5°, 202.5°, 292.5°) are given, but all 4 solutions are found, penalise one A mark only. Ignore solutions out of range. | [12] |

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| 8. | <p>(a)</p> $\begin{aligned} gf(x) &= e^{2(x+\ln 2)} \\ &= e^{4x} e^{2\ln 2} \\ &= e^{4x} e^{\ln 4} \\ &= 4e^{4x} \end{aligned}$ <p>(Hence $gf : x \mapsto 4e^{4x}, x \in \mathbb{R}$)</p> <p>(b)</p>  <p>(c)</p> <p>Range is \mathbb{R}_+</p> <p>(d)</p> $\begin{aligned} \frac{d}{dx}[gf(x)] &= 16e^{4x} \\ e^{4x} &= \frac{3}{16} \\ 4x &= \ln \frac{3}{16} \\ x &\approx -0.418 \end{aligned}$ | <p>M1 M1 M1 A1 (4)</p> <p>Give mark at this point, cso</p> <p>Shape and point B1 (1)</p> <p>Accept $gf(x) > 0, y > 0$ B1 (1)</p> <p>M1 A1 M1 A1 (4) [10]</p> |