

Mark Scheme 4721
January 2007

<p>1</p>	$\frac{5}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$ $= \frac{5(2+\sqrt{3})}{4-3}$ $= 10+5\sqrt{3}$	<p>M1</p> <p>A1</p> <p>A1 3 3</p>	<p>Multiply top and bottom by $\pm(2+\sqrt{3})$</p> <p>$(2+\sqrt{3})(2-\sqrt{3})=1$ (may be implied)</p> <p>$10+5\sqrt{3}$</p>
<p>2(i)</p> <p>(ii)</p>	<p>1</p> $\frac{1}{2} \times 2^4$ <p>= 8</p>	<p>B1 1</p> <p>M1</p> <p>M1</p> <p>A1 3 4</p>	<p>$2^{-1} = \frac{1}{2}$ or $32^{\frac{1}{5}} = 2$ or $2^5 = 32$ soi</p> <p>$32^{\frac{4}{5}} = 2^4$ or 16 seen or implied</p> <p>8</p>
<p>3(i)</p> <p>(ii)</p>	$3x-15 \leq 24$ $3x \leq 39$ $x \leq 13$ <p>or</p> $x-5 \leq 8$ $x \leq 13$ $5x^2 > 80$ $x^2 > 16$ $x > 4$ <p>or $x < -4$</p>	<p>M1</p> <p>A1 2</p> <p>M1</p> <p>B1</p> <p>A1 3</p> <p>5</p>	<p>Attempt to simplify expression by multiplying out brackets</p> <p>$x \leq 13$</p> <p>Attempt to simplify expression by dividing through by 3</p> <p>Attempt to rearrange inequality or equation to combine the constant terms</p> <p>$x > 4$</p> <p>fully correct, not wrapped, not 'and'</p> <p>SR B1 for $x \geq 4, x \leq -4$</p>

4	Let $y = x^{\frac{1}{3}}$ $y^2 + 3y - 10 = 0$ $(y - 2)(y + 5) = 0$ $y = 2, y = -5$ $x = 2^3, x = (-5)^3$ $x = 8, x = -125$	*M1 DM1 A1 DM1 A1 ft 5 5	Attempt a substitution to obtain a quadratic or factorise with $\sqrt[3]{x}$ in each bracket Correct attempt to solve quadratic Both values correct Attempt cube Both answers correctly followed through SR B2 $x = 8$ from T & I
5 (i)		M1 A1 2	Reflection in either axis Correct reflection in x axis
(ii)	(1, 3)	B1 B1 2	Correct x coordinate Correct y coordinate SR B1 for (3, 1)
(iii)	Translation 2 units in negative x direction	B1 B1 2 6	
6 (i)	$2(x^2 - 12x + 40)$ $= 2[(x - 6)^2 - 36 + 40]$ $= 2[(x - 6)^2 + 4]$ $= 2(x - 6)^2 + 8$	B1 B1 M1 A1 4	$a = 2$ $b = 6$ $80 - 2b^2$ or $40 - b^2$ or $80 - b^2$ or $40 - 2b^2$ (their b) $c = 8$
(ii)	$x = 6$	B1 ft 1	
(iii)	$y = 8$	B1 ft 1 6	

7(i)	$\frac{dy}{dx} = 5$	B1 1	
(ii)	$y = 2x^{-2}$ $\frac{dy}{dx} = -4x^{-3}$	B1 B1 B1 3	x^{-2} soi $-4x^c$ kx^{-3}
(iii)	$y = 10x^2 - 14x + 5x - 7$ $y = 10x^2 - 9x - 7$ $\frac{dy}{dx} = 20x - 9$	M1 A1 B1 ft B1 ft 4 8	Expand the brackets to give an expression of form $ax^2 + bx + c$ ($a \neq 0, b \neq 0, c \neq 0$) Completely correct (allow 2 x -terms) 1 term correctly differentiated Completely correct (2 terms)
8 (i)	$\frac{dy}{dx} = 9 - 6x - 3x^2$ At stationary points, $9 - 6x - 3x^2 = 0$ $3(3 + x)(1 - x) = 0$ $x = -3$ or $x = 1$ $y = 0, 32$	*M1 A1 M1 DM1 A1 A1ft 6	Attempt to differentiate y or $-y$ (at least one correct term) 3 correct terms Use of $\frac{dy}{dx} = 0$ (for y or $-y$) Correct method to solve 3 term quadratic $x = -3, 1$ $y = 0, 32$ (1 correct pair www A1 A0)
(ii)	$\frac{d^2y}{dx^2} = -6x - 6$ When $x = -3, \frac{d^2y}{dx^2} > 0$ When $x = 1, \frac{d^2y}{dx^2} < 0$	M1 A1 A1 3	Looks at sign of $\frac{d^2y}{dx^2}$, derived correctly from $k \frac{dy}{dx}$, or other correct method $x = -3$ minimum $x = 1$ maximum
(iii)	$-3 < x < 1$	M1 A1 2 11	Uses the x values of both turning points in inequality/inequalities Correct inequality or inequalities. Allow \leq

