

4723 Core Mathematics 3

1 (i)	Obtain integral of form ke^{-2x} Obtain $-4e^{-2x}$	M1 A1	any constant k different from 8 or (unsimplified) equiv
(ii)	Obtain integral of form $k(4x+5)^7$ Obtain $\frac{1}{28}(4x+5)^7$ Include $\dots + c$ at least once	M1 A1 B1	any constant k in simplified form in either part
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2 (i)	Form expression involving attempts at y values and addition Obtain $k(\ln 4 + 4 \ln 6 + 2 \ln 8 + 4 \ln 10 + \ln 12)$ Use value of k as $\frac{1}{3} \times 2$ Obtain 16.27	M1 A1 A1 A1	with coeffs 1, 4 and 2 present at least once any constant k or unsimplified equiv 4 or 16.3 or greater accuracy (16.27164...)
(ii)	State 162.7 or 163	B1	1 following their answer to (i), maybe rounded
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3 (i)	Attempt use of identity for $\tan^2 \theta$ Replace $\frac{1}{\cos \theta}$ by $\sec \theta$ Obtain $2(\sec^2 \theta - 1) - \sec \theta$	M1 B1 A1	using $\pm \sec^2 \theta \pm 1$; or equiv 3 or equiv
(ii)	Attempt soln of quadratic in $\sec \theta$ or $\cos \theta$ Relate $\sec \theta$ to $\cos \theta$ and attempt at least one value of θ Obtain $60^\circ, 131.8^\circ$ Obtain $60^\circ, 131.8^\circ, 228.2^\circ, 300^\circ$	M1 M1 A1 A1	as far as factorisation or substitution in correct formula may be implied allow 132 or greater accuracy 4 allow 132, 228 or greater accuracy; and no others between 0° and 360°
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4 (i)	Obtain derivative of form $kx(4x^2+1)^4$ Obtain $40x(4x^2+1)^4$ State $x = 0$	M1 A1 A1	any constant k or (unsimplified) equiv 3 and no other; following their derivative of form $kx(4x^2+1)^4$
(ii)	Attempt use of quotient rule Obtain $\frac{2x \ln x - x^2 \cdot \frac{1}{x}}{(\ln x)^2}$ Equate to zero and attempt solution Obtain $e^{\frac{1}{2}}$	M1 A1 M1 A1	or equiv or equiv as far as solution involving e 4 or exact equiv; and no other; allow from \pm (correct numerator of derivative)
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5 (i)	State 40	B1	
	Attempt value of k using 21 and 80	M1	or equiv
	Obtain $40e^{21k} = 80$ and hence 0.033	A1	or equiv such as $\frac{1}{21} \ln 2$
	Attempt value of M for $t = 63$	M1	using established formula or using exponential property
	Obtain 320	A1	5 or value rounding to this

(ii)	Differentiate to obtain $ce^{0.033t}$ or $40ke^{kt}$	M1	any constant c different from 40
	Obtain $40 \times 0.033e^{0.033t}$	A1√	following their value of k
	Obtain 2.64	A1	3 allow 2.6 or 2.64 ± 0.01 or greater accuracy (2.64056...)
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6 (i)	Attempt correct process for finding inverse	M1	maybe in terms of y so far
	Obtain $2x^3 - 4$	A1	or equiv; in terms of x now
	State $\sqrt[3]{2}$ or 1.26	B1	3

(ii)	State reflection in $y = x$	B1	or clear equiv
	Refer to intersection of $y = x$ and $y = f(x)$ and hence confirm $x = \sqrt[3]{\frac{1}{2}x + 2}$	B1	2 AG; or equiv

