C3 JAN 09

1) $u = x^2 \quad v = (5x-1)^{\frac{1}{2}} \quad \frac{dy}{dx} = vu' + uv'$

$u' = 2x \quad v' = \frac{1}{2} \times 5(5x-1)^{-\frac{1}{2}}$

$$\frac{dy}{dx} = 2x(5x-1)^{\frac{1}{2}} + \frac{5}{2}x^2(5x-1)^{-\frac{1}{2}}$$

$x=2 \quad \frac{dy}{dx} = 4(9)^{\frac{1}{2}} + 10(9)^{-\frac{1}{2}} = 12 + \frac{10}{3} = \frac{46}{3}$

b) $u = \sin 2x \quad v = x^2 \quad \frac{dy}{dx} = \frac{vu' - uv'}{v^2}$

$u' = 2\cos 2x \quad v' = 2x$

$$\frac{dy}{dx} = \frac{2x^2\cos 2x - 2x\sin 2x}{x^4}$$

2) $\frac{2\cancel{(x+1)}}{\cancel{(x+1)}(x-3)} - \frac{(x+1)}{(x-3)} = \frac{2-x-1}{x-3} = \frac{1-x}{x-3}$

b) $u = 1-x \quad v = x-3 \quad \frac{dy}{dx} = \frac{-(x-3)-(1-x)}{(x-3)^2}$

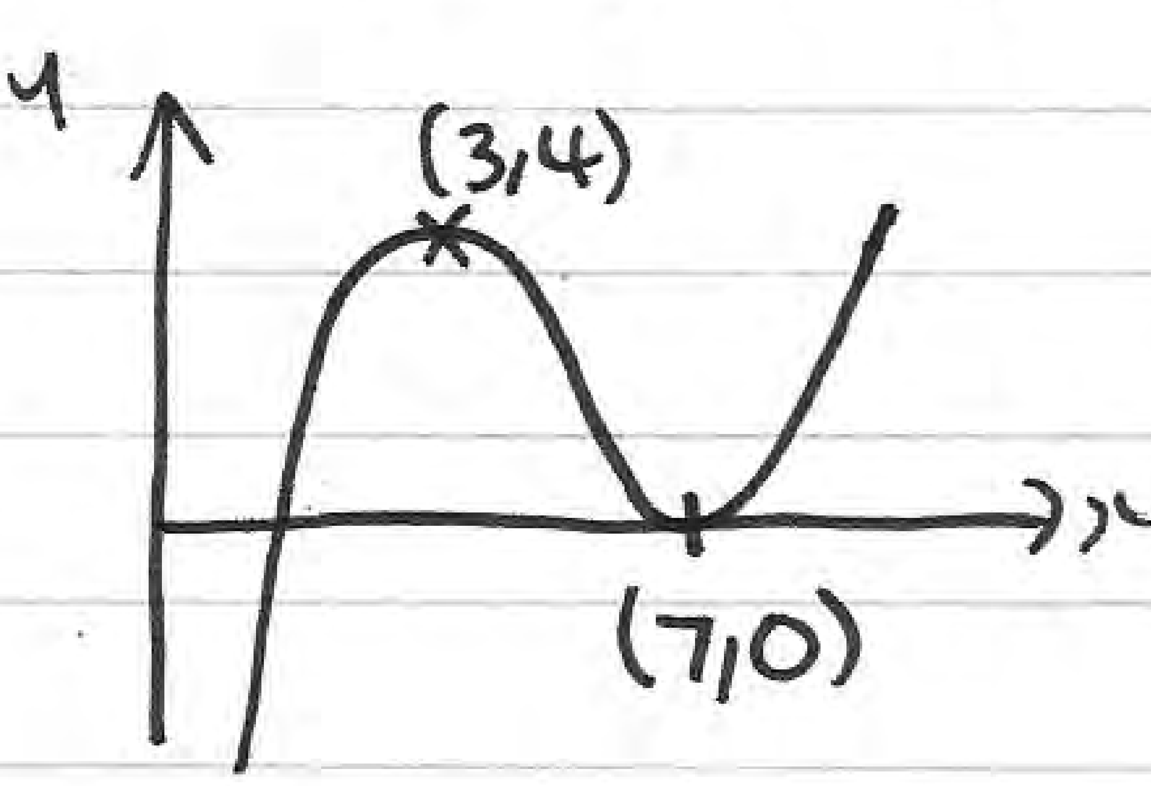
