

# Mark Scheme (Results)

## January 2008

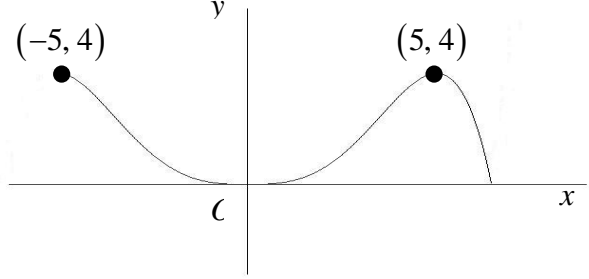
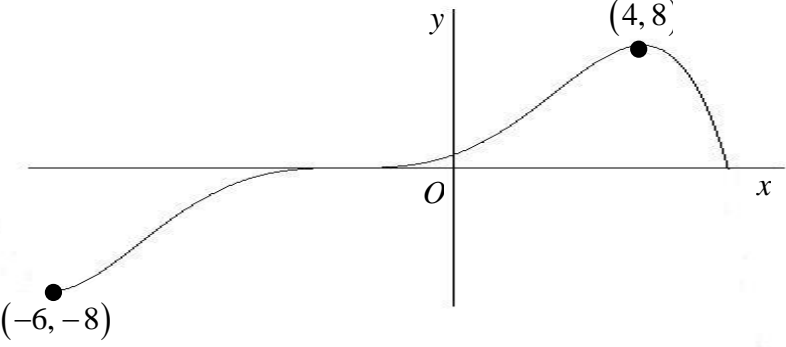
GCE

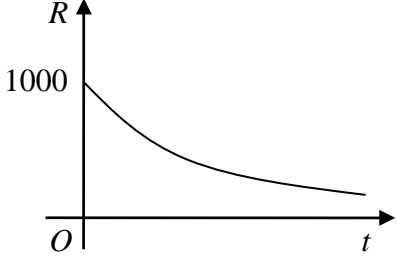
GCE Mathematics (6665/01)

January 2008  
6665 Core Mathematics C3  
Mark Scheme

Question Number	Scheme	Marks
<b>1.</b>	<div style="text-align: center;"> <math display="block">x^2 - 1 \begin{array}{r} \overline{2x^2 \quad -1} \\ 2x^4 \quad -3x^2 + x + 1 \\ \underline{2x^4 \quad -2x^2} \\ \quad -x^2 + x + 1 \\ \quad \underline{-x^2 \quad +1} \\ \qquad \qquad x \end{array}</math> </div> <div style="text-align: center; margin-top: 20px;"> <math display="block">2x^2 - 1 + \frac{x}{x^2 - 1}</math> </div> <div style="text-align: center; margin-top: 10px;"> <math display="block">a = 2, b = 0, c = -1, d = 1, e = 0</math> <p><math>d = 1</math> and <math>b = 0, e = 0</math> stated or implied</p> </div>	<p>M1 A1 A1</p> <p style="margin-top: 30px;">A1</p> <p style="text-align: right; margin-top: 10px;"><b>[4]</b></p>
<b>2.</b>	<p>(a)</p> <div style="text-align: center; margin-top: 20px;"> <math display="block">\frac{dy}{dx} = 2e^{2x} \tan x + e^{2x} \sec^2 x</math> </div> <div style="text-align: center; margin-top: 10px;"> <math display="block">\frac{dy}{dx} = 0 \Rightarrow 2e^{2x} \tan x + e^{2x} \sec^2 x = 0</math> </div> <div style="text-align: center; margin-top: 10px;"> <math display="block">2 \tan x + 1 + \tan^2 x = 0</math> </div> <div style="text-align: center; margin-top: 5px;"> <math display="block">(\tan x + 1)^2 = 0</math> </div> <div style="text-align: center; margin-top: 5px;"> <math display="block">\tan x = -1 \quad *</math> </div> <p style="text-align: right; margin-right: 50px;">cso</p> <p>(b)</p> <div style="text-align: center; margin-top: 20px;"> <math display="block">\left( \frac{dy}{dx} \right)_0 = 1</math> </div> <div style="text-align: center; margin-top: 20px;"> <p>Equation of tangent at (0, 0) is <math>y = x</math></p> </div>	<p>M1 A1+A1</p> <p>M1</p> <p>A1</p> <p>A1 (6)</p> <p>M1</p> <p>A1 (2)</p> <p style="text-align: right; margin-top: 10px;"><b>[8]</b></p>

Question Number	Scheme	Marks
3.	(a) $f(2) = 0.38 \dots$ $f(3) = -0.39 \dots$ Change of sign (and continuity) $\Rightarrow$ root in $(2, 3)$ *	M1 A1 (2) cso
	(b) $x_1 = \ln 4.5 + 1 \approx 2.50408$ $x_2 \approx 2.50498$ $x_3 \approx 2.50518$	M1 A1 A1 (3)
	(c) Selecting $[2.5045, 2.5055]$ , or appropriate tighter range, and evaluating at both ends. $f(2.5045) \approx 6 \times 10^{-4}$ $f(2.5055) \approx -2 \times 10^{-4}$ Change of sign (and continuity) $\Rightarrow$ root $\in (2.5045, 2.5055)$ $\Rightarrow$ root = 2.505 to 3 dp *	M1 A1 (2) cso <b>[7]</b>
Note: The root, correct to 5 dp, is 2.50524		

