# Mark Scheme 4724
# January 2007

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|---|---|---|---|---|
| **1** | Factorise numerator and denominator | M1 | | <u>or</u> Attempt long division |
| | Num = $(x+6)(x-4)$ or denom = $x(x-4)$ | A1 | | Result = $1+\dfrac{6x-24}{x^2-4x}$ |
| | Final answer = $\dfrac{x+6}{x}$ or $1+\dfrac{6}{x}$ | A1 | **3** | = $1+\dfrac{6}{x}$ |
| **2** | Use parts with $u = \ln x, \mathrm{d}v = x$ | M1 | | & give 1st stage in form $f(x) +/- \int g(x)(\mathrm{d}x)$ |
| | Obtain $\frac{1}{2}x^2 \ln x - \int \frac{1}{x}.\frac{1}{2}x^2(\mathrm{d}x)$ | A1 | | or $\frac{1}{2}x^2 \ln x - \int \frac{1}{2}x(\mathrm{d}x)$ |
| | = $\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2$ (+c) | A1 | | |
| | Use limits correctly | M1 | | |
| | Exact answer $2\ln 2 - \frac{3}{4}$ | A1 | **5** | AEF ISW |
| **3** | (i) Find $\boldsymbol{a}-\boldsymbol{b}$ or $\boldsymbol{b}-\boldsymbol{a}$ irrespective of label | M1 | | (expect 11**i** − 2 **j** − 6 **k** or −11**i** + 2**j** + 6**k**) |
| | Method for magnitude of any vector | M1 | | |
| | $\sqrt{161}$ or $12.7(12.688578)$ | A1 | **3** | |
| | (ii) Using $(\overline{\mathrm{AO}}$ or $\overline{\mathrm{OA}})$ and $(\overline{AB}$ or $\overline{\mathrm{BA}})$ | B1 | | Do not class angle *AOB* as MR |
| | $\cos\theta = \dfrac{\text{scalar product of any two vectors}}{\text{product of their moduli}}$ | M1 | | |
| | 43 or better (42.967…), 0.75 or better (0.7499218..) | A1 | **3** | If 137 obtained, followed by 43, award A0<br>Common answer 114 probably → B0 M1 A0 |
| **4** | Attempt to connect $\mathrm{d}x$ and $\mathrm{d}u$ | M1 | | but not just $\mathrm{d}x = \mathrm{d}u$ |
| | For $\mathrm{d}u = 2\,\mathrm{d}x$ AEF correctly used | A1 | | sight of $\frac{1}{2}$ ( $\mathrm{d}u$ ) necessary |
| | $\int u^8 + u^7\,(\mathrm{d}u)$ | A1 | | or $\int u^7(u+1)(\mathrm{d}u)$ |
| | Attempt new limits for $u$ at any stage (expect 0,1) | M1 | | or re-substitute & use ( $\frac{5}{2}$,3 ) |
| | $\frac{17}{72}$ | A1 | **5** | **AG** WWW |
| | S.R. If M1 A0 A0 M1 A0, award S.R. B1 for answer $\frac{68}{72}, \frac{34}{36}$ or $\frac{17}{18}$ | | | ISW |
| **5** | (i) Show clear knowledge of binomial expansion | M1 | | $-3x$ should appear but brackets can be missing; $-\frac{1}{3}.-\frac{4}{3}$ should appear, not $-\frac{1}{3}.\frac{2}{3}$ |
| | = $1+x$ | B1 | | Correct first 2 terms; not dep on M1 |
| | $+2x^2$ | A1 | | |
| | $+\frac{14}{3}x^3$ | A1 | **4** | |
| | (ii) Attempt to substitute $x+x^3$ for $x$ in (i) | M1 | | Not just in the $\frac{14}{3}x^3$ term |
| | Clear indication that $(x+x^3)^2$ has no term in $x^3$ | A1 | | |
| | $\frac{17}{3}$ | √A1 | **3** | f.t. cf$(x)$ + cf$(x^3)$ in part (i) |
| **6** | (i) $2x+1 = / \equiv A(x-3)+B$ | M1 | | |
| | $A=2$ | A1 | | |
| | $B=7$ | A/B 1 | **3** | Cover-up rule acceptable for B1 |
| | (ii) $\int \frac{1}{x-3}(\mathrm{d}x) = \ln(x-3)$ or $\ln\lvert x-3\rvert$ | B1 | | Accept *A* or $\frac{1}{A}$ as a multiplier |
| | $\int \frac{1}{(x-3)^2}(\mathrm{d}x) = -\frac{1}{x-3}$ | B1 | | Accept *B* or $\frac{1}{B}$ as a multiplier |
| | 6 + 2 ln 7 Follow-through $\dfrac{6}{7}B + A\ln 7$ | √B2 | **4** | |

