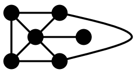
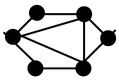
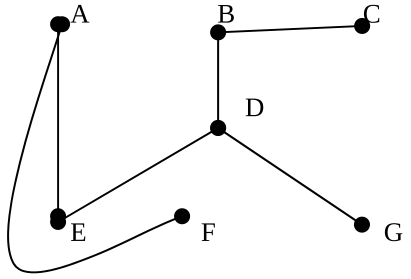
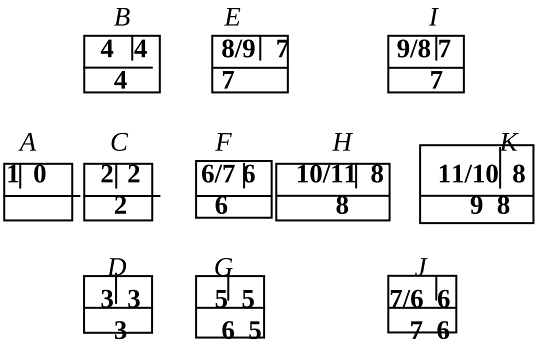
Mark Scheme 4736
January 2007

1	**(i)**	10 4 2 3 5 13 7 2 2 4 5 8 5 3 10 5 5 3	M1 M1 A1	First bundle starting with 10 4 2 and has at least one more bag in it Second bundle correct All bundles correct	[3]
	(ii)	Decreasing order: 13 10 10 8 7 5 5 5 5 5 4 4 3 3 3 2 2 2 13 10 2 10 8 7 5 5 5 5 5 4 4 3 3 3 2 2	M1 M1 A1	A value missing from written out list may be treated as a misread and lose the A mark only Sorting into decreasing order (may be implied from first bundle starting with 13) If each row sorted, award first M1 only Second and third bundles correct All bundles correct	[3]
	(iii)	Each person has roughly the same number of bags <u>or</u> the total weights are more evenly spread	B1	Saying that (i) gives a more even/equal allocation Five bundles in either part ⌖ B0	[1]
				Total =	**7**
2	**(i)**	*a* = number of apple cakes *b* = number of banana cakes *c* = number of cherry cakes	B1 B1	Identifying variables as 'number of cakes' Indicating *a* as apple, *b* as banana and *c* as cherry.	[2]
	(ii)	4×30 = 3×40 = 4×30 = 120 $\frac{a}{30}+\frac{b}{40}+\frac{c}{30}=30\times40\times30$ $4a + 3b + 4c \leq 120$ or $X = 4$, $Y = 3$, $Z = 4$	M1 A1	Any reasonable attempt 4, 3 and 4	[2]
	(iii)	$a + b + c \geq 30$ (or $a + b + c = 30$) $0 \leq a \leq 20$, $0 \leq b \leq 25$, $0 \leq c \leq 10$ (no need to say 'all integer')	B1 M1 A1	Constraint from total number of cakes correct All three upper constraints correct All three lower constraints correct also	[3]
	(iv)	$4a + 3b + 2c$	B1	Any multiple of this expression	[1]
				Total =	**8**
3	**(i) a**	9×2 = 18	B1	18	[1]
	b	Since the graph is simple, the two nodes of order 5 are each connected to every other node and hence every node has order at least 2 (exactly 2)	B1 B1	Explicitly using the fact that the graph is simple Deducing that each node has order at least 2 or that all other nodes have order 2 A diagram on its own is not enough.	[2]
	c	3×5 = 15 and 18 – 15 = 3 but the orders of the other nodes must sum to at least 3×3 = 9 (must sum to more than 3)	B1 B1	Or, the nodes of order 5 contribute 5+4+3 = 12 arcs But there are only 9 arcs available	[2]
	(ii)	 or equivalent	M1 A1	A simply connected graph with 6 nodes and 9 arcs, with at least one odd node For such a graph with node orders 1, 3, 3, 3, 3, 5	[2]
	(iii)	 or equivalent	M1 A1	A simply connected graph with 6 nodes and 9 arcs, with at least one even node For such a graph with node orders 2, 2, 2, 4, 4, 4	[2]
				Total =	**9**

