Mark Scheme (Results)

January 2009

GCE

GCE Mathematics (6663/01)

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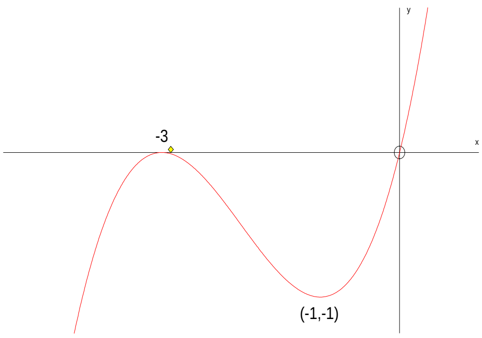
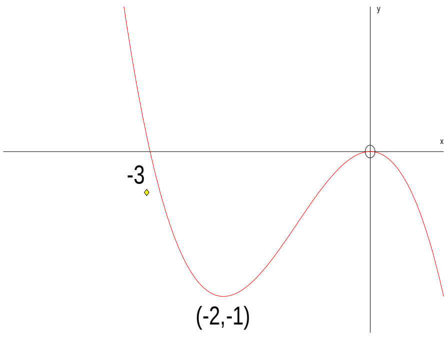
January 2009
6663 Core Mathematics C1
Mark Scheme

Question Number	Scheme	Marks
1 (a)	**5** (± 5 is B0)	B1 (1)
(b)	$\dfrac{1}{(\text{their } 5)^2}$ or $\left(\dfrac{1}{\text{their } 5}\right)^2$	M1
	$= \dfrac{1}{25}$ **or 0.04** ($\pm\dfrac{1}{25}$ is A0)	A1 (2) **[3]**
(b)	M1 follow through their value of 5. Must have reciprocal and square. 5^{-2} is <u>not</u> sufficient to score this mark, unless $\dfrac{1}{5^2}$ follows this. A negative introduced at any stage can score the M1 but not the A1, e.g. $125^{-2/3} = \left(-\dfrac{1}{5}\right)^2 = \dfrac{1}{25}$ scores M1 A0 $125^{-2/3} = -\left(\dfrac{1}{5}\right)^2 = -\dfrac{1}{25}$ scores M1 A0. Correct answer with no working scores both marks. <u>Alternative</u>: $\dfrac{1}{\sqrt[3]{125^2}}$ or $\dfrac{1}{(125^2)^{1/3}}$ M1 (reciprocal and the correct number squared) $\left(= \dfrac{1}{\sqrt[3]{15625}}\right)$ $= \dfrac{1}{25}$ A1	

Question Number	Scheme	Marks
2	$(I=)\dfrac{12}{6}x^6-\dfrac{8}{4}x^4+3x+c$ $=2x^6-2x^4+3x+c$	M1 A1A1A1 **[4]**
	M1 for an attempt to integrate $x^n \to x^{n+1}$ (i.e. ax^6 or ax^4 or ax, where a is any non-zero constant). Also, this M mark can be scored for just the + c (seen at some stage), even if no other terms are correct. 1st A1 for $2x^6$ 2nd A1 for $-2x^4$ 3rd A1 for $3x + c$ (or $3x+k$, etc., any appropriate letter can be used as the constant) Allow $3x^1+c$, but <u>not</u> $\dfrac{3x^1}{1}+c$. Note that the A marks can be awarded at separate stages, e.g. $\dfrac{12}{6}x^6-2x^4+3x$ scores 2nd A1 $\dfrac{12}{6}x^6-2x^4+3x+c$ scores 3rd A1 $2x^6-2x^4+3x$ scores 1st A1 (even though the c has now been lost). Remember that all the A marks are dependent on the M mark. If applicable, isw (ignore subsequent working) after a correct answer is seen. Ignore wrong notation if the intention is clear, e.g. Answer $\int 2x^6-2x^4+3x+c\,dx$.	