$u' = -1 \quad v' = 1$

$$\frac{dy}{dx} = \frac{-x+3-1+x}{(x-3)^2} = \frac{2}{(x-3)^2} \quad \#$$

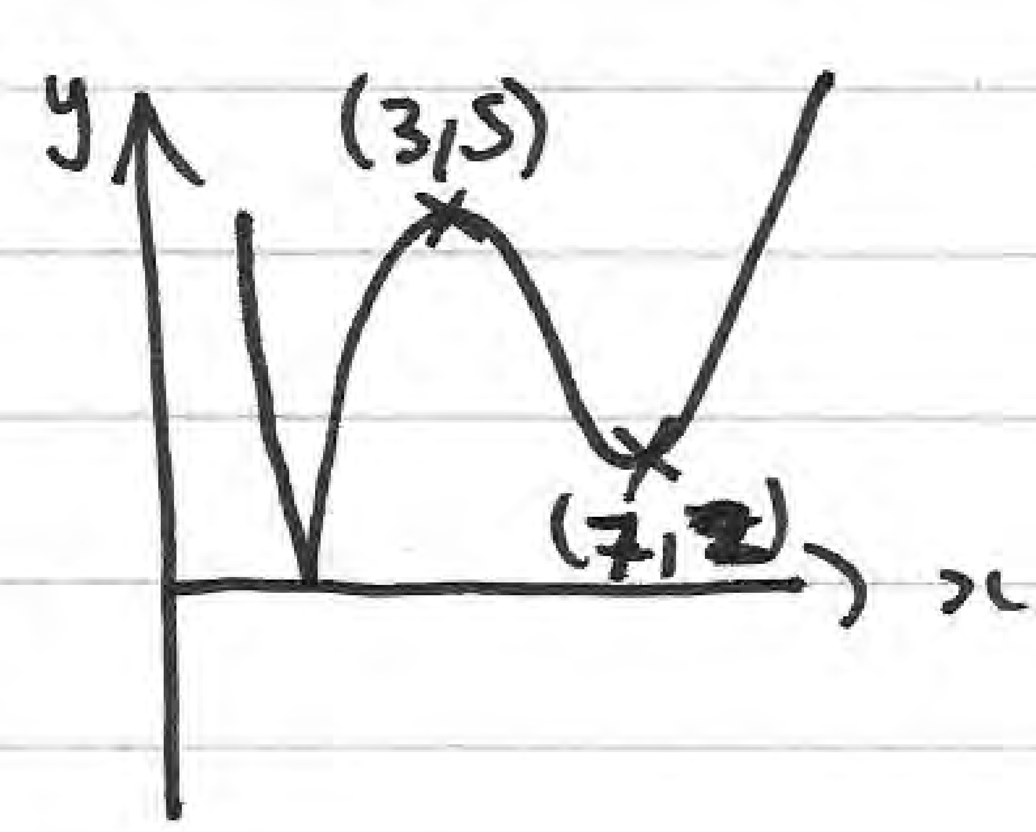
3) $y = 2f(x)\ 4$

$2 \updownarrow \quad \downarrow 4$

$y \times 2$ then -4



b) $|f(x)|$



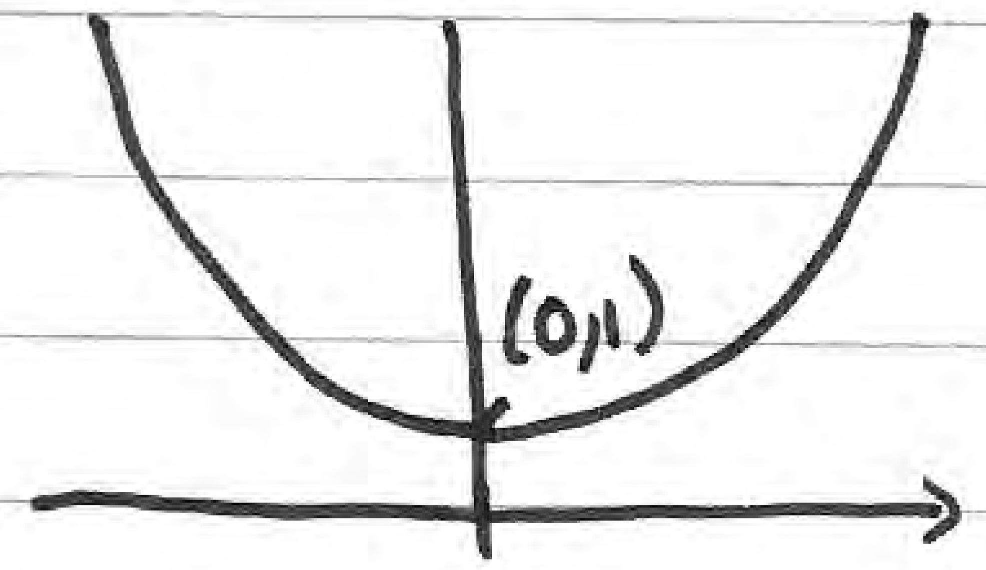
4) $x = \cos(2y+\pi) \qquad \left(0, \frac{\pi}{4}\right)$

$\frac{dx}{dy} = -2\sin(2y+\pi) \Rightarrow \frac{dy}{dx} = \frac{-1}{2\sin(2y+\pi)}$

$y = \frac{\pi}{4} \Rightarrow m_t = \frac{-1}{2\sin\left(\frac{\pi}{2}+\pi\right)} = \frac{-1}{-2} = \frac{1}{2}$

$\Rightarrow y - \frac{\pi}{4} = \frac{1}{2}(x-0) \Rightarrow y = \frac{1}{2}x + \frac{\pi}{4}$

5) $g(x) = e^{x^2}$



range $y \geq 1$

