ADVANCED SUBSIDIARY GCE UNIT

MATHEMATICS (MEI)

4751/01

Introduction to Advanced Mathematics (C1)

TUESDAY 16 JANUARY 2007

Morning
Time: 1 hour 30 minutes

Additional materials:
Answer booklet (8 pages)
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are **not** permitted to use a calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- There is an **insert** for use in Question **11**.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

WARNING

You are not allowed to use a calculator in this paper

This document consists of **4** printed pages and an insert.

Section A (36 marks)

1 Find, in the form $y = ax + b$, the equation of the line through $(3, 10)$ which is parallel to $y = 2x + 7$. [3]

2 Sketch the graph of $y = 9 - x^2$. [3]

3 Make a the subject of the equation

$$2a + 5c = af + 7c.$$ [3]

4 When $x^3 + kx + 5$ is divided by $x - 2$, the remainder is 3. Use the remainder theorem to find the value of k. [3]

5 Calculate the coefficient of x^4 in the expansion of $(x + 5)^6$. [3]

6 Find the value of each of the following, giving each answer as an integer or fraction as appropriate.

(i) $25^{\frac{3}{2}}$ [2]

(ii) $\left(\frac{7}{3}\right)^{-2}$ [2]

7 You are given that $a = \frac{3}{2}$, $b = \frac{9-\sqrt{17}}{4}$ and $c = \frac{9+\sqrt{17}}{4}$. Show that $a + b + c = abc$. [4]

8 Find the set of values of k for which the equation $2x^2 + kx + 2 = 0$ has no real roots. [4]

9 **(i)** Simplify $3a^3b \times 4(ab)^2$. [2]

(ii) Factorise $x^2 - 4$ and $x^2 - 5x + 6$.

Hence express $\dfrac{x^2 - 4}{x^2 - 5x + 6}$ as a fraction in its simplest form. [3]

10 Simplify $(m^2 + 1)^2 - (m^2 - 1)^2$, showing your method.

Hence, given the right-angled triangle in Fig. 10, express p in terms of m, simplifying your answer. [4]

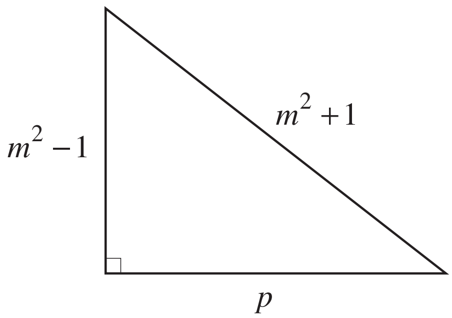


Fig. 10

Section B (36 marks)

11 **There is an insert for use in this question.**

The graph of $y = x + \frac{1}{x}$ is shown on the insert. The lowest point on one branch is $(1, 2)$. The highest point on the other branch is $(-1, -2)$.

(i) Use the graph to solve the following equations, showing your method clearly.

(A) $x + \frac{1}{x} = 4$ [2]

(B) $2x + \frac{1}{x} = 4$ [4]

(ii) The equation $(x - 1)^2 + y^2 = 4$ represents a circle. Find in exact form the coordinates of the points of intersection of this circle with the y-axis. [2]

(iii) State the radius and the coordinates of the centre of this circle.

Explain how these can be used to deduce from the graph that this circle touches one branch of the curve $y = x + \frac{1}{x}$ but does not intersect with the other. [4]

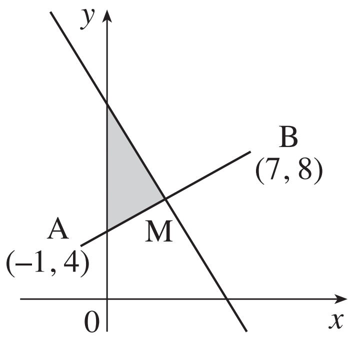
12 Use coordinate geometry to answer this question. Answers obtained from accurate drawing will receive no marks.

A and B are points with coordinates $(-1, 4)$ and $(7, 8)$ respectively.

(i) Find the coordinates of the midpoint, M, of AB.

Show also that the equation of the perpendicular bisector of AB is $y + 2x = 12$. [6]

(ii) Find the area of the triangle bounded by the perpendicular bisector, the y-axis and the line AM, as sketched in Fig. 12. [6]



Not to scale

Fig. 12

13

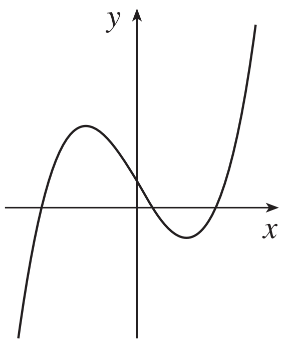


Fig. 13

Fig. 13 shows a sketch of the curve $y = f(x)$, where $f(x) = x^3 - 5x + 2$.

(i) Use the fact that $x = 2$ is a root of $f(x) = 0$ to find the exact values of the other two roots of $f(x) = 0$, expressing your answers as simply as possible. [6]

(ii) Show that $f(x - 3) = x^3 - 9x^2 + 22x - 10$. [4]

(iii) Write down the roots of $f(x - 3) = 0$. [2]