Question Number	Scheme	Marks
3	$\sqrt{7}^2 + 2\sqrt{7} - 2\sqrt{7} - 2^2$, or $7 - 4$ or an exact equivalent such as $\sqrt{49} - 2^2$ $= \mathbf{3}$	M1 A1 **[2]**
	M1 for an expanded expression. At worst, there can be <u>one wrong term</u> and <u>one wrong sign</u>, or <u>two wrong signs</u>. e .g. $7 + 2\sqrt{7} - 2\sqrt{7} - 2$ is M1 (one wrong term -2) $7 + 2\sqrt{7} + 2\sqrt{7} + 4$ is M1 (two wrong signs $+2\sqrt{7}$ and $+4$) $7 + 2\sqrt{7} + 2\sqrt{7} + 2$ is M1 (one wrong term $+2$, one wrong sign $+2\sqrt{7}$) $\sqrt{7} + 2\sqrt{7} - 2\sqrt{7} + 4$ is M1 (one wrong term $\sqrt{7}$, one wrong sign $+4$) $\sqrt{7} + 2\sqrt{7} - 2\sqrt{7} - 2$ is M0 (two wrong terms $\sqrt{7}$ and -2) $7 + \sqrt{14} - \sqrt{14} - 4$ is M0 (two wrong terms $\sqrt{14}$ and $-\sqrt{14}$) If only 2 terms are given, they must be correct, i.e. (7 – 4) or an equivalent unsimplified version to score M1. The terms can be seen <u>separately</u> for the M1. Correct answer with <u>no working</u> scores both marks.	

Question Number	Scheme	Marks
4	$\left(\text{f}(x)=\right)\dfrac{3x^3}{3}-\dfrac{3x^{\frac{3}{2}}}{\frac{3}{2}}-7x\left(+c\right)$	M1
	$= \; x^3-2x^{\frac{3}{2}}-7x \quad (+c)$	A1A1
	f(4) = 22 $\Rightarrow$ $22=64-16-28+c$	M1
	$\boldsymbol{c}$ **= 2**	A1cso (5)
		[5]
	1st M1 for an attempt to integrate (x^3 or $x^{\frac{3}{2}}$ seen). The *x* term is insufficient for this mark and similarly the + *c* is insufficient. 1st A1 for $\frac{3}{3}x^3$ or $-\dfrac{3x^{\frac{3}{2}}}{\frac{3}{2}}$ (An unsimplified or simplified correct form) 2nd A1 for all three *x* terms correct and simplified… (the simplification may be seen later). The + *c* is not required for this mark. Allow $-7x^1$, but <u>not</u> $-\dfrac{7x^1}{1}$. 2nd M1 for an attempt to use *x* = 4 <u>and</u> *y* = 22 in a changed function (even if differentiated) to form an equation in *c*. 3rd A1 for *c* = 2 with no earlier incorrect work (a final expression for f(*x*) is not required).	

Question Number	Scheme	Marks
5 (a)	 Shape  , touching the *x*-axis at its maximum.	M1
	Through (0,0) & -3 marked on *x*-axis, or $(-3,0)$ seen. Allow $(0,-3)$ if marked on the *x*-axis. Marked in the correct place, but 3, is A0.	A1
	Min at $(-1,-1)$	A1 (3)
(b)	 Correct shape  (top left - bottom right)	B1
	Through -3 and max at (0, 0). Marked in the correct place, but 3, is B0.	B1
	Min at $(-2,-1)$	B1 (3)
		[6]
(a)	M1 as described above. Be generous, even when the curve seems to be composed of straight line segments, but there must be a discernible 'curve' at the max. and min. 1st A1 for curve passing through -3 and the origin. Max at $(-3,0)$ 2nd A1 for minimum at $(-1,-1)$. Can simply be indicated on sketch.	
(b)	1st B1 for the correct shape. A negative cubic passing from top left to bottom right. Shape: Be generous, even when the curve seems to be composed of straight line segments, but there must be a discernible 'curve' at the max. and min. 2nd B1 for curve passing through $(-3,0)$ having a max at (0, 0) and no other max. 3rd B1 for minimum at $(-2,-1)$ and no other minimum. If in correct quadrant but labelled, e.g. $(-2,1)$, this is B0. In each part the (0, 0) does <u>not</u> need to be written to score the second mark… having the curve pass through the origin is sufficient. The last mark (for the minimum) in each part is dependent on a sketch being attempted, and the sketch must show the minimum in approximately the correct place (not, for example, $(-2,-1)$ marked in the wrong quadrant). The mark for the minimum is <u>not</u> given for the coordinates just marked on the axes <u>unless</u> these are clearly linked to the minimum by vertical and horizontal lines.	