b) $fg(x) = f(e^{x^2}) = 3e^{x^2} + \cancel{\ln e}^{x^2}$

$fg(x) = 3e^{x^2} + x^2$

c) $x \longrightarrow g \longrightarrow f \longrightarrow fg(x)$

>0 and ≥ 1 so ≥ 1

$fg(x) \geq 3(1) + \ln(1)$

range $y \geq 3$

d) $\frac{d}{dx}\left[3e^{x^2} + x^2\right] = 6xe^{x^2} + 2x = x(xe^{x^2}+2)$

$\Rightarrow 6xe^{x^2} + \cancel{2x} = x^2e^{x^2} + \cancel{2x}$

$\Rightarrow (x^2 - 6x)e^{x^2} = 0$

$\Rightarrow x(x-6)e^{x^2} = 0 \qquad \underline{x=0}, \underline{x=6}$

6) $\sin(3\theta) = \sin(2\theta + \theta) = \sin 2\theta\cos\theta + \cos 2\theta\sin\theta$

$= 2\sin\theta\cos^2\theta + (1 - 2\sin^2\theta)\sin\theta$

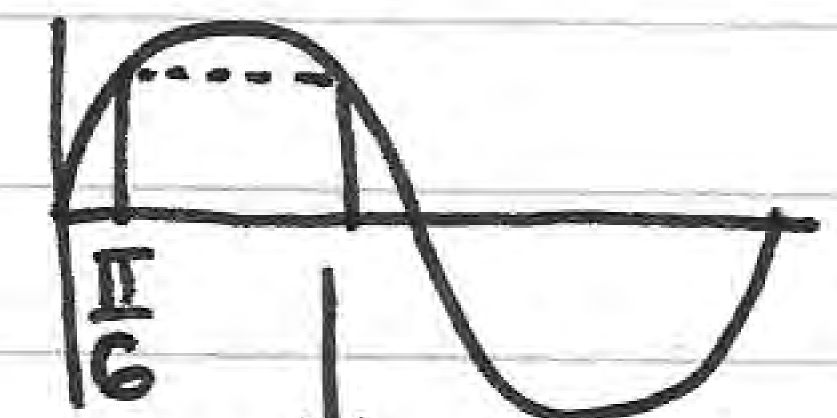
$= 2\sin\theta(1 - \sin^2\theta) + \sin\theta - 2\sin^3\theta$

