MEI STRUCTURED MATHEMATICS   **4752**

Concepts for Advanced Mathematics (C2)

Monday   **16 JANUARY 2006**   Morning   1 hour 30 minutes

Additional materials:
     8 page answer booklet
     Graph paper
     MEI Examination Formulae and Tables (MF2)

TIME   1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

·   Write your name, centre number and candidate number in the spaces provided on the answer booklet.

·   Answer **all** the questions.

·   You are permitted to use a graphical calculator in this paper.

·   Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

·   The number of marks is given in brackets [ ] at the end of each question or part question.

·   You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

·   The total number of marks for this paper is 72.

This question paper consists of 5 printed pages and 3 blank pages.

Section A (36 marks)

1 Given that $140° = k\pi$ radians, find the exact value of k. [2]

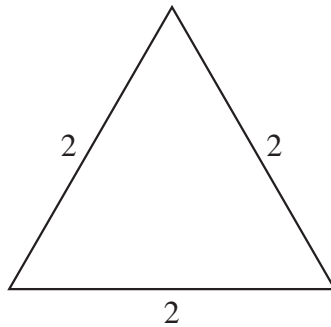**2** Find the numerical value of $\sum_{k=2}^{5} k^3$. [2]

3

Fig. 3

Beginning with the triangle shown in Fig. 3, prove that $\sin 60° = \dfrac{\sqrt{3}}{2}$. [3]
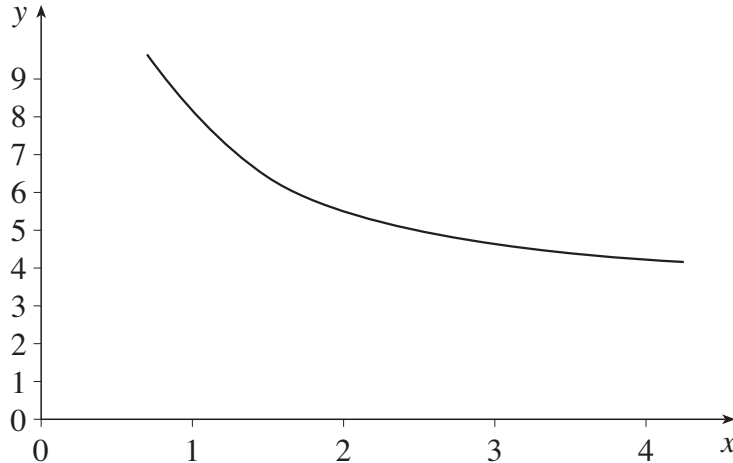
4

Fig. 4

Fig. 4 shows a curve which passes through the points shown in the following table.

x	1	1.5	2	2.5	3	3.5	4
y	8.2	6.4	5.5	5.0	4.7	4.4	4.2

Use the trapezium rule with 6 strips to estimate the area of the region bounded by the curve, the lines $x = 1$ and $x = 4$, and the x-axis.

State, with a reason, whether the trapezium rule gives an overestimate or an underestimate of the area of this region. [5]

5 **(i)** Sketch the graph of $y = \tan x$ for $0° \leqslant x \leqslant 360°$. [2]

    (ii) Solve the equation $4 \sin x = 3 \cos x$ for $0° \leqslant x \leqslant 360°$. [3]

6 A curve has gradient given by $\dfrac{\mathrm{d}y}{\mathrm{d}x} = x^2 - 6x + 9$. Find $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$.

Show that the curve has a stationary point of inflection when $x = 3$. [4]

7 In Fig. 7, A and B are points on the circumference of a circle with centre O.
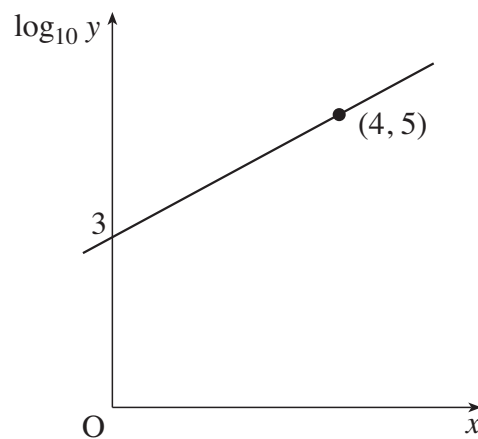
Angle AOB = 1.2 radians.