Question Number	Scheme	Marks
6 (a)	$2x^{3/2}$ or $p = \frac{3}{2}$ (<u>Not</u> $2x\sqrt{x}$)	B1
	$-x$ or $-x^1$ or $q = 1$	B1 (2)
(b)	$\left(\frac{dy}{dx} =\right) 20x^3 + 2\times\frac{3}{2}x^{1/2} - 1$	M1
	$= \underline{20x^3 + 3x^{\frac{1}{2}} - 1}$	A1A1ftA1ft (4) **[6]**
(a)	1^{st} B1 for $p = 1.5$ or exact equivalent 2^{nd} B1 for $q = 1$	
(b)	M1 for an attempt to differentiate $x^n \to x^{n-1}$ (for any of the 4 terms) 1^{st} A1 for $20x^3$ (the -3 must 'disappear') 2^{nd} A1ft for $3x^{\frac{1}{2}}$ or $3\sqrt{x}$. Follow through their p but they must be differentiating $2x^p$, where p is a <u>fraction</u>, and the coefficient must be simplified if necessary. 3^{rd} A1ft for -1 (<u>not</u> the unsimplified $-x^0$), or follow through for correct differentiation of their $-x^q$ (i.e. coefficient of x^q is -1). If ft is applied, the coefficient must be simplified if necessary. 'Simplified' coefficient means $\frac{a}{b}$ where a and b are integers with no common factors. Only a single + or – sign is allowed (e.g. – – must be replaced by +). If there is a 'restart' in part (b) it can be marked independently of part (a), but marks for part (a) cannot be scored for work seen in (b). <u>Multiplying</u> by $\sqrt{x}$: (assuming this is a restart) e.g. $y = 5x^4\sqrt{x} - 3\sqrt{x} + 2x^2 - x^{3/2}$ $\left(\frac{dy}{dx} =\right)\frac{45}{2}x^{7/2} - \frac{3}{2}x^{-1/2} + 4x - \frac{3}{2}x^{1/2}$ scores M1 A0 A0 (p not a fraction) A1ft. <u>Extra term</u> included: This invalidates the final mark. e.g. $y = 5x^4 - 3 + 2x^2 - x^{3/2} - x^{1/2}$ $\left(\frac{dy}{dx} =\right)20x^3 + 4x - \frac{3}{2}x^{1/2} - \frac{1}{2}x^{-1/2}$ scores M1 A1 A0 (p not a fraction) A0. <u>Numerator and denominator differentiated separately</u>: For this, neither of the last two (ft) marks should be awarded. <u>Quotient/product rule</u>: Last two terms must be correct to score the last 2 marks. (If the M mark has not already been earned, it can be given for the quotient/product rule attempt.)	

Question Number	Scheme	Marks
7 (a)	$b^2-4ac>0\Rightarrow\quad 16-4k(5-k)>0$ or equiv., e.g. $16>4k(5-k)$	M1A1
	So $k^2-5k+4>0$ (Allow any order of terms, e.g. $4-5k+k^2>0$) (*)	A1cso (3)
(b)	Critical Values $(k-4)(k-1)=0$ $k=\ldots.$	M1
	$k = 1$ or 4	A1
	Choosing "outside" region	M1
	<u>$\boldsymbol{k<1}$ or $\boldsymbol{k>4}$</u>	A1 (4) **[7]**

For this question, ignore (a) and (b) labels and award marks wherever correct work is seen.

(a)

M1 for attempting to use the discriminant of the initial equation (> 0 not required, but substitution of a, b and c in the correct formula is required).
If the formula b^2-4ac is seen, at least 2 of a, b and c must be correct.
If the formula b^2-4ac is <u>not</u> seen, all 3 (a, b and c) must be correct.
This mark can still be scored if substitution in b^2-4ac is within the quadratic formula.
This mark can also be scored by comparing b^2 and $4ac$ (with substitution).
However, use of b^2+4ac is M0.

1st A1 for fully correct expression, possibly unsimplified, with > symbol. NB must appear before the last line, even if this is simply in a statement such as $b^2-4ac>0$ or 'discriminant positive'.
Condone a bracketing slip, e.g. $16-4\times k\times 5-k$ if subsequent work is correct and convincing.

2nd A1 for a fully correct derivation with no incorrect working seen.
Condone a bracketing slip if otherwise correct and convincing.

<u>Using</u> $\sqrt{b^2-4ac}>0$:
Only available mark is the first M1 (unless recovery is seen).