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4.	<p>(a)</p>  <p>(b) For the purpose of marking this paper, the graph is identical to (a)</p> <p>(c)</p>  <p>General shape – unchanged Translation to left</p> <p>In all parts of this question ignore any drawing outside the domains shown in the diagrams above.</p>	<p>Shape (5, 4) B1 (-5, 4) B1 (3)</p> <p>Shape (5, 4) B1 (-5, 4) B1 (3)</p> <p>General shape – unchanged B1 Translation to left B1 (4, 8) B1 (-6, -8) B1 (4)</p> <p>[10]</p>

Question Number	Scheme	Marks
5.	<p>(a) 1000</p> <p>(b) <math>1000e^{-5730c} = 500</math>  <math>e^{-5730c} = \frac{1}{2}</math>  <math>-5730c = \ln \frac{1}{2}</math>  <math>c = 0.000121</math></p> <p>(c) <math>R = 1000e^{-22920c} = 62.5</math></p> <p>(d)</p> <div style="text-align: center;">  </div>	<p>B1 (1)</p> <p>M1 A1 M1 cao A1 (4)</p> <p>Accept 62-63 M1 A1 (2)</p> <p>Shape 1000 B1 B1 (2)</p> <p><b>[9]</b></p>

Question Number	Scheme	Marks
6.	<p>(a) <math>\cos(2x+x) = \cos 2x \cos x - \sin 2x \sin x</math>  <math>= (2\cos^2 x - 1)\cos x - (2\sin x \cos x)\sin x</math>  <math>= (2\cos^2 x - 1)\cos x - 2(1 - \cos^2 x)\cos x</math> any correct expression  <math>= 4\cos^3 x - 3\cos x</math></p> <p>(b)(i) <math>\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x} = \frac{\cos^2 x + (1+\sin x)^2}{(1+\sin x)\cos x}</math>  <math>= \frac{\cos^2 x + 1 + 2\sin x + \sin^2 x}{(1+\sin x)\cos x}</math>  <math>= \frac{2(1+\sin x)}{(1+\sin x)\cos x}</math>  <math>= \frac{2}{\cos x} = 2\sec x</math> *      cso</p> <p>(c) <math>\sec x = 2</math> or <math>\cos x = \frac{1}{2}</math>  <math>x = \frac{\pi}{3}, \frac{5\pi}{3}</math>      accept awrt 1.05, 5.24</p>	<p>M1 M1 A1 A1      (4)</p> <p>M1 A1 M1 A1      (4)</p> <p>M1 A1, A1      (3)</p> <p><b>[11]</b></p>
7.	<p>(a) <math>\frac{dy}{dx} = 6\cos 2x - 8\sin 2x</math>  <math>\left(\frac{dy}{dx}\right)_0 = 6</math>  <math>y - 4 = -\frac{1}{6}x</math>      or equivalent</p> <p>(b) <math>R = \sqrt{(3^2 + 4^2)} = 5</math>  <math>\tan \alpha = \frac{4}{3}, \alpha \approx 0.927</math>      awrt 0.927</p> <p>(c) <math>\sin(2x + \text{their } \alpha) = 0</math>  <math>x = -2.03, -0.46, 1.11, 2.68</math></p> <p>First A1 any correct solution; second A1 a second correct solution; third A1 all four correct and to the specified accuracy or better.  Ignore the y-coordinate.</p>	<p>M1 A1 B1 M1 A1      (5)</p> <p>M1 A1 M1 A1      (4)</p> <p>M1 A1 A1 A1      (4)</p> <p><b>[13]</b></p>

Question Number	Scheme	Marks
8.	(a) $x = 1 - 2y^3 \Rightarrow y = \left(\frac{1-x}{2}\right)^{\frac{1}{3}}$ or $\sqrt[3]{\frac{1-x}{2}}$ $f^{-1} : x \mapsto \left(\frac{1-x}{2}\right)^{\frac{1}{3}}$	M1 A1 (2)
		Ignore domain
	(b) $gf(x) = \frac{3}{1-2x^3} - 4$ $= \frac{3-4(1-2x^3)}{1-2x^3}$ $= \frac{8x^3-1}{1-2x^3}$ *	M1 A1 M1
		cso
	$gf : x \mapsto \frac{8x^3-1}{1-2x^3}$	A1 (4) Ignore domain
	(c) $8x^3 - 1 = 0$	Attempting solution of numerator = 0 M1
	$x = \frac{1}{2}$	Correct answer and no additional answers A1 (2)
	(d) $\frac{dy}{dx} = \frac{(1-2x^3) \times 24x^2 + (8x^3-1) \times 6x^2}{(1-2x^3)^2}$ $= \frac{18x^2}{(1-2x^3)^2}$	M1 A1 A1
	Solving their numerator = 0 and substituting to find y.	M1
$x = 0, y = -1$	A1 (5) [13]	