$= 2\sin\theta - 2\sin^3\theta + \sin\theta - 2\sin^3\theta$

$= 3\sin\theta - 4\sin^3\theta$ #

ii) $6\sin\theta - 8\sin^3\theta = 1 \Rightarrow 3\sin\theta - 4\sin^3\theta = \frac{1}{2} \Rightarrow \sin 3\theta = \frac{1}{2}$

$\Rightarrow 3\theta = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$



$\pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$3\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \cancel{\frac{25\pi}{6}}, \cancel{\frac{29\pi}{6}} \ldots$

$+2\pi = \frac{12\pi}{6}$ $\quad \cancel{+2\pi = \frac{12\pi}{6}}$

$(\div 3)$ $\theta = \frac{\pi}{18}, \frac{5\pi}{18}$

b) $\sin 15 = \sin(45-30) = \sin 45 \cos 30 - \cos 45 \sin 30$

$= \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \times \frac{1}{2} = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$= \frac{1}{4}(\sqrt{6} - \sqrt{2})$ #

7) $f(x) = 3xe^x - 1$

$u = 3x \quad v = e^x$
$u' = 3 \quad v' = e^x$

$f'(x) = 3e^x + 3xe^x$

TP when $f'(x) = 0 \Rightarrow 3e^x(1+x) = 0$

$\Rightarrow \underline{x = -1}$

$P\left(-1, -\frac{3}{e} - 1\right)$

b) $x_0 = 0.25 \quad x_1 = 0.2596 \quad x_2 = 0.2571 \quad x_3 = 0.2578$

c) $f(0.25765) = 0.000109 > 0$ $\quad$ Change of sign
$f(0.25755) = -0.00038 < 0$ $\quad \Rightarrow \alpha \in \underline{0.2576}$ (4dp)

8) $R\cos(\theta-\alpha) = R\cancel{\cos\theta}\cos\alpha + R\cancel{\sin\theta}\sin\alpha$

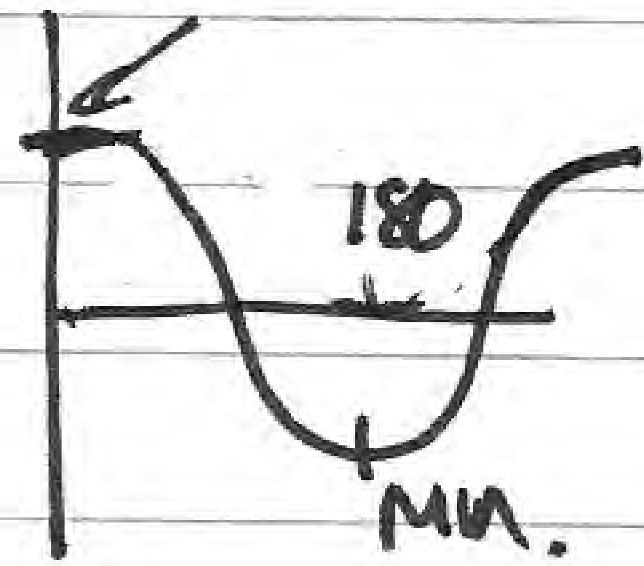
$3\cancel{\cos\theta} \quad + 4\cancel{\sin\theta}$

$\frac{R\sin\alpha = 4}{R\cos\alpha = 3} \Rightarrow \tan\alpha = \frac{4}{3} \Rightarrow \alpha = 53.1°$

$R^2 = 3^2 + 4^2$
$\Rightarrow R = 5$

$5\cos(\theta - 53.1°)$

b) max = 5



max occurs when $\theta - 53.1 = 0$
$\Rightarrow \theta = \underline{53.1}$

c) $f(t) = \underbrace{3\cos(15t) + 4\sin(15t)}_{}\ + 10 \quad \Rightarrow \text{Min } f(t) = 10 - 5 = \underline{5}$

min = −5

min when $\theta - 53.1 = 180$
$\Rightarrow \theta = 233.1$

$15t = 233.1 \Rightarrow t = \underline{15.54}$

(0332)