The arc length AB is 6 cm.
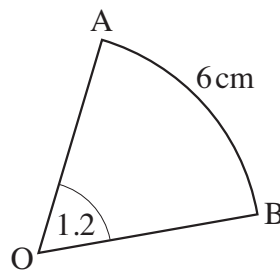
Fig. 7

    (i) Calculate the radius of the circle. [2]

    (ii) Calculate the length of the chord AB. [3]

8 Find $\displaystyle\int \left(x^{\frac{1}{2}} + \dfrac{6}{x^3} \right) \mathrm{d}x$. [5]

[Turn over

9

Fig. 9

The graph of $\log_{10} y$ against x is a straight line as shown in Fig. 9.

(i) Find the equation for $\log_{10} y$ in terms of x. [3]

(ii) Find the equation for y in terms of x. [2]

Section B (36 marks)

10 The equation of a curve is $y = 7 + 6x - x^2$.

(i) Use calculus to find the coordinates of the turning point on this curve.

Find also the coordinates of the points of intersection of this curve with the axes, and sketch the curve. [8]

(ii) Find $\int_1^5 (7 + 6x - x^2)\, \mathrm{d}x$, showing your working. [3]

(iii) The curve and the line $y = 12$ intersect at $(1, 12)$ and $(5, 12)$. Using your answer to part **(ii)**, find the area of the finite region between the curve and the line $y = 12$. [1]
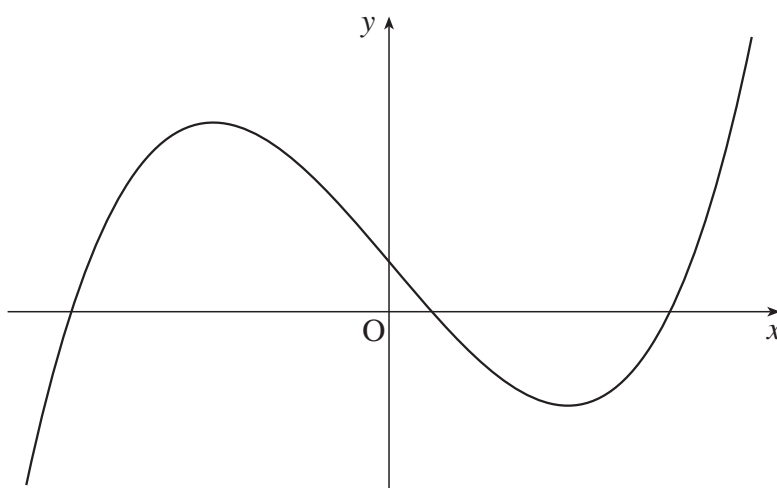
11

Fig. 11

The equation of the curve shown in Fig. 11 is $y = x^3 - 6x + 2$.

(i) Find $\dfrac{dy}{dx}$. [2]

(ii) Find, in exact form, the range of values of x for which $x^3 - 6x + 2$ is a decreasing function. [3]

(iii) Find the equation of the tangent to the curve at the point $(-1, 7)$.

Find also the coordinates of the point where this tangent crosses the curve again. [6]

12 **(i)** Granny gives Simon £5 on his 1st birthday. On each successive birthday, she gives him £2 more than she did the previous year.

   (*A*) How much does she give him on his 10th birthday? [2]

   (*B*) How old is he when she gives him £51? [2]

   (*C*) How much has she given him **in total** when he has had his 20th birthday present? [2]

 (ii) Grandpa gives Simon £5 on his 1st birthday and increases the amount by 10% each year.

   (*A*) How much does he give Simon on his 10th birthday? [2]

   (*B*) Simon first gets a present of over £50 from Grandpa on his nth birthday. Show that

$$n > \frac{1}{\log_{10} 1.1} + 1.$$

   Find the value of n. [5]