(b)

1st M1 for attempt to solve an appropriate 3TQ

1st A1 for both k = 1 and 4 (only the critical values are required, so accept, e.g. $k>1$ and $k>4$). **

2nd M1 for choosing the "outside" region. A diagram or table alone is not sufficient.
Follow through their values of k.
The set of values must be 'narrowed down' to score this M mark… listing everything $k<1$, $1<k<4$, $k>4$ is M0.

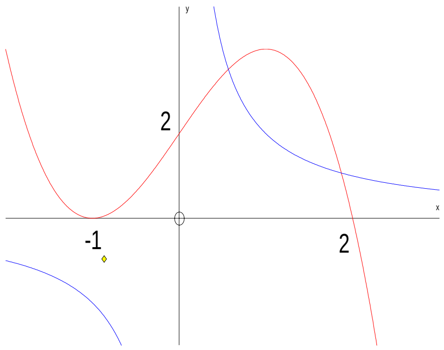
2nd A1 for correct answer only, condone "$k<1$, $k>4$" and even "$k<1$ and $k>4$", but "$1>k>4$" is A0.

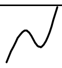
** Often the statement $k>1$ and $k>4$ is followed by the correct final answer. Allow full marks.

<u>Seeing 1 and 4 used as critical values</u> gives the first M1 A1 by implication.

In part (b), condone working with x's except for the final mark, where the set of values must be a set of values of k (i.e. 3 marks out of 4).

Use of ≤ (or ≥) in the final answer loses the final mark.

Question Number	Scheme	Marks	
8 (a)	$(a=)\ \ (1+1)^2(2-1)=\underline{\mathbf{4}}$ (1, 4) or $y = 4$ is also acceptable	B1	(1)
(b)			
	(i) Shape  or  anywhere	B1	
	Min at $(-1, 0)$ … can be -1 on x-axis. Allow $(0, -1)$ if marked on the x-axis. Marked in the correct place, but 1, is B0.	B1	
	(2, 0) and (0, 2) can be 2 on axes	B1	
	(ii) Top branch in 1st quadrant with 2 intersections	B1	
	Bottom branch in 3rd quadrant (ignore any intersections)	B1	(5)
(c)	(2 intersections therefore) **<u>2</u>** (roots)	B1ft	(1) **[7]**

(b) 1st B1 for shape  or  Can be anywhere, but there must be one max. and one min. and no further max. and min. turning points.
Shape: Be generous, even when the curve seems to be composed of straight line segments, but there must be a discernible 'curve' at the max. and min.

2nd B1 for minimum at $(-1, 0)$ (even if there is an additional minimum point shown)

3rd B1 for the sketch meeting axes at (2, 0) and (0, 2). They can simply mark 2 on the axes.

The marks for minimum and intersections are dependent upon having a sketch.

Answers on the diagram for min. and intersections take precedence over answers seen elsewhere.

4th B1 for the branch fully within 1st quadrant having 2 intersections with (not just ‘touching’) the other curve. The curve can ‘touch’ the axes.
A curve of (roughly) the correct shape is required, but be very generous, even when the arc appears to turn 'inwards' rather than approaching the axes, and when the curve looks like two straight lines with a small curve at the join.
Allow, for example, shapes like these:

5th B1 for a branch fully in the 3rd quadrant (ignore any intersections with the other curve for this branch). The curve can ‘touch’ the axes.
A curve of (roughly) the correct shape is required, but be very generous, even when the arc appears to turn 'inwards' rather than approaching the axes.

(c) B1ft for a statement about the number of roots - compatible with their sketch. No sketch is B0.
The answer 2 <u>incompatible with the sketch</u> is B0 (ignore any algebra seen).
If the sketch shows the 2 correct intersections <u>and</u>, for example, one other intersection, the answer here should be 3, not 2, to score the mark.

Question Number	Scheme	Marks
9 (a)	$a+17d=25$ or equiv. (for 1st B1), $a+20d=32.5$ or equiv. (for 2nd B1),	B1, B1 (2)
(b)	Solving (Subtract) $3d = 7.5$ so $\mathbf{d = \underline{2.5}}$	M1
	$a = 32.5 - 20\times 2.5$ so $\mathbf{a = \underline{-17.5}}$ (*)	A1cso (2)
(c)	$2750 = \frac{n}{2}\left[-35+\frac{5}{2}(n-1)\right]$	M1A1ft
	{ $4\times 2750 = n(5n-75)$ }	
	$4\times 550 = n(n-15)$	M1
	$\underline{n^2 - 15n = 55\times 40}$ (*)	A1cso (4)
(d)	$n^2 - 15n - 55\times 40 = 0$ or $n^2 - 15n - 2200 = 0$	M1
	$(n-55)(n+40)=0$ $n = \ldots$	M1
	$\underline{\mathbf{n = 55}}$ (ignore - 40)	A1 (3) **[11]**

Mark parts (a) and (b) as 'one part', ignoring labelling.