<table>
<tr><td>4</td><td>(i)</td><td>
<table>
<tr><td></td><td>1</td><td>4</td><td>5</td><td>3</td><td>2</td><td>7</td><td>6</td></tr>
<tr><td></td><td><i>A</i></td><td><i>B</i></td><td><i>C</i></td><td><i>D</i></td><td><i>E</i></td><td><i>F</i></td><td><i>G</i></td></tr>
<tr><td><i>A</i></td><td>0</td><td>4</td><td>5</td><td>3</td><td>2</td><td>5</td><td>6</td></tr>
<tr><td><i>B</i></td><td>4</td><td>0</td><td>1</td><td>2</td><td>4</td><td>7</td><td>6</td></tr>
<tr><td><i>C</i></td><td>5</td><td>1</td><td>0</td><td>3</td><td>4</td><td>6</td><td>7</td></tr>
<tr><td><i>D</i></td><td>3</td><td>2</td><td>3</td><td>0</td><td>2</td><td>6</td><td>4</td></tr>
<tr><td><i>E</i></td><td>2</td><td>4</td><td>4</td><td>2</td><td>0</td><td>6</td><td>6</td></tr>
<tr><td><i>F</i></td><td>5</td><td>7</td><td>6</td><td>6</td><td>6</td><td>0</td><td>10</td></tr>
<tr><td><i>G</i></td><td>6</td><td>6</td><td>7</td><td>4</td><td>6</td><td>10</td><td>0</td></tr>
</table>
</td><td>M1
M1 dep
A1</td><td>FIRST THREE MARKS ARE FOR WORK ON THE TABLE ONLY
(Starting by) choosing row E in column A
Choosing more than one entry from column A
Correct entries chosen (or all transposed)</td><td></td></tr>
<tr><td></td><td></td><td>Order: <i>A E D B C G F</i></td><td>B1</td><td>Correct order, listed or marked on arrows or table, or arcs listed <i>AE ED DB BC DG AF</i></td><td></td></tr>
<tr><td></td><td></td><td>Minimum spanning tree:

 </td><td>B1</td><td>Tree (correct or follow through from table, provided solution forms a spanning tree)</td><td></td></tr>
<tr><td></td><td></td><td>Total weight: 16 (or 1600 m)</td><td>B1</td><td>16 or 1600m (or follow through from table or diagram, provided solution forms a spanning tree)</td><td>[6]</td></tr>
<tr><td></td><td>(ii)</td><td>Travelling salesperson (problem)</td><td>B1</td><td>Identifying TSP by name</td><td>[1]</td></tr>
<tr><td></td><td>(iii)</td><td>Two shortest arcs from <i>H</i>: 12 + 13 = 25</td><td>B1</td><td>12 + 13 or 25, or implied from final answer</td><td></td></tr>
<tr><td></td><td></td><td>25 + 16 = 41</td><td>M1</td><td>Adding their 25 to their 16 or for 41 (must be using two arcs from <i>H</i>)</td><td></td></tr>
<tr><td></td><td></td><td>4100 m</td><td>A1</td><td>4100 m or 4.1 km (correct and with units)</td><td>[3]</td></tr>
<tr><td></td><td>(iv)</td><td><i>H A E D B C F G H</i></td><td>M1</td><td>(<i>H</i>) <i>A E D B C</i> …</td><td></td></tr>
<tr><td></td><td></td><td></td><td>A1</td><td>Correct tour</td><td></td></tr>
<tr><td></td><td></td><td>12+2+2+2+1+6+10+16 = 51</td><td>M1</td><td>A substantially correct attempt at sum</td><td></td></tr>
<tr><td></td><td></td><td>5100 m</td><td>A1</td><td>5100m or 5.1 km (correct and with units)</td><td>[4]</td></tr>
<tr><td colspan="5" align="right">Total =</td><td>14</td></tr>
</table>