9 (i)	<p>Gradient = 4</p> $y - 7 = 4(x - 2)$ $y = 4x - 1$	<p>B1</p> <p>M1</p> <p>A1 3</p>	<p>Gradient of 4 soi</p> <p>Attempts equation of straight line through (2, 7) with any gradient</p>
(ii)	$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ $= \sqrt{(2 - 1)^2 + (7 - 2)^2}$ $= \sqrt{3^2 + 9^2}$ $= \sqrt{90}$ $= 3\sqrt{10}$	<p>M1</p> <p>A1</p> <p>A1 3</p>	<p>Use of correct formula for d or d^2 (3 values correctly substituted)</p> $\sqrt{3^2 + 9^2}$ <p>Correct simplified surd</p>
(iii)	<p>Gradient of AB = 3</p> <p>Gradient of perpendicular line = $-\frac{1}{3}$</p> <p>Midpoint of AB = $\left(\frac{1}{2}, \frac{5}{2}\right)$</p> $y - \frac{5}{2} = -\frac{1}{3}\left(x - \frac{1}{2}\right)$ $x + 3y - 8 = 0$	<p>B1</p> <p>B1 ft</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1 6</p> <p>12</p>	<p>SR Allow B1 for $-\frac{1}{4}$</p> <p>Attempts equation of straight line through their midpoint with any non-zero gradient</p> $y - \frac{5}{2} = -\frac{1}{3}\left(x - \frac{1}{2}\right)$ $x + 3y - 8 = 0$

10 (i)	Centre $(-1, 2)$ $(x + 1)^2 - 1 + (y - 2)^2 - 4 - 8 = 0$ $(x + 1)^2 + (y - 2)^2 = 13$ Radius $\sqrt{13}$	B1 M1 A1 3	Correct centre Attempt at completing the square Correct radius <u>Alternative method:</u> Centre $(-g, -f)$ is $(-1, 2)$ B1 $g^2 + f^2 - c$ M1 Radius $= \sqrt{13}$ A1
(ii)	$(2)^2 + (k - 2)^2 = 13$ $(k - 2)^2 = 9$ $k - 2 = \pm 3$ $k = -1$	M1 M1 A1 3	Attempt to substitute $x = -3$ into circle equation Correct method to solve quadratic $k = -1$ (negative value chosen)
(iii)	EITHER $y = 6 - x$ $(x + 1)^2 + (6 - x - 2)^2 = 13$ $(x + 1)^2 + (4 - x)^2 = 13$ $x^2 + 2x + 1 + 16 - 8x + x^2 = 13$ $2x^2 - 6x + 4 = 0$ $2(x - 1)(x - 2) = 0$ $x = 1, 2$ $\therefore y = 5, 4$ OR $x = 6 - y$ $(6 - y + 1)^2 + (y - 2)^2 = 13$ $(7 - y)^2 + (y - 2)^2 = 13$ $49 - 14y + y^2 + y^2 - 4y + 4 = 13$ $2y^2 - 18y + 40 = 0$ $2(y - 4)(y - 5) = 0$ $y = 4, 5$ $\therefore x = 2, 1$	M1 M1 A1 M1 A1 A1 6	Attempt to solve equations simultaneously Substitute into their circle equation for x/y or attempt to get an equation in 1 variable only Obtain correct 3 term quadratic Correct method to solve quadratic of form $ax^2 + bx + c = 0$ ($b \neq 0$) Both x values correct Both y values correct <u>or</u> one correct pair of values www B1 second correct pair of values B1 SR T & I M1 A1 One correct x (or y) value A1 Correct associated coordinate
		12	