4723 Core Mathematics 3

1 (i) Show correct process for composition of functions **M1** numerical or algebraic; the right way round

Obtain (–3 and hence) –23 **A1 2**

(ii) Either: State or imply $x^3 + 4 = 12$ **B1**

Attempt solution of equation involving x^3 **M1** as far as $x = \ldots$

Obtain 2 **A1 3** and no other value

Or: Attempt expression for f^{-1} **M1** involving x or y; involving cube root

Obtain $\sqrt[3]{x-4}$ or $\sqrt[3]{y-4}$ **A1**

Obtain 2 **A1 (3)** and no other value

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2 (i) Obtain correct first iterate 2.864 **B1** or greater accuracy 2.864327…; condone 2 dp here and in working

Carry out correct iteration process **M1** to find at least 3 iterates in all

Obtain 2.877 **A1 3** after at least 4 steps; answer required to exactly 3 dp

[3 → 2.864327 → 2.878042 → 2.876661 → 2.876800]

(ii) State or imply $x = \sqrt[3]{31 - \frac{5}{2}x}$ **B1**

Attempt rearrangement of equation in x **M1** involving cubing and grouping non-zero terms on LHS

Obtain equation $2x^3 + 5x - 62 = 0$ **A1 3** or equiv with integers

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3 (a) State correct equation involving $\cos\frac{1}{2}\alpha$ **B1** such as $\cos\frac{1}{2}\alpha = \frac{1}{4}$ or $\frac{1}{\cos\frac{1}{2}\alpha} = 4$ or …

Attempt to find value of α **M1** using correct order for the steps

Obtain 151 **A1 3** or greater accuracy; and no other values between 0 and 180

(b) State or imply $\cot\beta = \frac{1}{\tan\beta}$ **B1**

Rearrange to the form $\tan\beta = k$ **M1** or equiv involving $\sin\beta$ only or $\cos\beta$ only; allow missing ±

Obtain 69.3 **A1**

Obtain 111 **A1 4** or greater accuracy; and no others between 0 and 180

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4 (i) Obtain derivative of form $kh^5(h^6+16)^n$ **M1** any constant k; any $n < \frac{1}{2}$; allow if – 4 term retained

Obtain correct $3h^5(h^6+16)^{-\frac{1}{2}}$ **A1** or (unsimplified) equiv; no –4 now

Substitute to obtain 10.7 **A1 3** or greater accuracy or exact equiv

(ii) Attempt multn or divn using 8 and answer from (i) **M1**

Attempt 8 divided by answer from (i) **M1**

Obtain 0.75 **A1√ 3** or greater accuracy; allow 0.75 ± 0.01; following their answer from (i)

5	**(a)**	Obtain integral of form $k(3x+7)^{10}$	**M1**		any constant k
		Obtain (unsimplified) $\frac{1}{10}\times\frac{1}{3}(3x+7)^{10}$	**A1**		or equiv
		Obtain (simplified) $\frac{1}{30}(3x+7)^{10}+c$	**A1**	**3**	
	(b)	State $\int \pi(\frac{1}{2\sqrt{x}})^2\,\mathrm{d}x$	**B1**		or equiv involving x; condone no dx
		Integrate to obtain $k\ln x$	**M1**		any constant k involving π or not; or equiv such as $k\ln 4x$ or $k\ln 2x$
		Obtain $\frac{1}{4}\pi\ln x$ or $\frac{1}{4}\ln x$ or $\frac{1}{4}\pi\ln 4x$ or $\frac{1}{4}\ln 4x$	**A1**		
		Show use of the log a – log b property	**M1**		not dependent on earlier marks
		Obtain $\frac{1}{4}\pi\ln 2$	**A1**	**5**	or similarly simplified equiv

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6	**(i)**	Either: Refer to translation and reflection	**B1**		in either order; allow clear equivs
		State translation by 1 in negative x-direction	**B1**		or equiv but now using correct terminology
		State reflection in x-axis	**B1**	**3**	using correct terminology
		Or: Refer to translation and reflection	**B1**		in either order; allow clear equivs
		State reflection in y-axis	**B1**		
		State translation by 1 in positive x-direction	**B1**	**(3)**	with order reflection then translation clearly intended
	(ii)	Show sketch with attempt at reflection of 'negative' part in x-axis	**M1**		and curve for $0<x<1$ unchanged
		Show (more or less) correct sketch	**A1**	**2**	with correct curvature
	(iii)	Attempt correct process for finding at least one value	**M1**		as far as x = …; accept decimal equivs (degrees or radians) or expressions involving $\sin(\frac{1}{3}\pi)$
		Obtain $1-\frac{1}{2}\sqrt{3}$	**A1**		or exact equiv
		Obtain $1+\frac{1}{2}\sqrt{3}$	**A1**	**3**	or exact equiv; give **A1A0** if extra incorrect solution(s) provided