(a) Alternative:

1st B1: $d = 2.5$ or equiv. or $d = \frac{32.5-25}{3}$. No method required, but $a = -17.5$ must not be assumed.

2nd B1: Either $a+17d=25$ or $a+20d=32.5$ seen, or used with a value of d…
or for 'listing terms' or similar methods, 'counting back' 17 (or 20) terms.

(b) M1: In main scheme: for a full method (allow numerical or sign slips) leading to solution for d or a without assuming $a = -17.5$
In alternative scheme: for using a d value to find a value for a.

A1: Finding correct values for both a and d (allowing equiv. fractions such as $d = \frac{15}{6}$), with no incorrect working seen.

In the main scheme, if the given a is used to find d from one of the equations, then allow M1A1 if both values are checked in the 2nd equation.

(c) 1st M1 for attempt to form equation with correct S_n formula and 2750, with values of a and d.
1st A1ft for a correct equation following through their d.
2nd M1 for expanding and simplifying to a 3 term quadratic.
2nd A1 for correct working leading to printed result (no incorrect working seen).

(d) 1st M1 forming the correct 3TQ = 0 . Can condone missing "= 0" but all terms must be on one side.
First M1 can be implied (perhaps seen in (c), but there must be an attempt at (d) for it to be scored).
2nd M1 for attempt to solve 3TQ, by factorisation, formula or completing the square (see general marking principles at end of scheme). If this mark is earned for the 'completing the square' method or if the factors are written down directly, the 1st M1 is given by implication.
A1 for n = 55 dependent on both Ms. Ignore – 40 if seen.
No working or 'trial and improvement' methods in (d) score all 3 marks for the answer 55, otherwise no marks.

Question Number	Scheme	Marks
10 (a)	$y-5=-\frac{1}{2}(x-2)$ or equivalent, e.g. $\frac{y-5}{x-2}=-\frac{1}{2}$, $\quad y=-\frac{1}{2}x+6$	M1A1, A1cao (3)
(b)	$x=-2 \Rightarrow \; y=-\frac{1}{2}(-2)+6 \;=\; 7$ (therefore B lies on the line) (or equivalent verification methods)	B1 (1)
(c)	$(AB^2=)\ (2--2)^2+(7-5)^2$, $\quad = 16 + 4 = 20$, $\quad AB=\sqrt{20}=2\sqrt{5}$	M1, A1, A1 (3)
(d)	C is $(p,\ -\frac{1}{2}p+6)$, so $\quad AC^2=(p-2)^2+\left(-\frac{1}{2}p+6-5\right)^2$ Therefore $\quad 25=p^2-4p+4+\frac{1}{4}p^2-p+1$ $25=1.25p^2-5p+5$ or $100=5p^2-20p+20$ (or better, RHS simplified to 3 terms) Leading to: $\quad 0=p^2-4p-16 \quad$ (*)	M1 M1 A1 A1cso (4) **[11]**
(a)	M1 A1 The version in the scheme above can be written down directly (for 2 marks), and M1 A0 can be allowed if there is just one slip (sign or number). If the 5 and 2 are the wrong way round the M mark can still be given if a correct formula (e.g. $y-y_1=m(x-x_1)$) is seen, otherwise M0. If (2, 5) is substituted into $y=mx+c$ to find c, the M mark is for attempting this and the 1st A mark is for c = 6. Correct answer without working or from a sketch scores full marks.	
(b)	A conclusion/comment is not required, except when the method used is to establish that the line through $(-2,7)$ with gradient $-\frac{1}{2}$ has the same eqn. as found in part (a), or to establish that the line through $(-2,7)$ and $(2,5)$ has gradient $-\frac{1}{2}$. In these cases a comment 'same equation' or 'same gradient' or 'therefore on same line' is sufficient.	
(c)	M1 for attempting AB^2 or AB. Allow one slip (sign or number) <u>inside</u> a bracket, i.e. do <u>not</u> allow $(2--2)^2-(7-5)^2$. 1st A1 for 20 (condone bracketing slips such as $-2^2=4$) 2nd A1 for $2\sqrt{5}$ or k = 2 (Ignore ± here).	
(d)	1st M1 for $(p-2)^2+(\text{linear function of } p)^2$. The linear function may be unsimplified but must be equivalent to $ap+b,\ a\neq 0,\ b\neq 0$. 2nd M1 (dependent on 1st M) for forming an equation in p (using 25 or 5) and attempting (perhaps not very well) to multiply out both brackets. 1st A1 for collecting like p terms and having a correct expression. 2nd A1 for correct work leading to printed answer. <u>Alternative, using the result</u>: Solve the quadratic $(p=2\pm 2\sqrt{5})$ and use one or both of the two solutions to find the length of AC^2 or $C_1C_2{}^2$: e.g. $AC^2=(2+2\sqrt{5}-2)^2+(5-\sqrt{5}-5)^2$ scores 1st M1, and 1st A1 if fully correct. Finding the length of AC or AC^2 for both values of p, or finding C_1C_2 with some evidence of halving (or intending to halve) scores the 2nd M1. Getting AC = 5 for both values of p, or showing $\frac{1}{2}C_1C_2=5$ scores the 2nd A1 (cso).	