5	**(i)**		M1 M1 A1 B1 B1	Correct temporary labels at *B* to *G*, no extras Correct temporary labels at *H* to *J*, no extras All temporary labels correct Order of becoming permanent correct (follow through their permanent labels) All permanent labels correct	
		Note: *H* may have only a temporary label if left until last			
		Route: *A D G J K* Number of speed cameras on route: 8	B1 B1	Correct route 8 (cao)	[7]
	(ii)	Odd nodes: *A I J K*	M1	Identifying or using *A I J K*	
		$AI = 7$ $AJ = 6$ $AK = 8$ $JK = \underline{2}$ $IK = \underline{4}$ $IJ = \underline{6}$ 9 10 14	A1 A1	Weight of *AI* + weight of *JK* = 9 Weight of *AJ* + weight of *IK* = 10 (follow through weight of *AI*, *AJ* from (i) if necessary)	
		Repeat *AI* and *JK* $\Rightarrow$ *AB BI* and *JK*			
		Route (example): *K J D A B I K J G K H G F H E F C G D C A B C E B I E K*	M1 A1	 A list of 28 nodes that starts and ends with *K* Such a list that includes each of *AB*, *BI*, *JK* (or reversed) twice	
		Number of speed cameras on route: 81	B1	72 + weight of their least pairing	[6]
	(iii)	The only odd nodes are *I* and *J* so she only needs to repeat *IJ* = 6	B1	Identifying *I* and *J* or *IJ* (not just implied from 6 or 72+6 or 78)	
		72 + 6 = 78	M1 A1	Correct calculation (may be implied from 78)	[3]
				Total =	**16**

6	**(i)**	$\begin{array}{c\|cccccc} P & x & y & z & s & t & \\ \hline 1 & -3 & 5 & -4 & 0 & 0 & 0 \\ \hline 0 & 1 & 2 & -3 & 1 & 0 & 12 \\ 0 & 2 & 5 & -8 & 0 & 1 & 40 \end{array}$	B1 B1 B1	Correct use of two slack variable columns ± (-3 5 -4) in objective row 1 2 -3 12 and 2 5 -8 40 in constraint rows	[3]
	(ii)	The entries in rows 2 and 3 of the *z* column are negative Pivot on 1 in *x* column *x* and *z* columns have negative entries in obj. row but no value in *z* column is positive so choose *x* 12 ÷ 1 = 12, 40 ÷ 2 = 20 Least positive ratio is 12 so pivot on the 1	B1 B1 B1	Entries for potential pivots are not positive Correct pivot choice (cao) (stated or entry ringed) Follow through their table 'Negative in top row for *x*' and a correct explanation of choice of row 'least ratio 12 ÷ 1'	[3]
	(iii)	$\begin{array}{c\|cccccc} P & x & y & z & s & t & \\ \hline 1 & 0 & 11 & -13 & 3 & 0 & 36 \\ \hline 0 & 1 & 2 & -3 & 1 & 0 & 12 \\ 0 & 0 & 1 & -2 & -2 & 1 & 16 \end{array}$ *x* = 12, *y* = 0, *z* = 0	M1 A1 B1	Follow through their tableau if possible Correct method evident Correct tableau (ft if reasonable and possible, column representing RHS of equations must contain non-negative entries) Correct non-negative values for their tableau	[3]
	(iv)	*z* can increase without limit and increasing *z* will increase *P*	B1	Discussing the effect of increasing *z* Not just referring to pivoting in tableau	[1]
	(v)	Initial tableau is unchanged except entry in *z* col of obj. row becomes +40 First iteration tableau is also unchanged except for this entry which becomes 31 36	B1 B1 B1 B1	Describing change to obj. row of initial tableau or showing tableau that results Identifying 31 instead of -13 (cao) No other changes 36 stated (cao)	[4]
	(vi)	Adding the constraints gives $3x - 5y + 7z \le 52$ so $Q \le 52$	B1	52	[1]
	(vii)	$x - 3z = 12$ and $2x + 10z = 40$ (Accept ≤) $\Rightarrow 10z + 6z = 40 - 24$ $\Rightarrow x = 15$ and $z = 1$	M1 M1 A1	Eliminating *y* terms (may be implied) Trying to solve simultaneous equations Correct values (may imply method marks)	[3]
				Total =	**18**