(iii)	Obtain correct first iterate	B1	
	Show correct process for iteration	M1	with at least one more step
	Obtain at least 3 correct iterates in all	A1	allowing recovery after error
	Obtain 1.39	A1	4 following at least 3 steps; answer required to exactly 2 d.p.
			$[0 \rightarrow 1.259921 \rightarrow 1.380330 \rightarrow 1.390784 \rightarrow 1.391684$ $1 \rightarrow 1.357209 \rightarrow 1.388789 \rightarrow 1.391512 \rightarrow 1.391747$ $1.26 \rightarrow 1.380337 \rightarrow 1.390784 \rightarrow 1.391684 \rightarrow 1.391761$ $1.5 \rightarrow 1.401020 \rightarrow 1.392564 \rightarrow 1.391837 \rightarrow 1.391775$ $2 \rightarrow 1.442250 \rightarrow 1.396099 \rightarrow 1.392141 \rightarrow 1.391801]$
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7 (i)	Refer to stretch and translation	M1	in either order; allow here informal terms
	State stretch, factor $\frac{1}{k}$, in x direction	A1	or equiv; now with correct terminology
	State translation in negative y direction by a	A1	3 or equiv; now with correct terminology
	[SC: If M0 but one transformation completely correct – B1]		

(ii)	Show attempt to reflect negative part in x -axis	M1	ignoring curvature
	Show correct sketch	A1	2 with correct curvature, no pronounced 'rounding' at x -axis and no obvious maximum point

(iii)	Attempt method with $x = 0$ to find value of a	M1	... other than (or in addition to) value -12
	Obtain $a = 14$	A1	and nothing else
	Attempt to solve for k	M1	using any numerical a with sound process
	Obtain $k = 3$	A1	4
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8 (i)	Attempt to express x or x^2 in terms of y	M1	
	Obtain $x^2 = \frac{1296}{(y+3)^4}$	A1	or (unsimplified) equiv
	Obtain integral of form $k(y+3)^{-3}$	M1	any constant k
	Obtain $-432\pi(y+3)^{-3}$ or $-432(y+3)^{-3}$	A1	or (unsimplified) equiv
	Attempt evaluation using limits 0 and p	M1	for expression of form $k(y+3)^{-n}$ obtained from integration attempt; subtraction correct way round
	Confirm $16\pi(1 - \frac{27}{(p+3)^3})$	A1	6 AG; necessary detail required, including appearance of π prior to final line

(ii)	State or obtain $\frac{dV}{dp} = 1296\pi(p+3)^{-4}$	B1	or equiv; perhaps involving y
	Multiply $\frac{dp}{dt}$ and attempt at $\frac{dV}{dp}$	*M1	algebraic or numerical
	Substitute $p = 9$ and attempt evaluation	M1	dep *M
	Obtain $\frac{1}{4}\pi$ or 0.785	A1	4 or greater accuracy

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9 (i)	State $\cos 2\theta \cos \theta - \sin 2\theta \sin \theta$	B1	
	Use at least one of $\cos 2\theta = 2\cos^2 \theta - 1$ and $\sin 2\theta = 2\sin \theta \cos \theta$	B1	
	Attempt to express in terms of $\cos \theta$ only	M1	using correct identities for $\cos 2\theta$, $\sin 2\theta$ and $\sin^2 \theta$
	Obtain $4\cos^3 \theta - 3\cos \theta$	A1	4 AG; necessary detail required

(ii)	<u>Either:</u> State or imply $\cos 6\theta = 2\cos^2 3\theta - 1$	B1	
	Use expression for $\cos 3\theta$ and attempt expansion	M1	for expression of form $\pm 2\cos^2 3\theta \pm 1$
	Obtain $32c^6 - 48c^4 + 18c^2 - 1$	A1	3 AG; necessary detail required
	<u>Or:</u> State $\cos 6\theta = 4\cos^3 2\theta - 3\cos 2\theta$	B1	maybe implied
	Express $\cos 2\theta$ in terms of $\cos \theta$ and attempt expansion	M1	for expression of form $\pm 2\cos^2 \theta \pm 1$
	Obtain $32c^6 - 48c^4 + 18c^2 - 1$	A1	(3) AG; necessary detail required

(iii)	Substitute for $\cos 6\theta$	*M1	with simplification attempted
	Obtain $32c^6 - 48c^4 = 0$	A1	or equiv
	Attempt solution for c of equation	M1	dep *M
	Obtain $c^2 = \frac{3}{2}$ and observe no solutions	A1	or equiv; correct work only
	Obtain $c = 0$, give at least three specific angles and conclude odd multiples of 90	A1	5 AG; or equiv; necessary detail required; correct work only

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