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| **7** | $\frac{d}{dx}(xy) = x\frac{dy}{dx} + y$ | B1 | | |
| | $\frac{d}{dx}(y^2) = 2y\frac{dy}{dx}$ | B1 | | |
| | $4x + x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0$ | B1 | | |
| | Put $\frac{dy}{dx} = 0$ | *M1 | | |
| | Obtain $4x + y = 0$ AEF | A1 | | and no other (different) result |
| | Attempt to solve simultaneously with eqn of curve | dep*M1 | | |
| | Obtain $x^2 = 1$ or $y^2 = 16$ from $4x + y = 0$ | A1 | | |
| | $(1,-4)$ and $(-1,4)$ and no other solutions | A1 | **8** | Accept $(\pm 1, \mp 4)$ but not $(\pm 1, \pm 4)$ |
| **8** | (i) Use $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ and $-\frac{1}{m}$ for grad of normal | M1 | | or change to cartesian.,diff & use $-\frac{1}{m}$ |
| | $= -p$ **AG** WWW | A1 | **2** | Not $-t$. |
| | (ii) Use correct formula to find gradient of line | M1 | | |
| | Obtain $\frac{2}{p+q}$ **AG** WWW | A1 | **2** | Minimum of denom = $2(p-q)(p+q)$ |
| | (iii) State $-p = \frac{2}{p+q}$ | M1 | | Or find eqn normal at P & subst $(2q^2, 4q)$ |
| | Simplify to $p^2 + pq + 2 = 0$ **AG** WWW | A1 | **2** | With sufficient evidence |
| | (iv) (8,8) $\rightarrow t$ or $p$ or $q = 2$ only | B1 | | No possibility of $-2$ |
| | Subst $p = 2$ in eqn (iii) to find $q_1$ | M1 | | Or eqn normal,solve simult with cartes/param |
| | Subst $p = q_1$ in eqn (iii) to find $q_2$ | M1 | | Ditto |
| | $q_2 = \frac{11}{3} \rightarrow \left(\frac{242}{9}, \frac{44}{3}\right)$ | A1 | **4** | No follow-through; accept (26.9 , 14.7) |
| **9** | (i) Separate variables as $\int \sec^2 y \, dy = 2\int \cos^2 2x \, dx$ | M1 | | seen or implied |
| | LHS = tan *y* | A1 | | |
| | RHS; attempt to change to double angle | M1 | | |
| | Correctly shown as 1 + cos 4*x* | A1 | | |
| | $\int \cos 4x \, dx = \frac{1}{4}\sin 4x$ | A1 | | |
| | Completely correct equation (other than +c) | A1 | | $\tan y = x + \frac{1}{4}\sin 4x$ |
| | +c on either side | A1 | **7** | <u>not</u> on both sides unless $c_1$ and $c_2$ |
| | (ii) Use boundary condition | M1 | | provided a sensible outcome would ensue |
| | c (on RHS) = 1 | A1 | | or $c_2 - c_1 = 1$; not fortuitously obtained |
| | Substitute $x = \frac{1}{6}\pi$ into their eqn, produce *y* = 1.05 | A1 | **3** | or 4.19 or 7.33 etc. Radians only |
| **10** | (i) For (either point) + *t*(diff between posn vectors) | M1 | | "**r** =" not necessary for the M mark |
| | **r** = (either point) + *t*(**i** -2**j** – 3**k** or –**i** + 2**j** + 3**k**) | A1 | **2** | **…** but it is essential for the A mark |
| | (ii) **r** = *s*(**i** + 2**j** – **k**) or (**i** + 2**j** – **k**) + *s*(**i** + 2**j** – **k**) | B1 | | Accept any parameter, including *t* |
| | Eval scalar product of **i**+2**j**-**k** & their dir vect in (i) | M1 | | |
| | Show as (1x1 or 1)+(2x-2 or -4)+(-1x-3 or 3) | A1 | | This is just one example of numbers involved |
| | = 0 <u>and</u> state perpendicular **AG** | A1 | **4** | |
| | (iii) For at least two equations with diff parameters | M1 | | e.g. 5 + *t* = *s*, 2 – 2*t* = 2*s*, $-9 - 3t = -s$ |
| | Obtain $t = -2$ or *s* = 3 (possibly -3 or 2 or -2) | A1 | | Check if $t = 2, 1$ or $-1$ |
| | Subst. into eqn *AB* or *OT* and produce 3**i** + 6**j** – 3**k** | A1 | **3** | |
| | (iv) Indicate that $\lvert OC \rvert$ is to be found | M1 | | where *C* is their point of intersection |
| | $\sqrt{54}$ ;f.t. $\sqrt{a^2 + b^2 + c^2}$ from *a***i** + *b***j** + *c***k** in (iii) | √A1 | **2** | |

<u>In the above question, accept any vectorial notation</u>
*t* and *s* may be interchanged, and values stated above need to be treated with caution.
In (iii), if the point of intersection is correct, it is more than likely that the whole part is correct – but check.