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7	**(i)**	Attempt use of product rule for $x\mathrm{e}^{2x}$	**M1**		obtaining … + …
		Obtain $\mathrm{e}^{2x}+2x\mathrm{e}^{2x}$	**A1**		or equiv; maybe within QR attempt
		Attempt use of quotient rule	**M1**		with or without product rule
		Obtain unsimplified $\dfrac{(x+k)(\mathrm{e}^{2x}+2x\mathrm{e}^{2x})-x\mathrm{e}^{2x}}{(x+k)^2}$	**A1**		
		Obtain $\dfrac{\mathrm{e}^{2x}(2x^2+2kx+k)}{(x+k)^2}$	**A1**	**5**	**AG**; necessary detail required
	(ii)	Attempt use of discriminant	**M1**		or equiv
		Obtain $4k^2-8k=0$ or equiv and hence $k=2$	**A1**		
		Attempt solution of $2x^2+2kx+k=0$	**M1**		using their numerical value of k or solving in terms of k using correct formula
		Obtain $x=-1$	**A1**		
		Obtain $-\mathrm{e}^{-2}$	**A1**	**5**	or exact equiv

8 (i)	State or imply $h = 1$	**B1**		
	Attempt calculation involving attempts at y values	**M1**		addition with each of coefficients 1, 2, 4 occurring at least once; involving at least 5 y values
	Obtain $a(1 + 4\times2 + 2\times4 + 4\times8 + 2\times16 + 4\times32 + 64)$	**A1**		any constant a
	Obtain 91	**A1**	**4**	
(ii)	State $e^{x\ln 2}$ or $k = \ln 2$	**B1**		allow decimal equiv such as $e^{0.69x}$
	Integrate e^{kx} to obtain $\frac{1}{k}e^{kx}$	**M1**		any constant k or in terms of general k
	Obtain $\frac{1}{\ln 2}(e^{6\ln 2} - e^0)$	**A1**		or exact equiv
	Simplify to obtain $\frac{63}{\ln 2}$	**A1**	**4**	allow if simplification in part (iii)
(iii)	Equate answers to (i) and (ii)	**M1**		provided ln 2 involved other than in power of e
	Obtain $\frac{63}{91}$ and hence $\frac{9}{13}$	**A1**	**2**	**AG**; necessary correct detail required

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9 (i)	State at least one of $\cos\theta\cos 60 - \sin\theta\sin 60$ and $\cos\theta\cos 30 - \sin\theta\sin 30$	**B1**		
	Attempt complete multiplication of identities of form $\pm\cos\cos \pm \sin\sin$	**M1**		with values $\frac{1}{2}\sqrt{3}$, $\frac{1}{2}$ involved
	Use $\cos^2\theta + \sin^2\theta = 1$ and $2\sin\theta\cos\theta = \sin 2\theta$	**M1**		
	Obtain $\sqrt{3} - 2\sin 2\theta$	**A1**	**4**	**AG**; necessary detail required
(ii)	Attempt use of 22.5 in right-hand side	**M1**		
	Obtain $\sqrt{3} - \sqrt{2}$	**A1**	**2**	or exact equiv
(iii)	Obtain 10.7	**B1**		or greater accuracy; allow ± 0.1
	Attempt correct process to find two angles	**M1**		from values of 2θ between 0 and 180
	Obtain 79.3	**A1**	**3**	or greater accuracy and no others between 0 and 90; allow ± 0.1
(iv)	Indicate or imply that critical values of $\sin 2\theta$ are -1 and 1	**M1**		
	Obtain both of $k > \sqrt{3} + 2$, $k < \sqrt{3} - 2$	**A1**		condoning decimal equivs, $\leq \geq$ signs
	Obtain complete correct solution	**A1**	**3**	now with exact values and unambiguously stated