Question Number	Scheme	Marks
11 (a)	$\left(\frac{dy}{dx}=\right) -4+8x^{-2}$ (4 or $8x^{-2}$ for M1… sign can be wrong)	M1A1
	$x=2 \Rightarrow \quad m=-4+2=-2$	M1
	$y=9-8-\frac{8}{2}=-3$ The first 4 marks could be earned in part (b)	B1
	Equation of tangent is: $y+3=-2(x-2) \rightarrow y=1-2x$ (*)	M1 A1cso (6)
(b)	Gradient of normal = $\frac{1}{2}$	B1ft
	Equation is: $\frac{y+3}{x-2}=\frac{1}{2}$ or better equivalent, e.g. $y=\frac{1}{2}x-4$	M1A1 (3)
(c)	$(A:)\ \frac{1}{2}$, $(B:)\ 8$	B1, B1
	Area of triangle is: $\frac{1}{2}(x_B \pm x_A)\times y_P$ with values for all of x_B, x_A and y_P	M1
	$\frac{1}{2}\left(8-\frac{1}{2}\right)\times 3 = \frac{45}{4}$ or 11.25	A1 (4) **[13]**

(a) 1st M1 for 4 or $8x^{-2}$ (ignore the signs).

1st A1 for both terms correct (including signs).

2nd M1 for substituting $x = 2$ into their $\frac{dy}{dx}$ (must be different from their y)

B1 for $y_P=-3$, but not if clearly found from the given equation of the tangent.

3rd M1 for attempt to find the equation of tangent at P, follow through their m and y_P.

Apply general principles for straight line equations (see end of scheme).

NO DIFFERENTIATION ATTEMPTED: Just assuming $m=-2$ at this stage is M0

2nd A1cso for correct work leading to printed answer (allow equivalents with $2x$, y, and 1 terms… such as $2x+y-1=0$).

(b) B1ft for correct use of the perpendicular gradient rule. Follow through their m, but if $m \neq -2$ there must be clear evidence that the m is thought to be the gradient of the tangent.

M1 for an attempt to find normal at P using their changed gradient and their y_P.

Apply general principles for straight line equations (see end of scheme).

A1 for any correct form as specified above (correct answer only).

(c) 1st B1 for $\frac{1}{2}$ and 2nd B1 for 8.

M1 for a full method for the area of triangle ABP. Follow through their x_A, x_B and their y_P, but the mark is to be awarded 'generously', condoning sign errors..

The final answer must be positive for A1, with negatives in the working condoned.

Determinant: Area = $\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 2 & -3 & 1 \\ 0.5 & 0 & 1 \\ 8 & 0 & 1 \end{vmatrix} = \ldots$ (Attempt to multiply out required for M1)

Alternative: $AP=\sqrt{(2-0.5)^2+(-3)^2}$, $BP=\sqrt{(2-8)^2+(-3)^2}$, Area = $\frac{1}{2}AP \times BP = \ldots$ M1

Intersections with y-axis instead of x-axis: Only the M mark is available B0 B0 M1 A0.